PHY489/1489: LECTURE 20

THE HIGGS BOSON

FINAL EXAMINATION

- Format:
  - 4 short answer questions.
    - Focus on some core concept
    - Answered mainly by explaining with minimal calculation.
  - 2 calculations
    - Evaluate amplitude, cross section/rate for process
- Equation sheet will be provided
  - additional information particular to a problem will be provided on the exam itself if needed.

LAST TIME:

- Last time, we introduced the Lagrangian formalism as an alternative to the equations of motion
- Local gauge symmetry works basically the same as before
  - e.g. for a U(1) gauge symmetry:

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi = 0 \qquad i\gamma^\mu\partial_\mu\psi - m\psi = 0$$

$$\mathcal{L} \to \mathcal{L} - q\bar{\psi}\gamma^\mu\psi\, A_\mu \qquad A_\mu \to A_\mu - \partial_\mu\theta$$

$$\partial_\mu \to D_\mu \equiv \partial_\mu + iqA_\mu$$

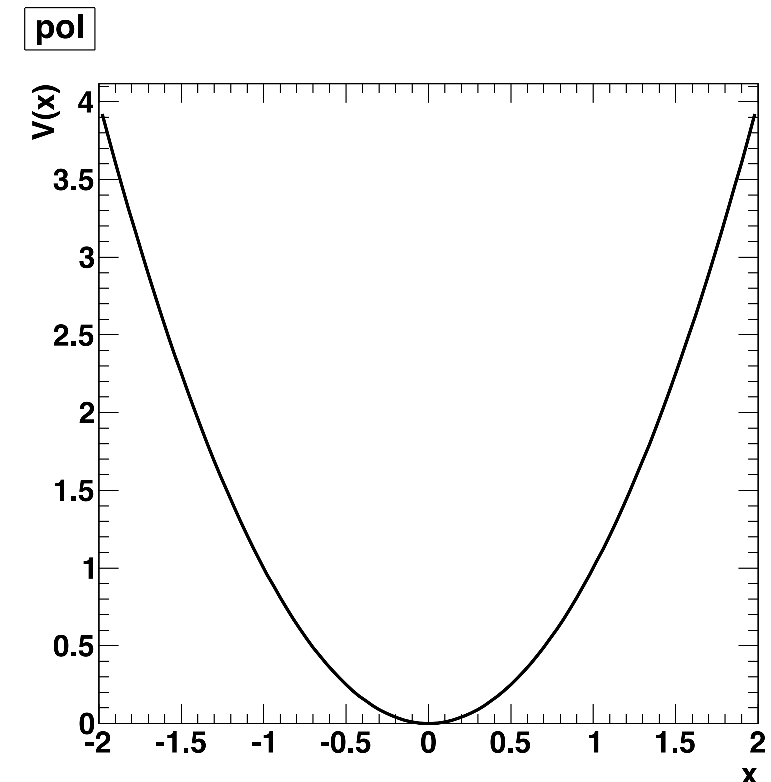
- We found that local gauge symmetry forbids the gauge bosons from having mass
- We also found that the $SU(2)_L$ x $U(1)_Y$ symmetry we introduced for the weak interaction also forbids fermion masses!

MORE ON THE MASS TERM

$$\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi = 0$$

$$\mathcal{L}_P = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2A^\mu A_\mu$$

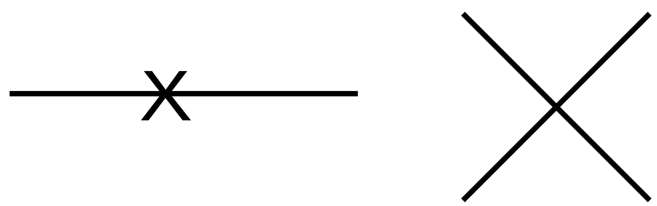


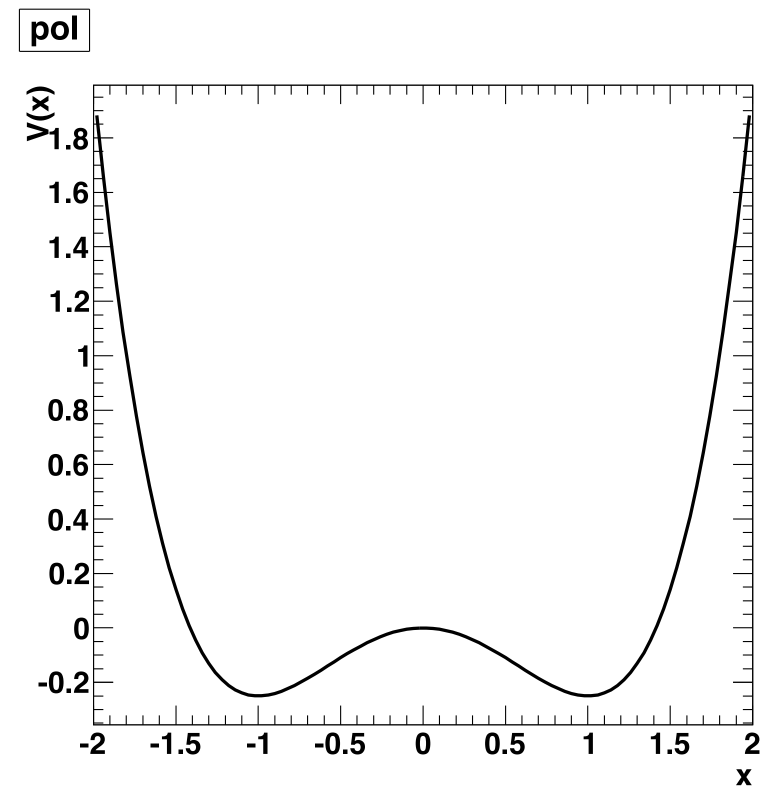
- mass terms are quadratic in the field
    - *i.e.* if there is a quadratic term in the lagrangian, it behaves as a mass.

