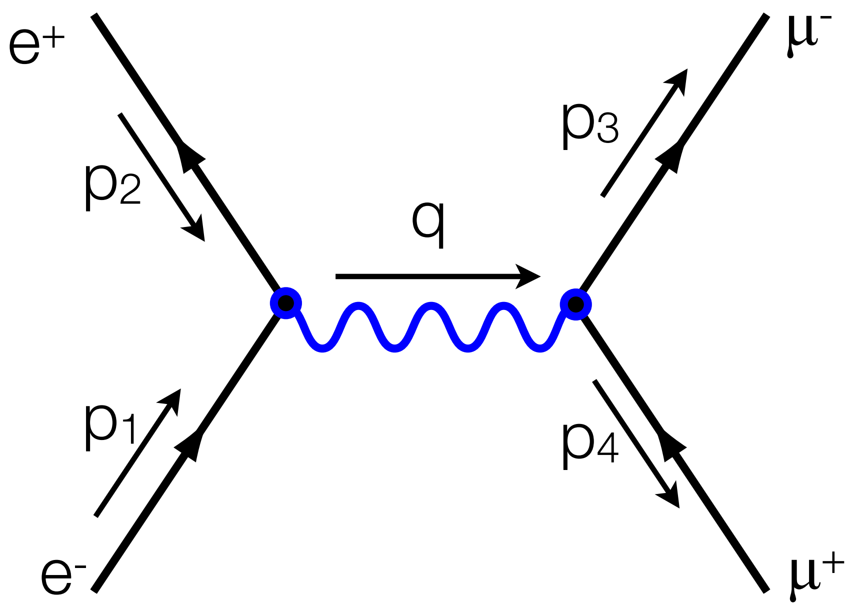
PHYSICS 489/1489

LECTURE 9:
QED EXPERIMENTS

LAST TIME

- We calculated the cross section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$



$$\mathcal{M} = -\frac{g_e^2}{(p_1+p_2)^2}\,[\bar{u}(3)\;\gamma^\mu\;v(4)]\,[\bar{v}(2)\;\gamma_\mu\;u(1)]$$

- Evaluated the matrix element with various helicity combinations in the massless limit

$$\mathcal{M}_{LR\to LR} = -\frac{e^2}{4E^2}[\bar{u}_{3L}\gamma^\mu v_{4R}]\;[\bar{v}_{2R}\gamma_\mu u_{1L}] \\ = e^2(1+\cos\theta) = \mathcal{M}_{RL\to RL}$$

$$\mathcal{M}_{LR\to RL} = -\frac{e^2}{4E^2}[\bar{u}_{3R}\gamma^\mu v_{4L}]\;[\bar{v}_{2R}\gamma_\mu u_{1L}] \\ = e^2\times(-\cos\theta+1) = \mathcal{M}_{RL\to LR}$$

- Obtain the differential (unpolarized, spin-summed) cross section

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{256\pi^2E^2}(1\pm\cos\theta)^2 \qquad \frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s}(1+\cos^2\theta)$$

A FEW NOTES:

- The derivation applies to any spin 1/2 fermion so long as
  - massless approximation(s) is appropriate
  - charge is appropriately scaled
- We can integrate over angles to get the total cross section

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s}(1+\cos^2\theta) \quad \Rightarrow \int d\phi \int d\cos\theta \; \frac{e^4}{64\pi^2 s}(1+\cos^2\theta)$$

$$\int d\cos\theta \; \frac{e^4}{32\pi s}(1+\cos^2\theta)$$

$$\frac{e^4}{12\pi s} \quad = \frac{4\pi\alpha^2}{3s}$$

- If we did not neglect the masses, we would obtain:

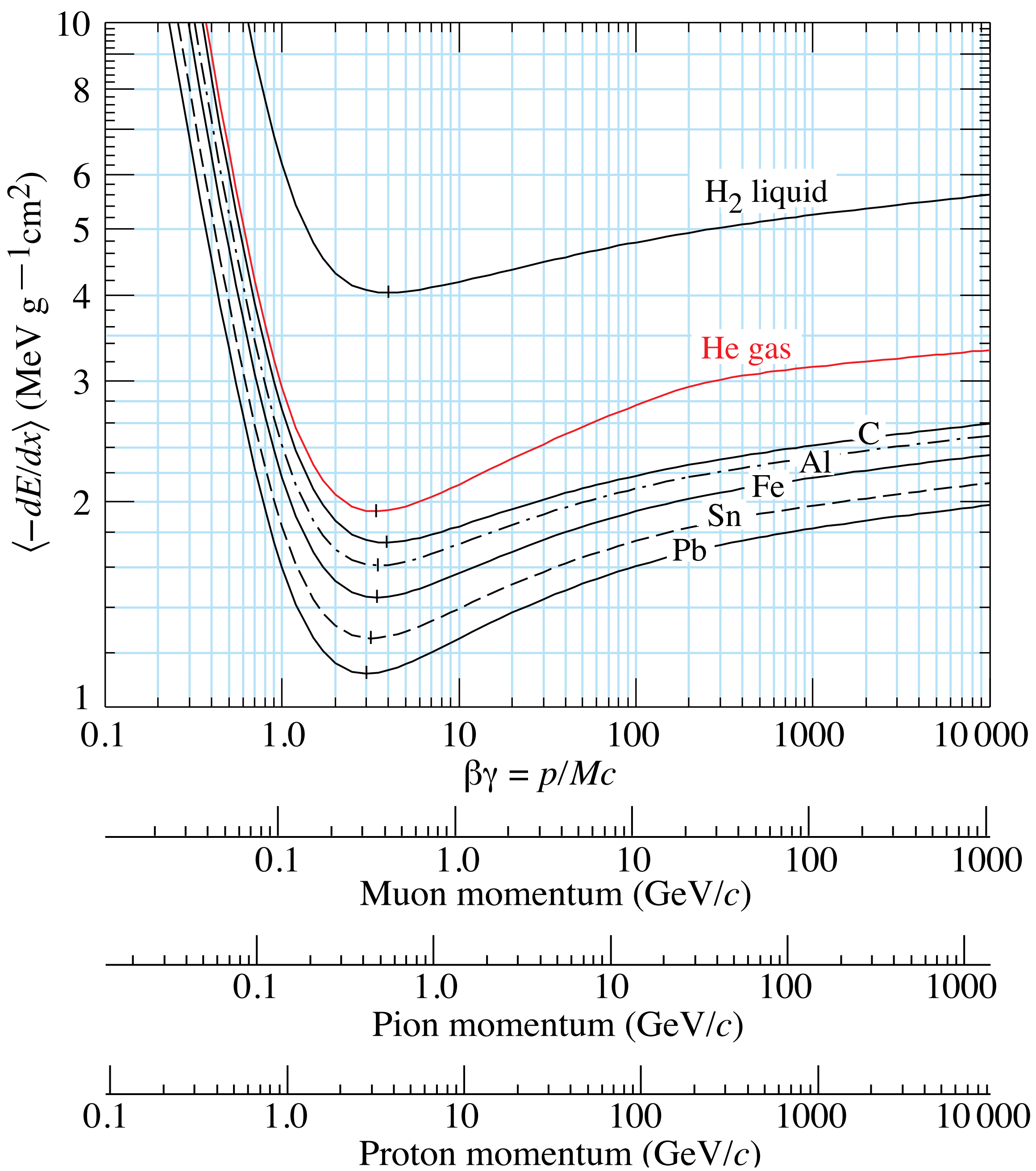
$$\langle|\mathcal{M}|^2\rangle = g_e^4\left[1+\left(\frac{mc^2}{E}\right)^2+\left(\frac{Mc^2}{E}\right)^2+\left[1-\left(\frac{mc^2}{E}\right)^2\right]\left[1-\left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right]$$

"spin averaged"

DETECTING PARTICLES

- For the most part, we can only detect charged particles
  - neutral particles can be detected if they
    - interact with charged particles which are in turn detected
    - decay to produce charged particles
- Detection methods:
  - ionization
  - scintillation
  - Cherenkov radiation
  - acoustic
  - . . . . ..

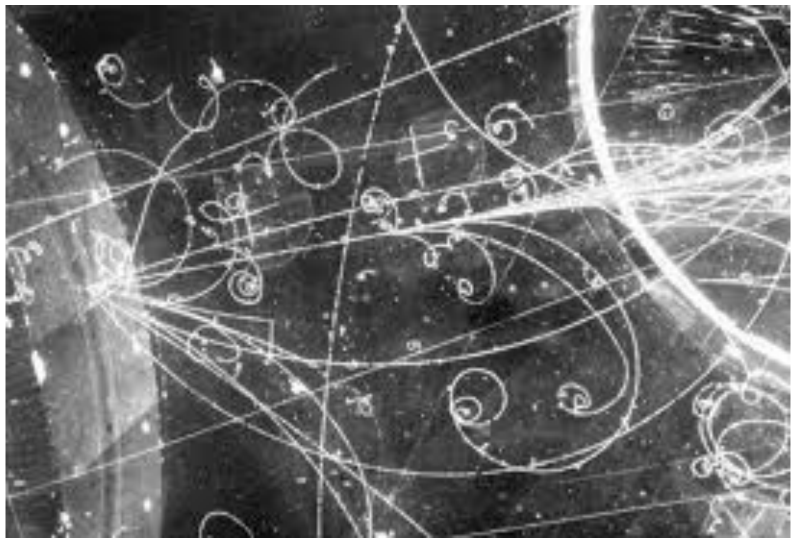
IONIZATION:

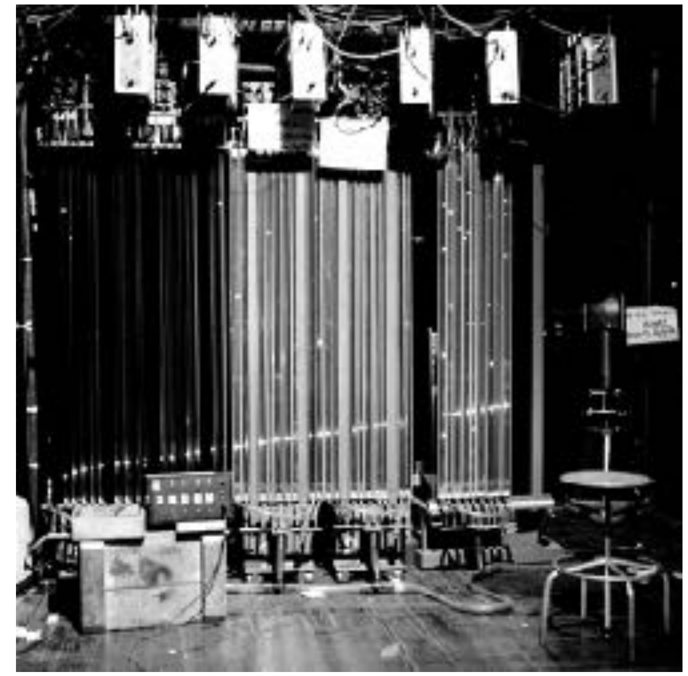


- Knock out of electrons from atom as a charged particle passes through a medium
- Ionization rate depends on velocity of particle
  - if we independently know the velocity of momentum of the particle, we can determine the particle identity
  - e.g. if the medium of
- "Tracking" detectors which determine the trajectory of a particle typically use ionization

HOW TO DETECT IONIZATION

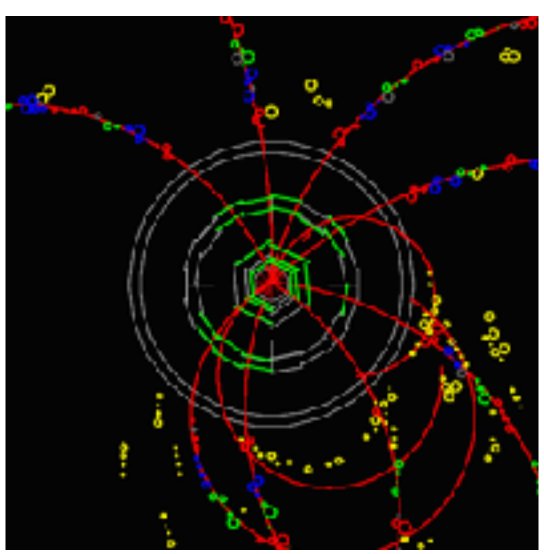
phase transitions

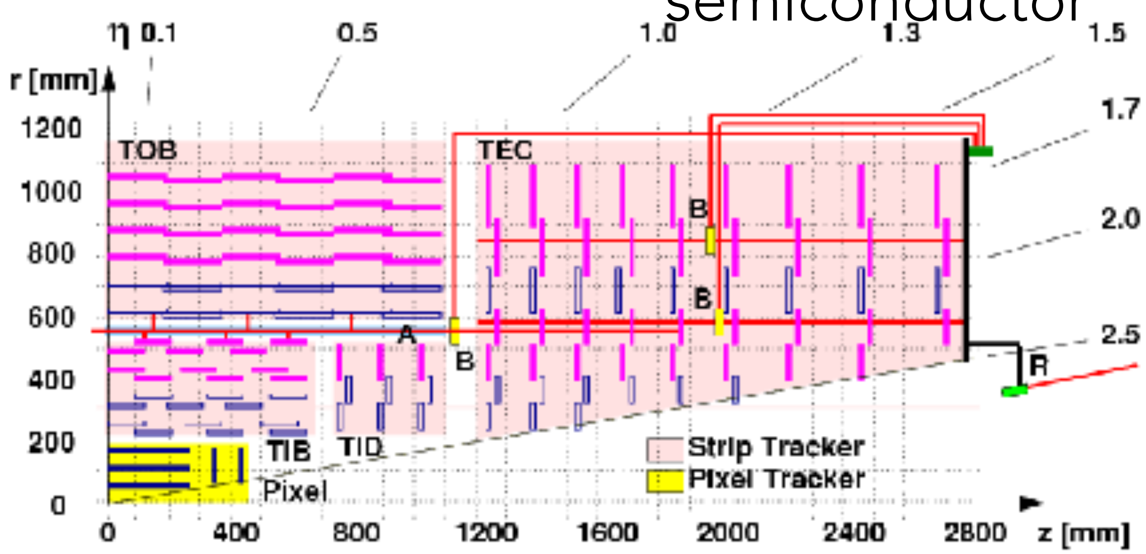


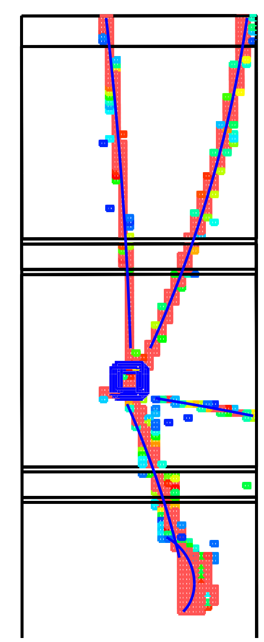


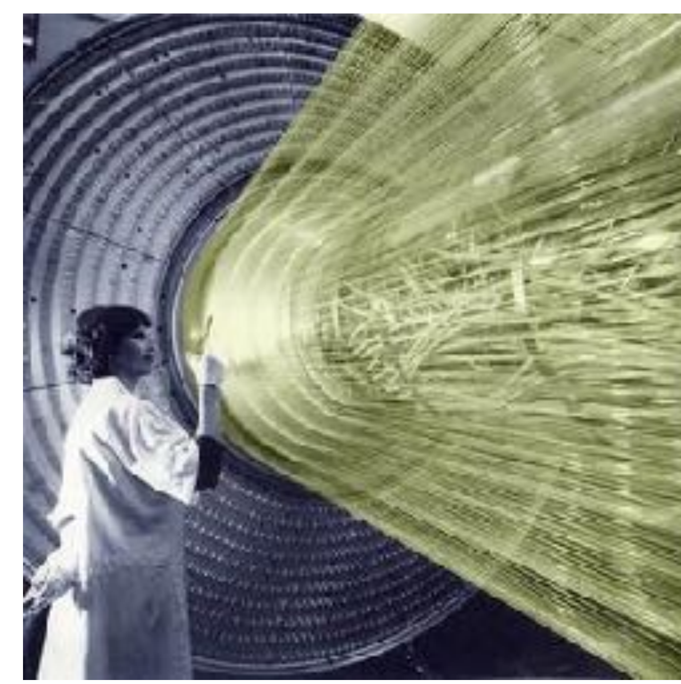
sparking, streaming

semiconductor



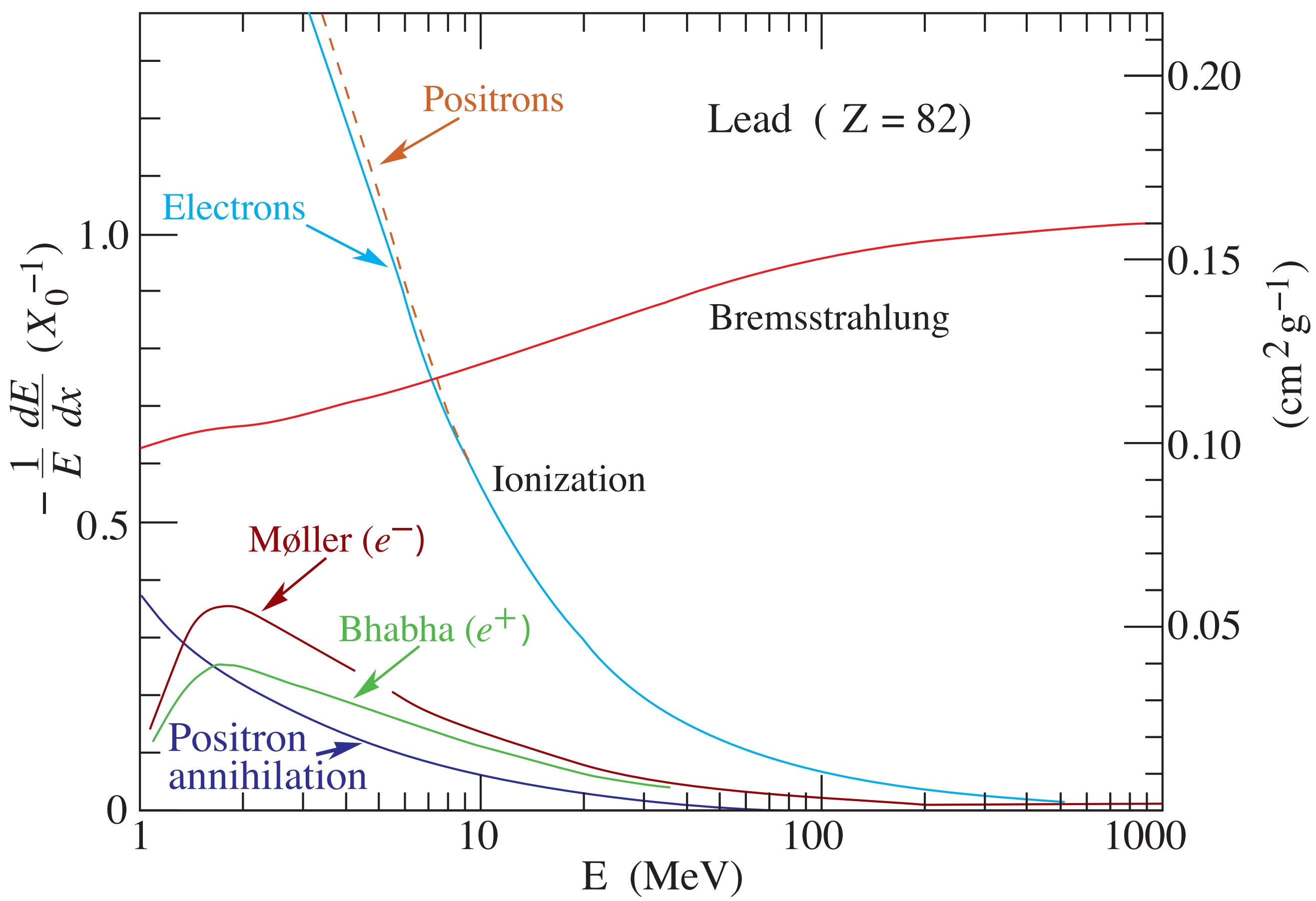


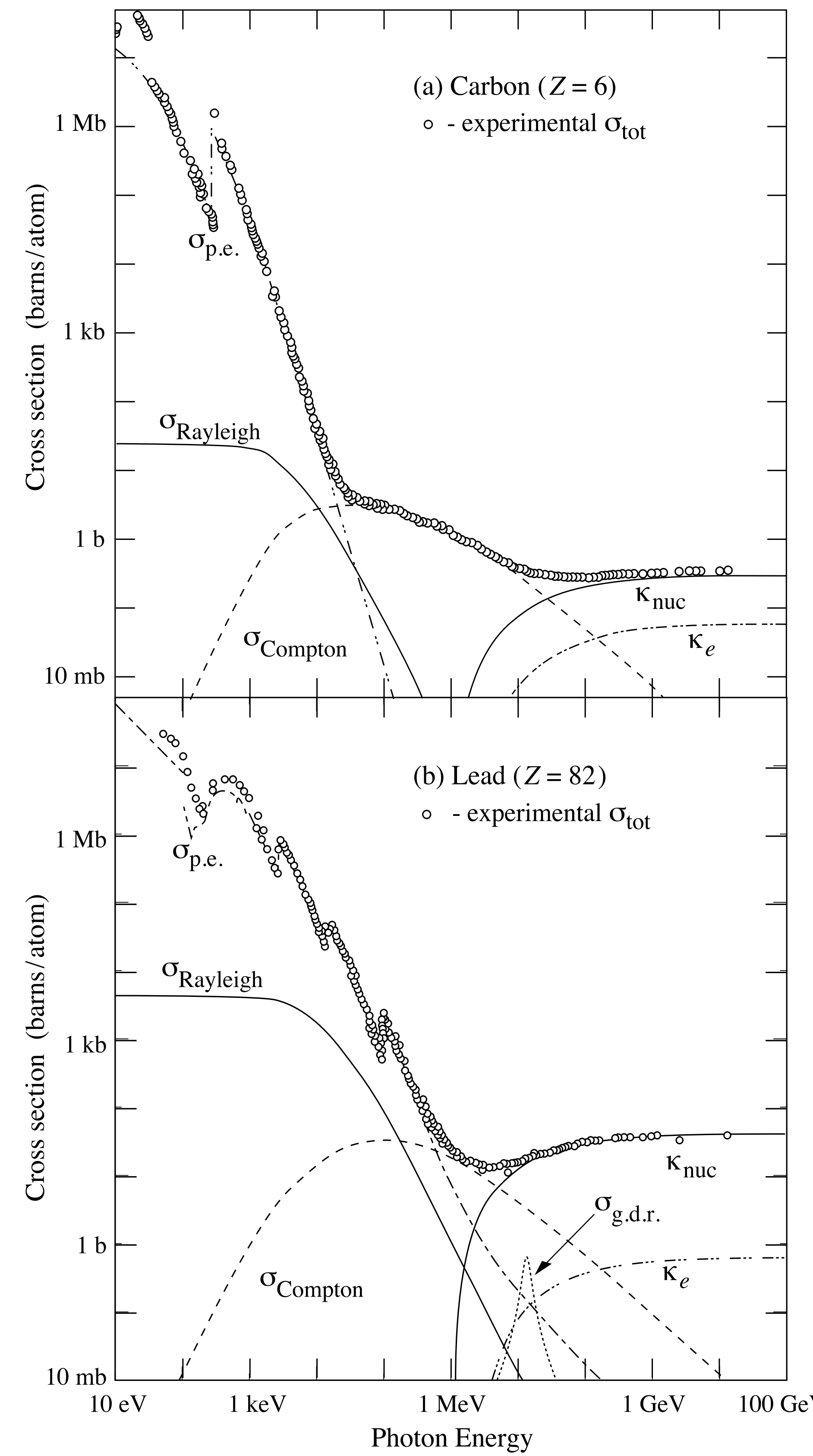




drifting in gas/liquid

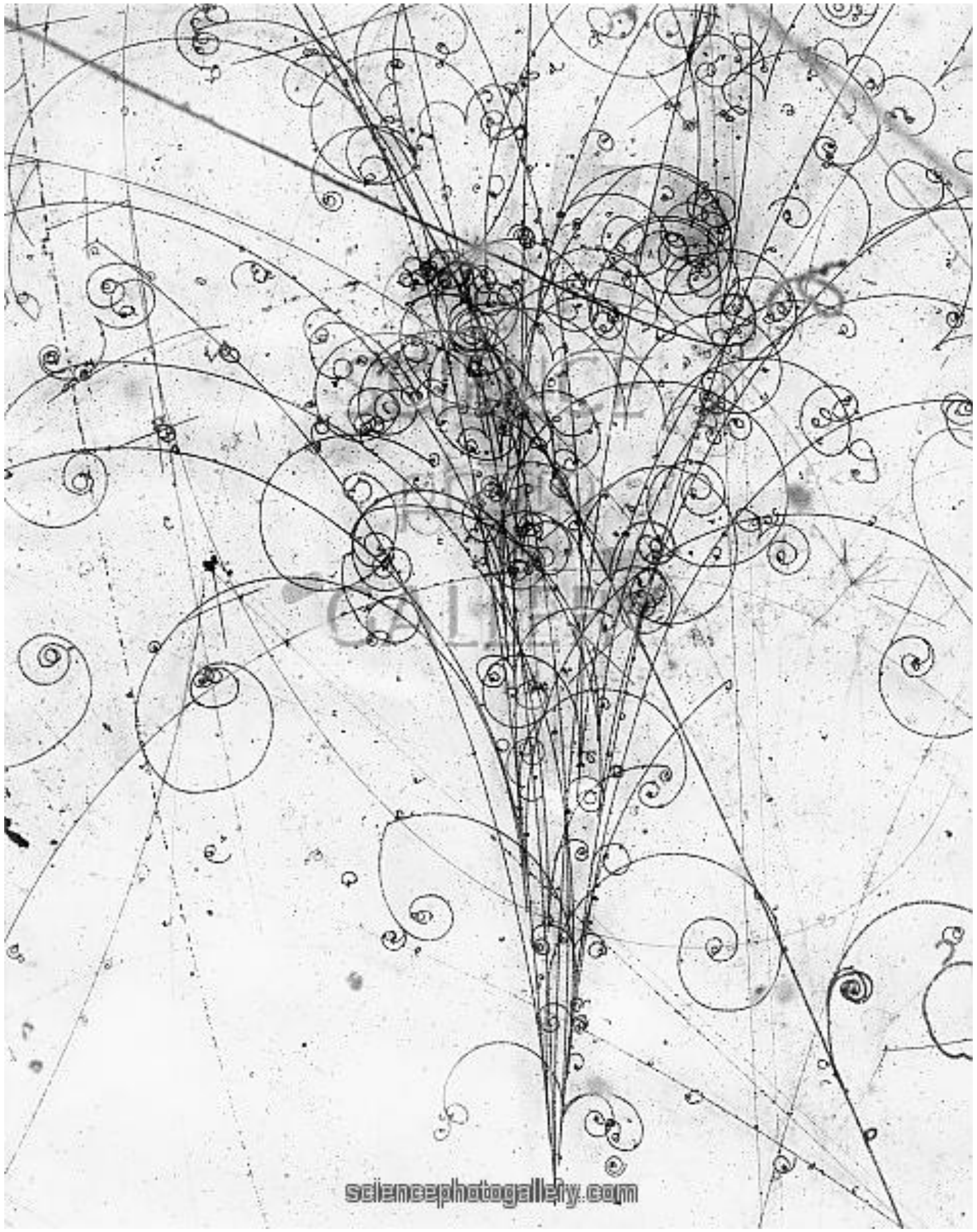
ELECTRONS AND PHOTONS

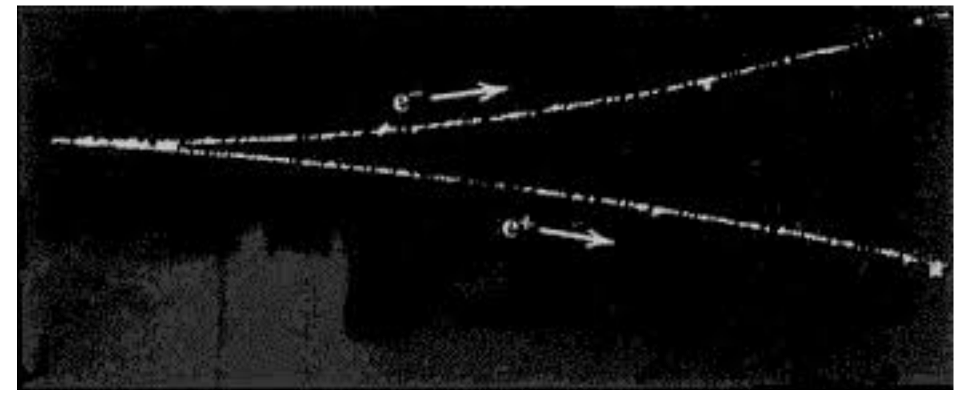




- electrons differ from other charged particles by their lightness and the presence of electrons in media
  - nuclear field can induce acceleration leading to radiation "bremsstrahlung"
- Photons will interact via Compton scattering or pair production at high energies

ELECTROMAGNETIC SHOWERS



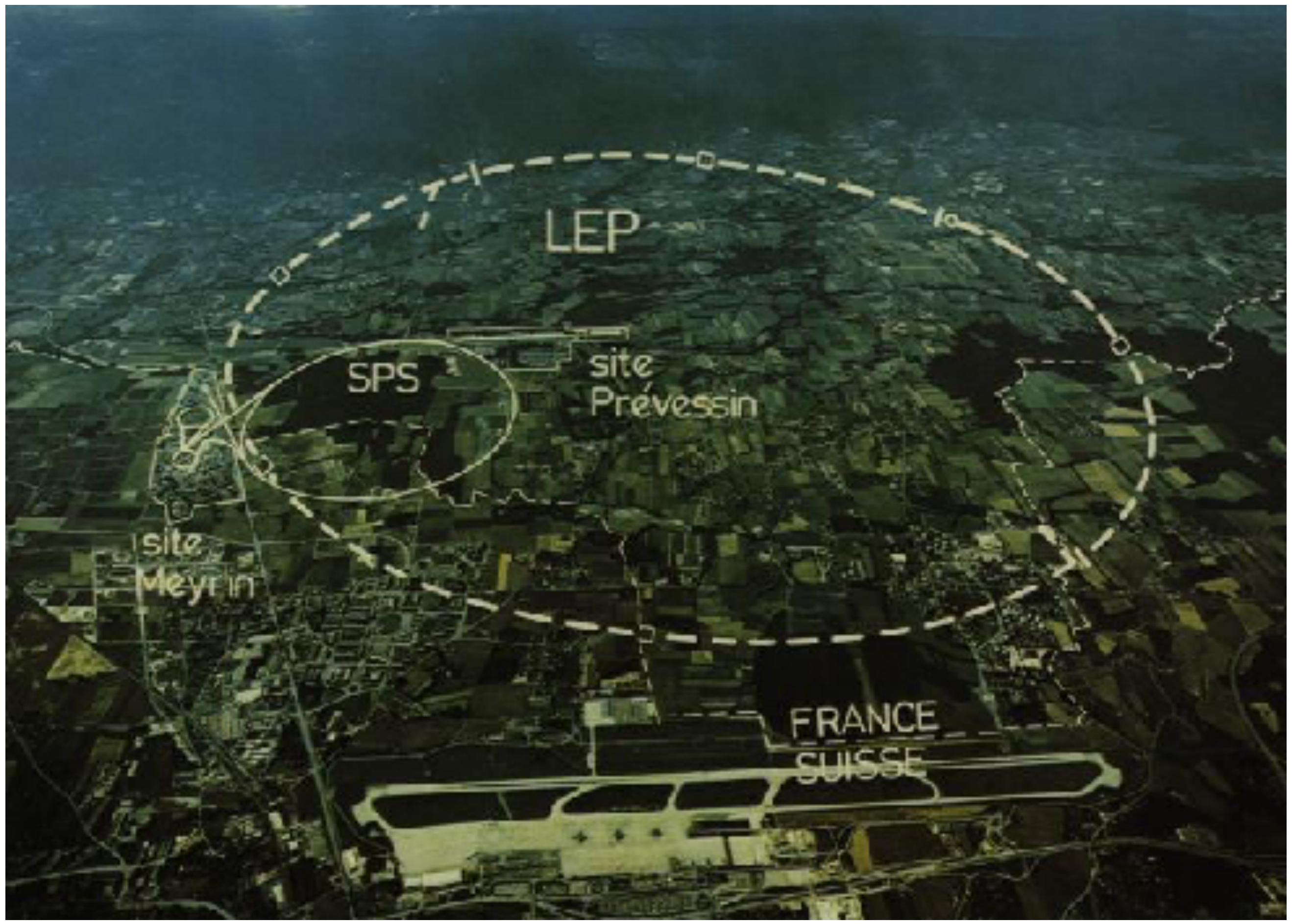


- Cascade of Bremsstrahlung, pair production, compton scattering, etc.

ACCELERATORS

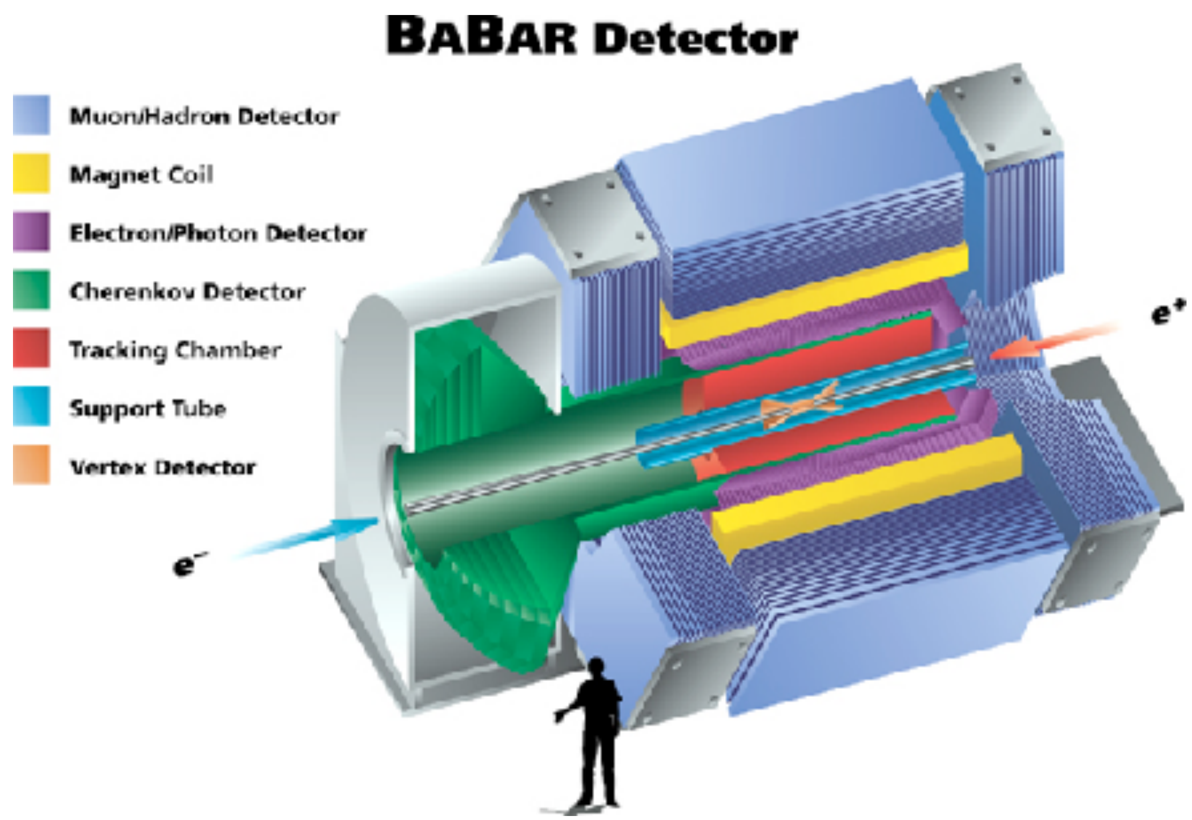
- Several generations of electron accelerators
  - CESR @ Cornell
  - SLAC linear accelerator
  - SLAC collier
- Also
  - PETRA at DESY (Hamburg, Germany)
  - TRISTAN at KEK (Tsukuba, Japan)
  - VEPP at BINP (Novosibirsk, Russia)
  - BES (Beijing, China)

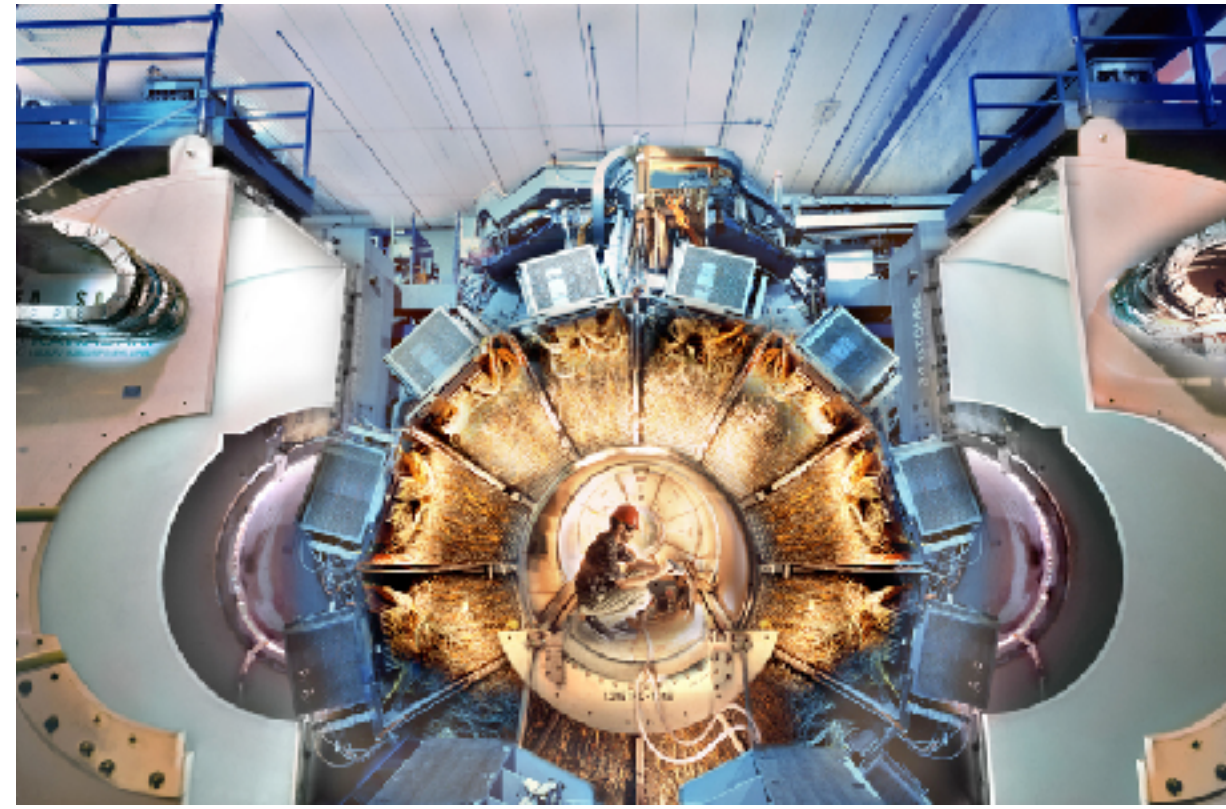
LEP



- Before the LHC there was LEP:
  - “large electron positron collider”
  - operated primarily at 91 GeV to study Z production and decays
- “LEP-II”:
  - increase of energy up to 209 GeV

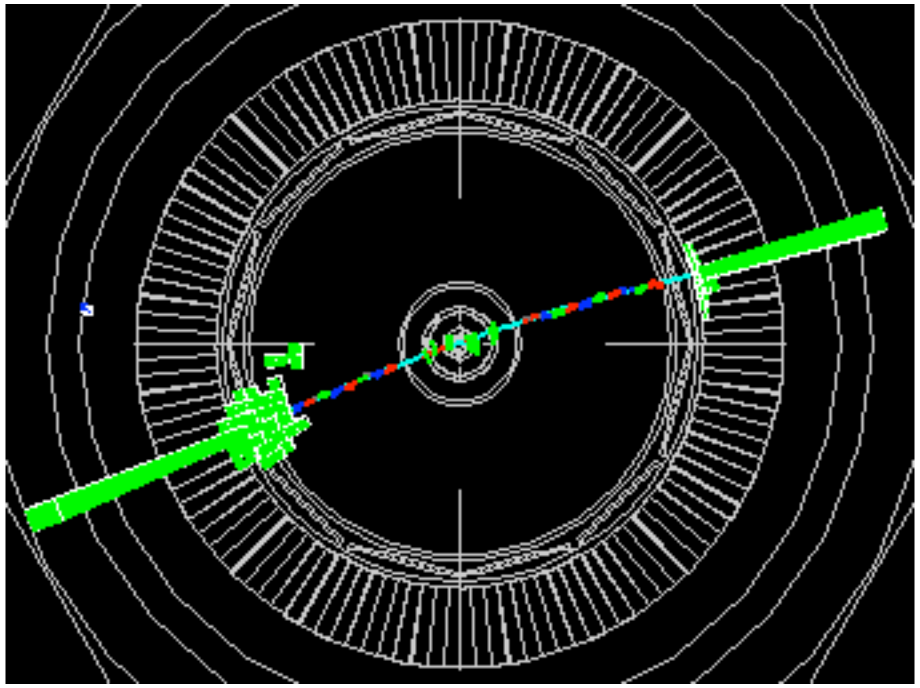
DETECTORS



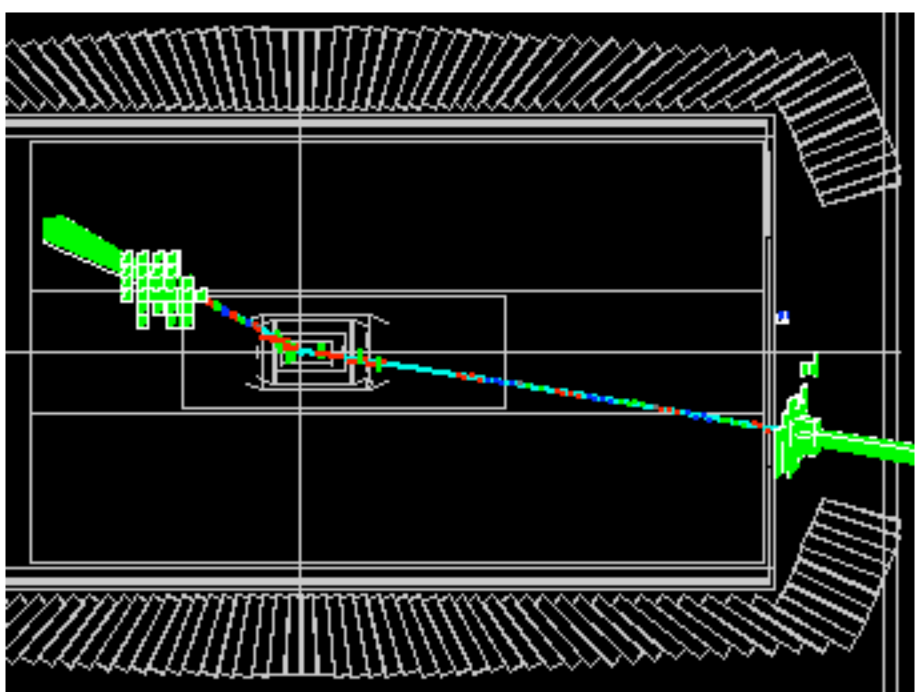


- Most collider detectors share a similar "cylindrical onion" design surrounding the interaction point
  - inner tracking region (silicon, drift chambers, etc.)
  - particle identification (Cherenkov counter, time-of-flight, etc.)
  - electromagnetic calorimeter (measure/identify electron/photon energy)
  - muon detector: identify muons by their penetration through lots of material
  - magnetic field throughout to bend particles and measure sign/momentum

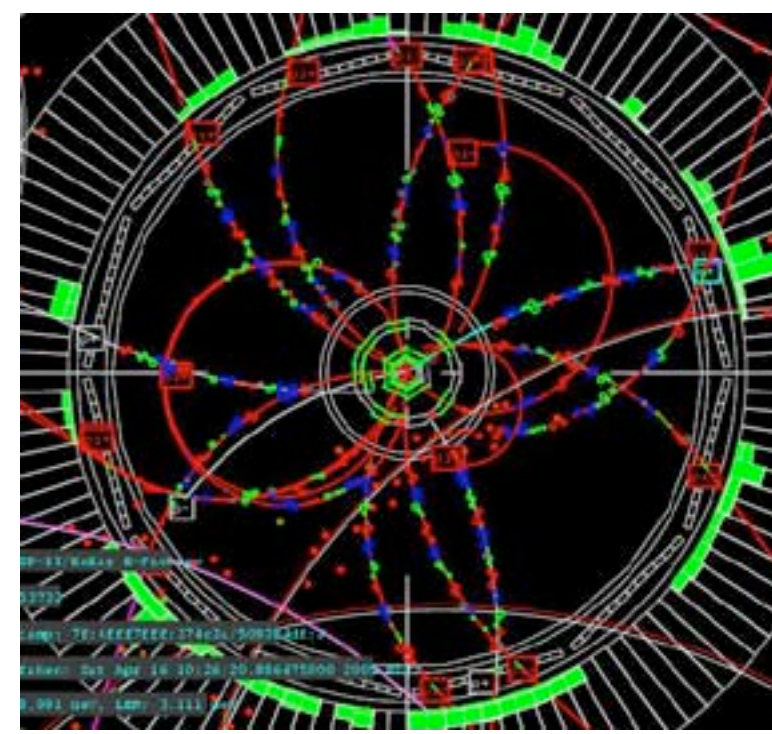
EVENTS AT BABAR



- $e^{+} + e^{-} \longrightarrow e^{+} + e^{-}$ event ("Bhabha scattering")
  - "straight" track: high momentum
  - large deposition in electromagnetic calorimeter (green)
- $e^{+} + e^{-} \longrightarrow \mu^{+} + \mu^{-}$ would look similar but with little calorimeter deposition



- "Hadronic" event at BaBar
  - $e^{+} + e^{-} \longrightarrow qq$
  - b and c quarks produced



τ PRODUCTION

- General expression without massless assumption:

$$\langle|\mathcal{M}|^2\rangle = g_e^4\left[1+\left(\frac{mc^2}{E}\right)^2+\left(\frac{Mc^2}{E}\right)^2+\left[1-\left(\frac{mc^2}{E}\right)^2\right]\left[1-\left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right]$$

- if we consider $e^+ + e^- \rightarrow \tau^+ + \tau^-$, we can still assume electron mass is ~0, but keep the mass of the τ.

$$\langle|\mathcal{M}|^2\rangle = g_e^4\left[1+\left(\frac{Mc^2}{E}\right)^2+\left[1-\left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right]$$

- Now putting into our cross section formulas

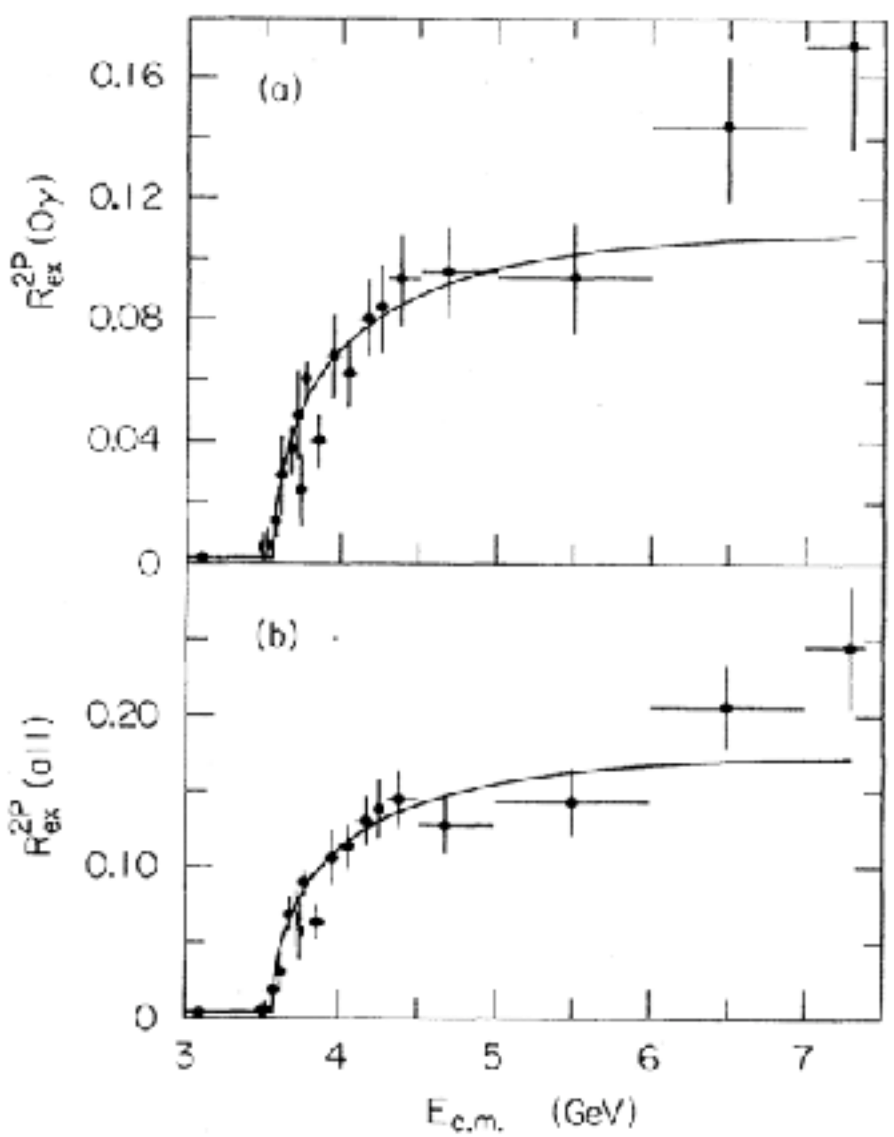
$$\frac{d\sigma}{d\cos\theta d\phi} = \left(\frac{\hbar c}{8\pi}\right)^2\frac{\langle|\mathcal{M}|^2\rangle}{4E^2}\frac{|p_f|}{|p_i|}$$

- Integrate to obtain total cross section

$$\sigma = \frac{\pi}{3}\left(\frac{\hbar c\alpha}{E}\right)^2\sqrt{1-(Mc^2/E)^2}\left[1+\frac{1}{2}\left(\frac{Mc^2}{E}\right)^2\right]$$

RATIO OF CROSS SECTIONS

- $e^{+} + e^{-} \rightarrow \mu^{+} + \mu^{-}$ has a very distinct signature in the detector.
- predict the ratio of **τ** production to μ production:

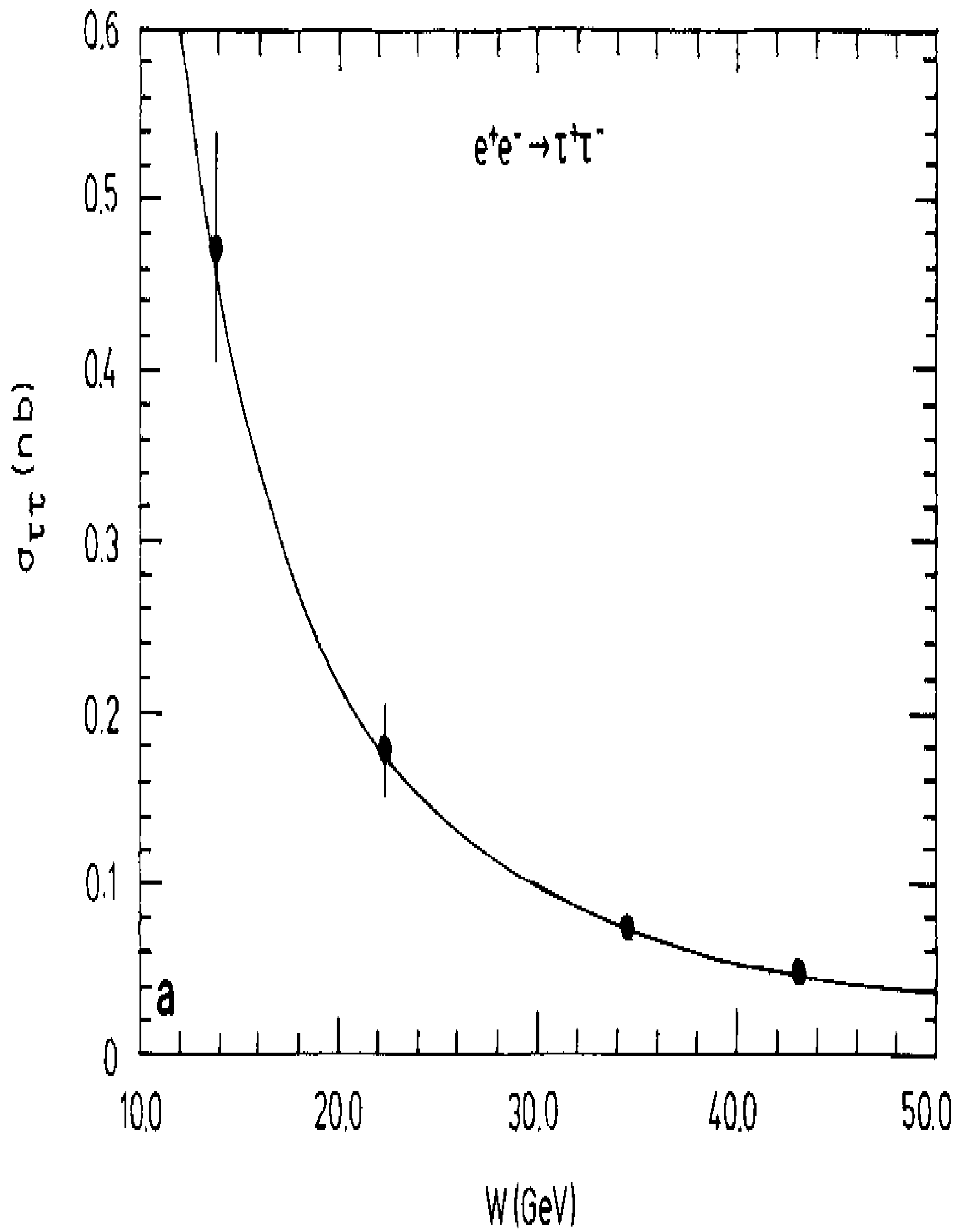


$$R_{\tau\mu} = \frac{\sigma_{\tau^+\tau^-}}{\sigma_{\mu^+\mu^-}} = \frac{\sqrt{1-(M_\tau c^2/E)^2}}{\sqrt{1-(M_\mu c^2/E)^2}} \times \frac{1+\frac{1}{2}(M_\tau c^2/E)^2}{1+\frac{1}{2}(M_\mu c^2/E)^2}$$

- Step the accelerator in energy
  - count the number of **τ** and μ produced at each energy
- Plot the ratio vs. beam energy
- Ratio depends on:
  - Dirac nature of **τ**
  - **τ** mass

TOTAL CROSS SECTION

- If we go to high energy ($E \gg m_\tau \sim 1.777$ GeV)



$$\sigma = \frac{\pi}{3}\left(\frac{\hbar c \alpha}{E}\right)^2 \sqrt{1-(Mc^2/E)^2}\left[1+\frac{1}{2}\left(\frac{Mc^2}{E}\right)^2\right]$$

$$\sigma = \frac{\pi}{3}\left(\frac{\hbar c \alpha}{E}\right)^2$$

$= 2.2\times10^{-5}$ mb/E^2(GeV^2)
$= 22$ nb/E^2(GeV^2)

E (GeV)	Cross section (nb)
14	0.44
22	0.18
34	0.075
43	0.047

ANGULAR DISTRIBUTION

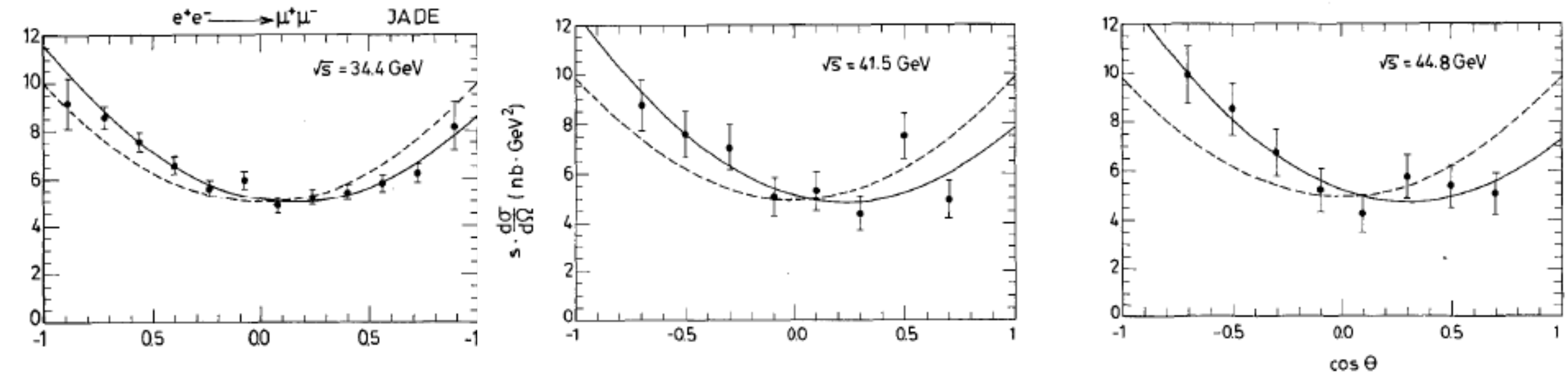


Fig. 2. Angular distribution of $e^+e^- \to \mu^+\mu^-$ at the highest PETRA energies. The dashed lines are the symmetric QED predictions. The full lines are fits to the data allowing for an asymmetry

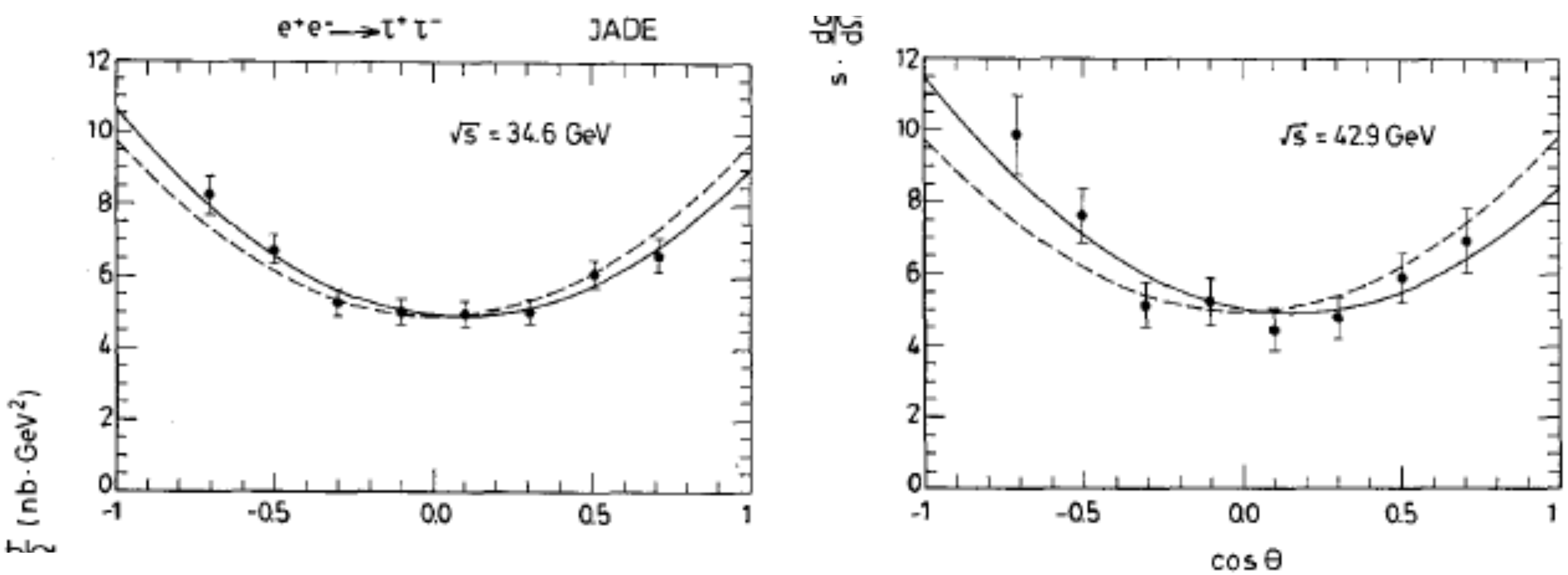
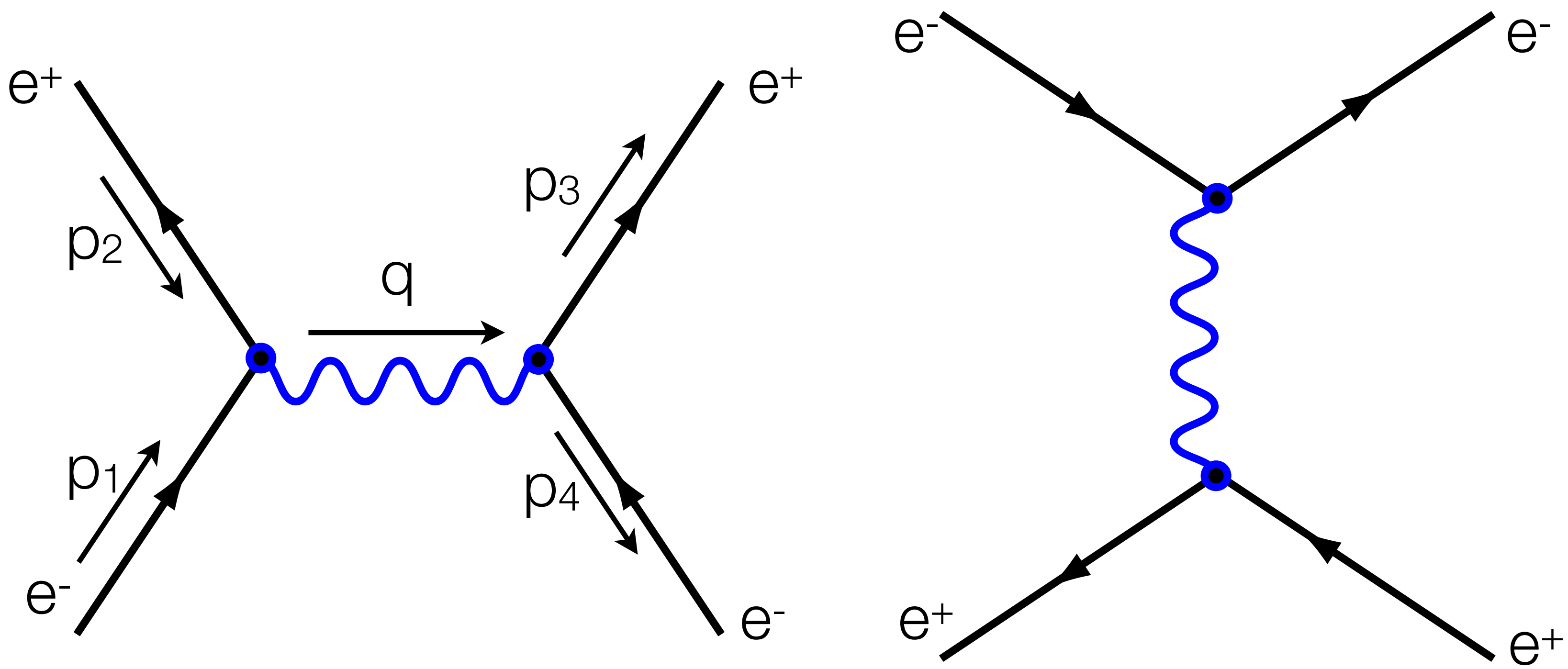


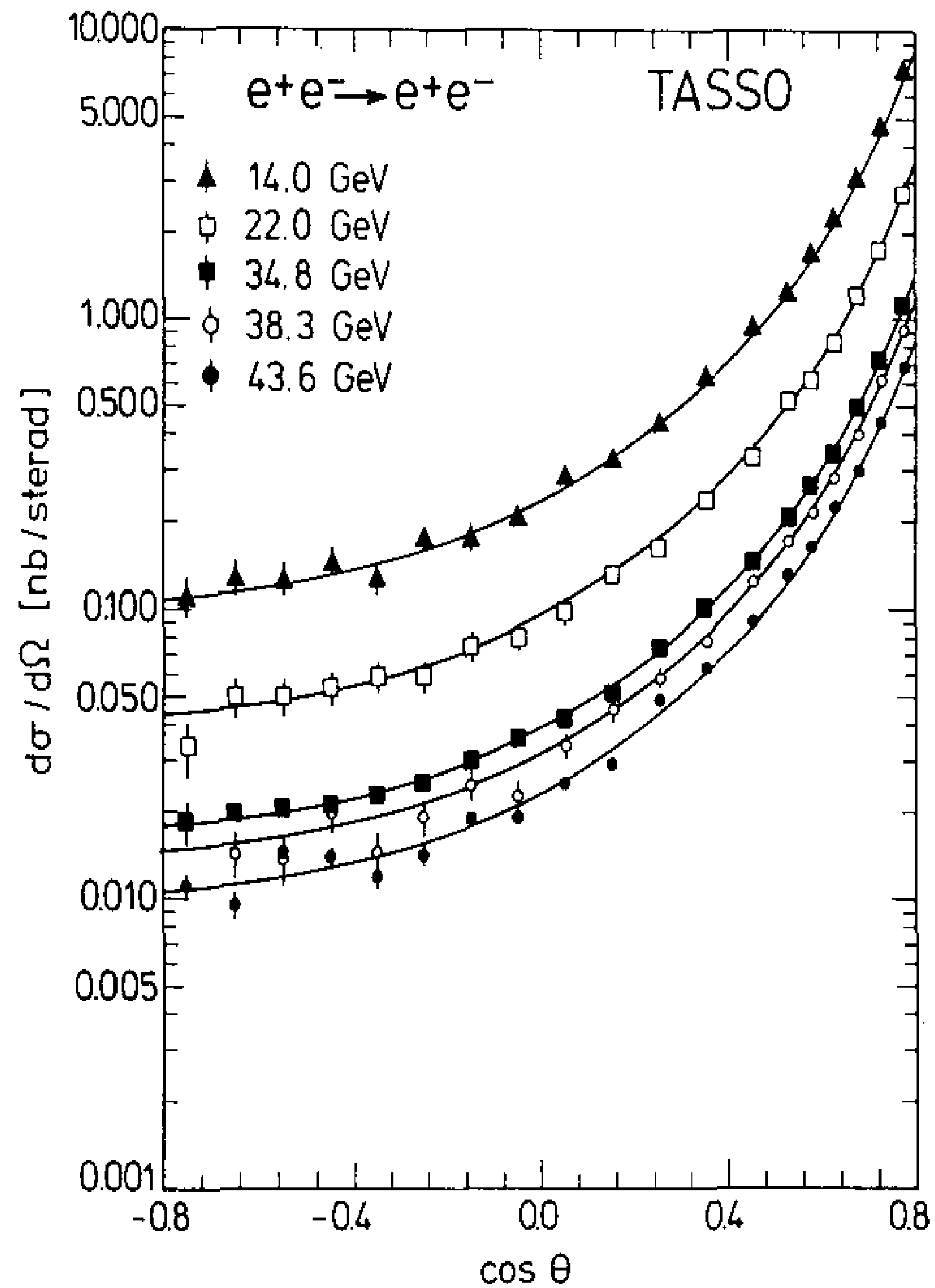
Fig. 3. $e^+e^- \to \tau^+\tau^-$. For explanations see Fig. 2

BHABHA SCATTERING

- $e^{+}+ e^{-} \rightarrow e^{+} + e^{-}$
- additional diagram contributes



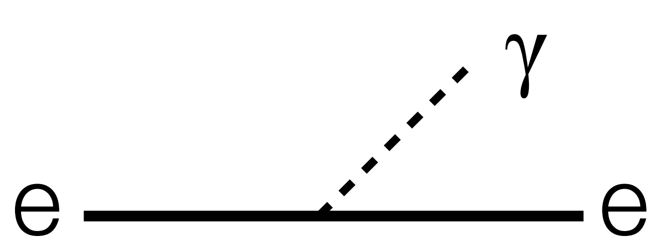
$$\frac{d\sigma}{d\cos\theta d\phi} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g_e^4}{4E^2}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$

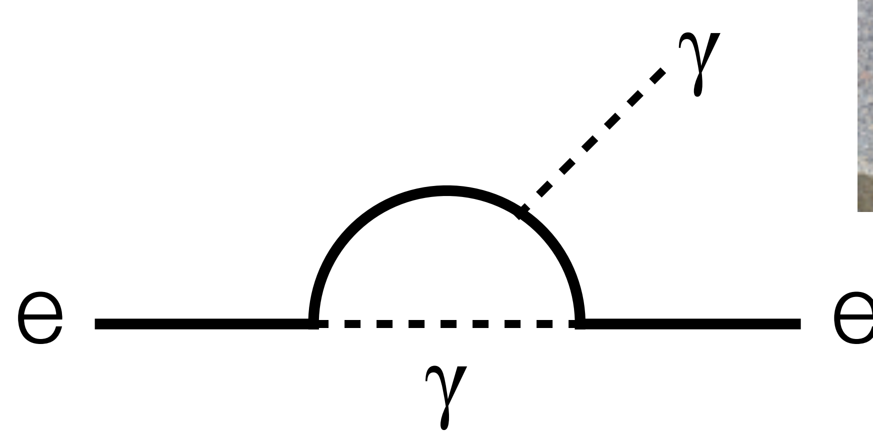


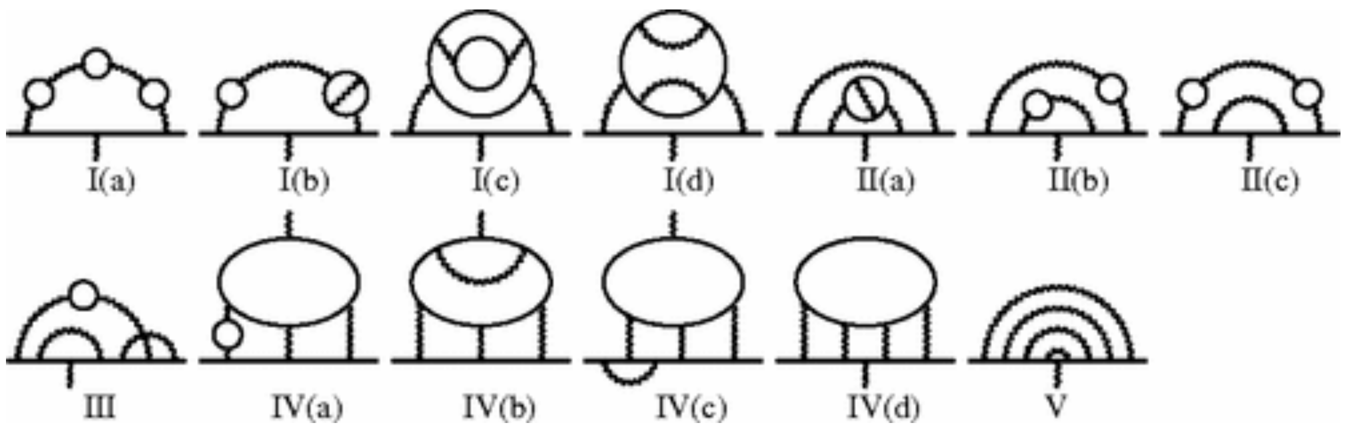
THE "GYROMAGNETIC RATIO"

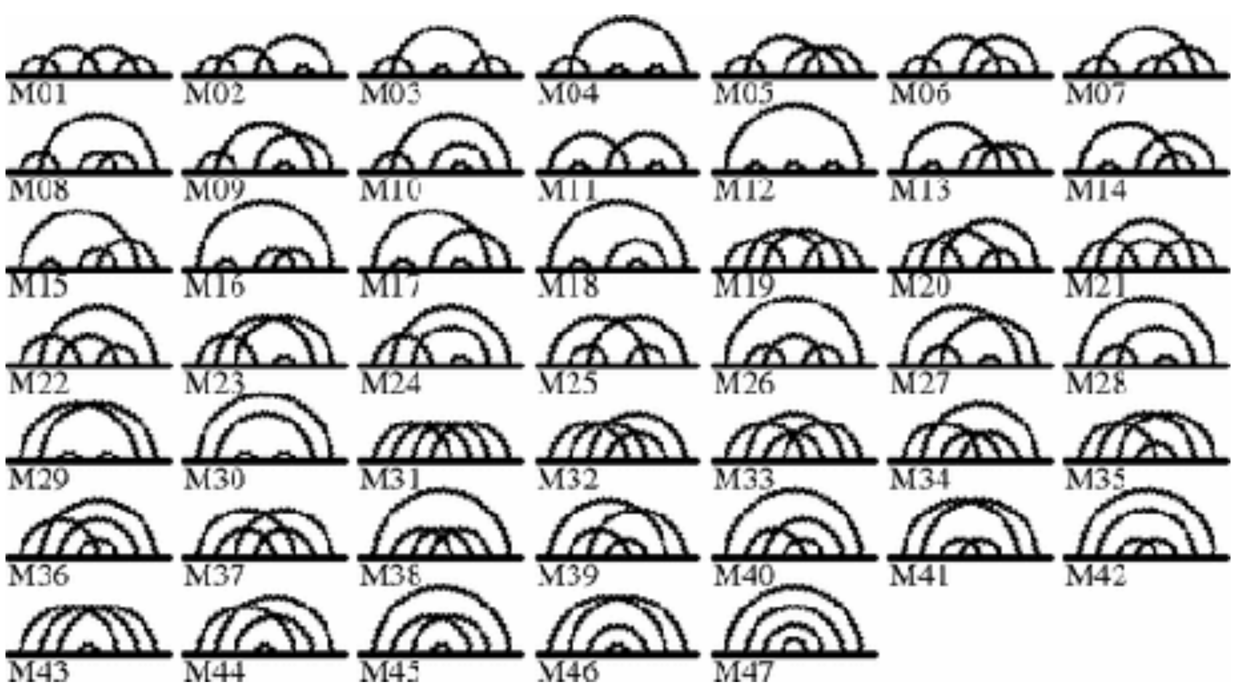
$$\mu = g\mu_B s/\hbar \qquad \mu_B = \frac{e\hbar}{2m}$$

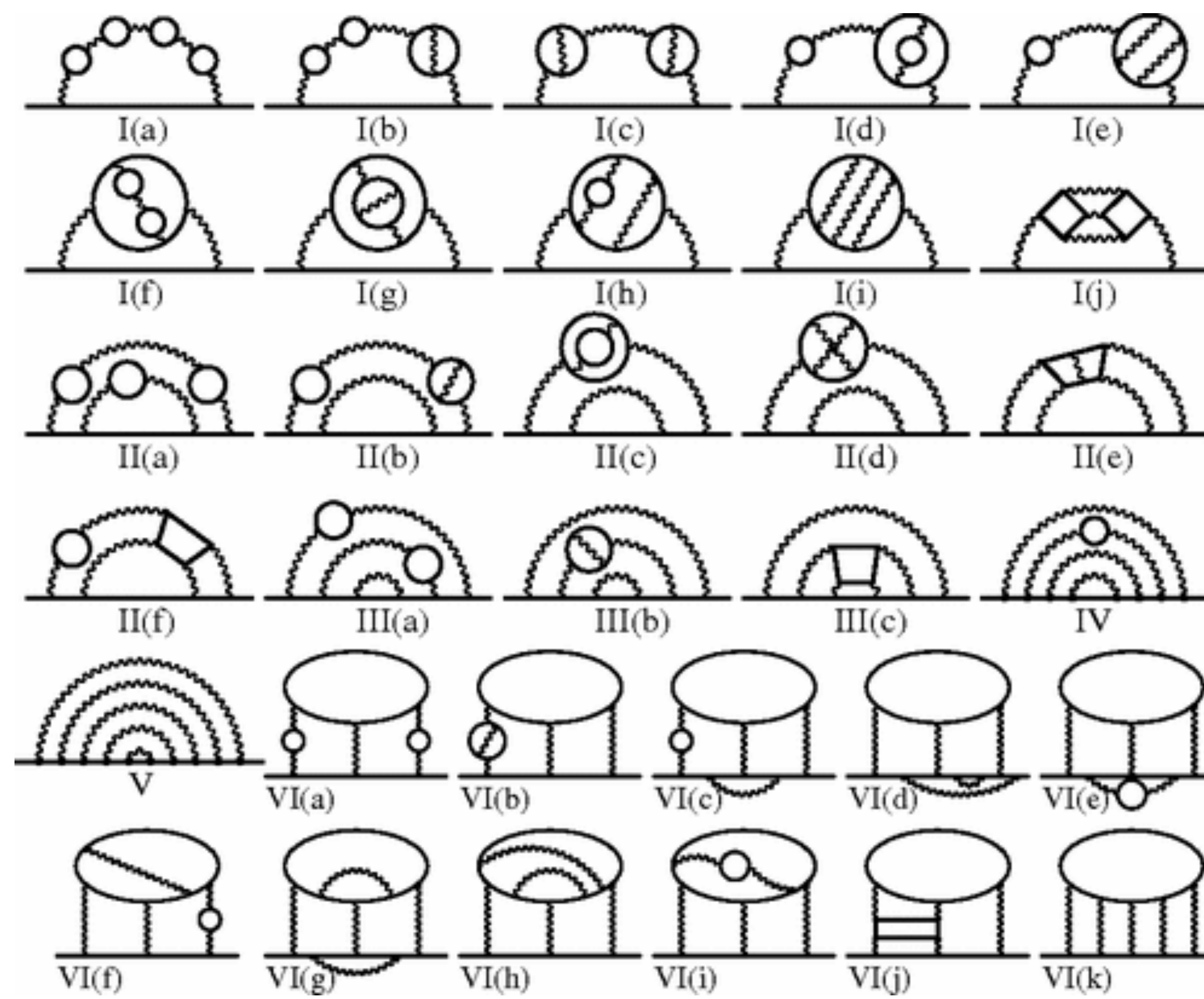
- Ratio of magnetic moment to the spin x Bohr magneton
- This is not exactly 2 for an electron
  - higher order electromagnetic corrections
  - a = (g-2)/2 = ~ 0.0011596521807328
  - "anomalous" moment
  - first calculated by Julian Schwinger in 1948
    - a ~$\alpha/2\pi$ = 0.0011614



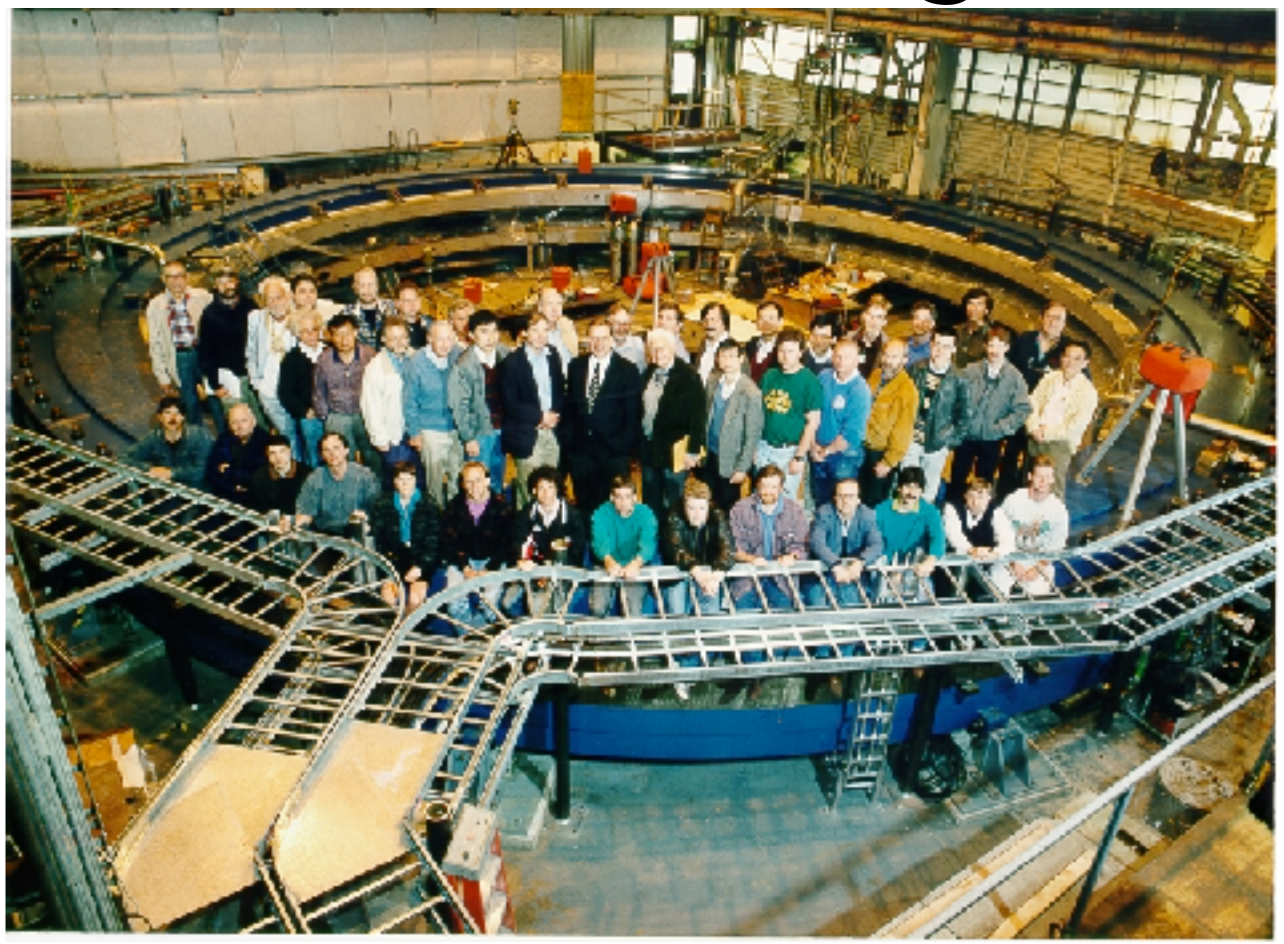



I(a) I(b) I(c) I(d) II(a) II(b) II(c)
III IV(a) IV(b) IV(c) IV(d) V


M01 M02 M03 M04 M05 M06 M07
M08 M09 M10 M11 M12 M13 M14
M15 M16 M17 M18 M19 M20 M21
M22 M23 M24 M25 M26 M27 M28
M29 M30 M31 M32 M33 M34 M35
M36 M37 M38 M39 M40 M41 M42
M43 M44 M45 M46 M47


I(a) I(b) I(c) I(d) I(e)
I(f) I(g) I(h) I(i) I(j)
II(a) II(b) II(c) II(d) II(e)
II(f) III(a) III(b) III(c) IV
V VI(a) VI(b) VI(c) VI(d) VI(e)
VI(f) VI(g) VI(h) VI(i) VI(j) VI(k)

THE MUON g-2 EXPERIMENT



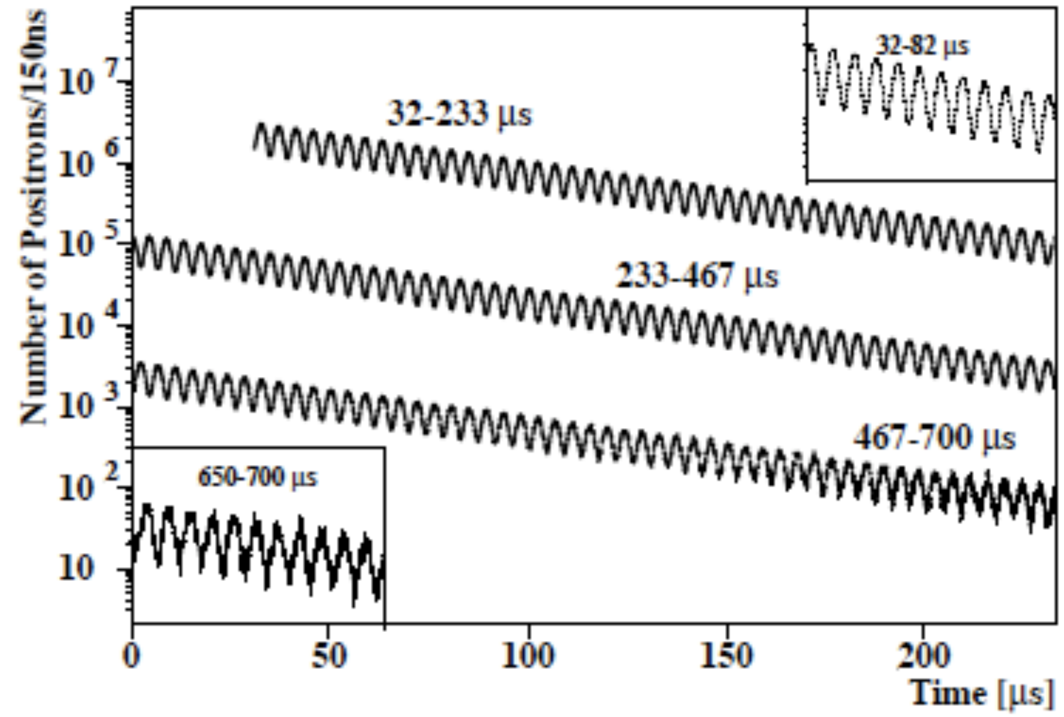
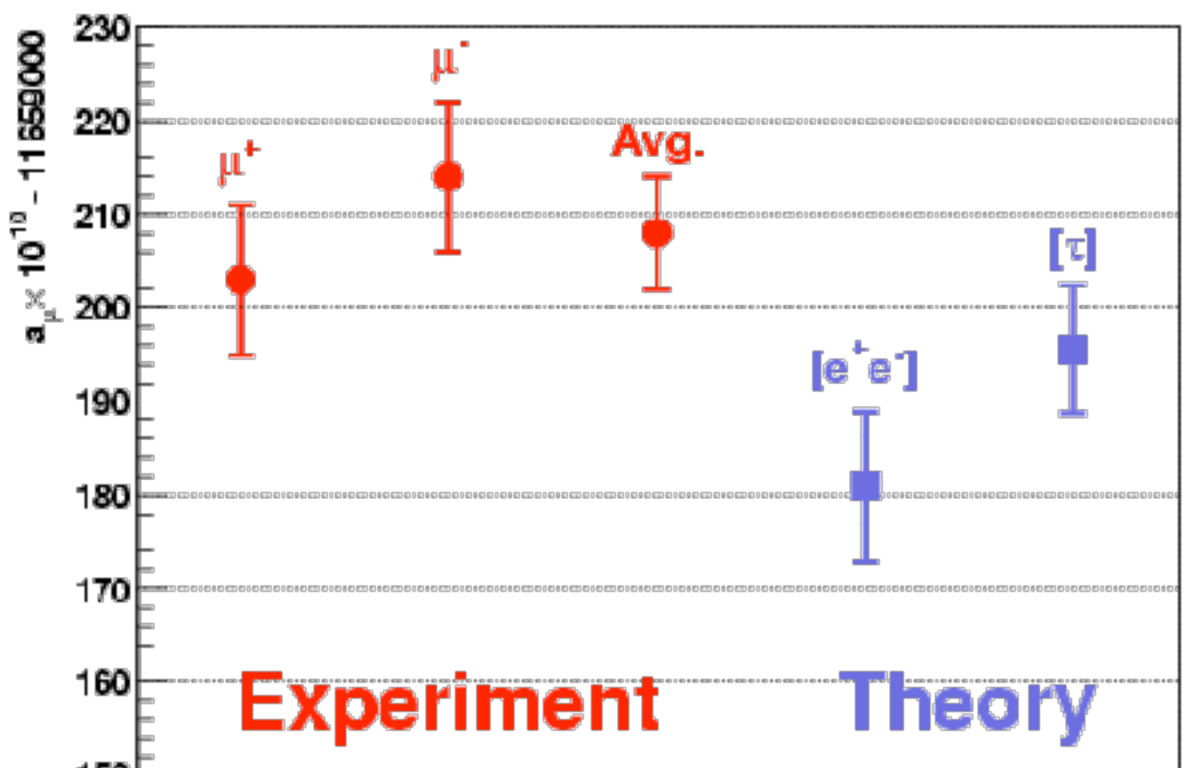


FIG. 3. Positron time spectrum overlaid with the fitted 10 parameter function (χ^2/dof= 3818/3799). The total event sample of 0.95×10^9 e^+ with $E \geq 2.0$ GeV is shown.



- Precess muon spin in a magnetic field as it circulates around a ring
    - direction of electron emerging from muon decay is correlated with its polarization
    - measure the precession of the spin to extract magnetic moment
- Predicted (g-2)/2=(1165918.81±0.38)x10^{-9}
- Measured (g-2)/2=(1165920.80±0.63)x10^{-9}

SUMMARY

- Please read Chapter 7 for next time