“VACUUM EXPECTATION VALUE”

- Consider the Lagrangian

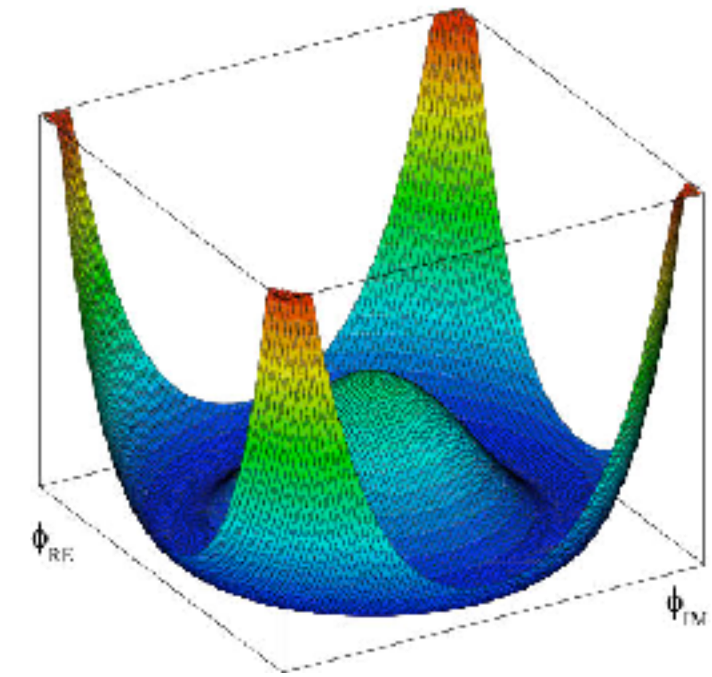
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$$





- Note that ϕ=0 is not a stable configuration
  - the vacuum (e.g. lowest energy state) actually happens when ϕ has some non-zero value
  - “vacuum expectation value” (VEV)
- Perturbation theory must start from a stable vacuum in order to work
  - choose a vacuum state
  - “spontaneous symmetry” breaking

TOY MODEL WITH CONTINUOUS SYMMETRY



- Consider a complex scalar Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^*)(\partial^\mu \phi) + \frac{1}{2}\mu^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2$$

$$\phi = \phi_1 + i\phi_2$$

- Instead of two potential vacuum configurations, we now have an infinite number of connected states

$$|\phi| = \frac{\mu}{\lambda}$$

- Expand about a vacuum point
    - let's also make it locally gauge invariant by introducing the "covariant derivative"
    - that means we get a gauge boson along for the ride

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi + \frac{1}{2}\mu^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2 - \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}$$

BREAK THE SYMMETRY

$$\mathcal{L} = \frac{1}{2}(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi + \frac{1}{2}\mu^2\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2 \quad -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

- choose a vacuum point: $\phi_0 = \frac{\mu}{\lambda}$
- and reparametrize the fields as:

$$\eta = \phi_1 - \frac{\mu}{\lambda} \qquad \chi = \phi_2$$

- and rewrite the Lagrangian focussing on the kinetic part

$$(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi$$

$$\Rightarrow \left[(\partial_\mu \boxed{- iqA_\mu})(\eta + \boxed{\frac{\mu}{\lambda}} - i\chi)\right]\left[(\partial^\mu + \boxed{iqA^\mu})(\eta + \boxed{\frac{\mu}{\lambda}} + i\chi)\right]$$

$$\longrightarrow \frac{1}{2}\left(\frac{\mu q}{\lambda}\right)^2 A_\mu A^\mu$$

this is a mass term for the vector particle $m_A = 2\sqrt{\pi}\frac{q\mu}{\lambda}$

HOW DID THIS HAPPEN:

- Recall that our gauge invariant Lagrangian

$$(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi$$

- has a term $q^2 A_\mu A^\mu \phi^* \phi$

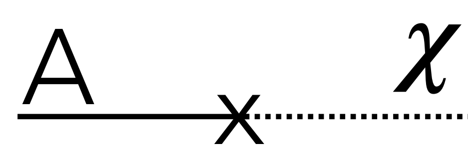
- Normally, ϕ is just a normal field
  - but the potential gives it a vacuum expectation (e.g. non-zero) base value that turns this into a mass term for A
  - we chose a particular vacuum configuration but the result is independent of our choice
  - the symmetry isn't "really" broken, just hidden by our choice

OTHER TERMS

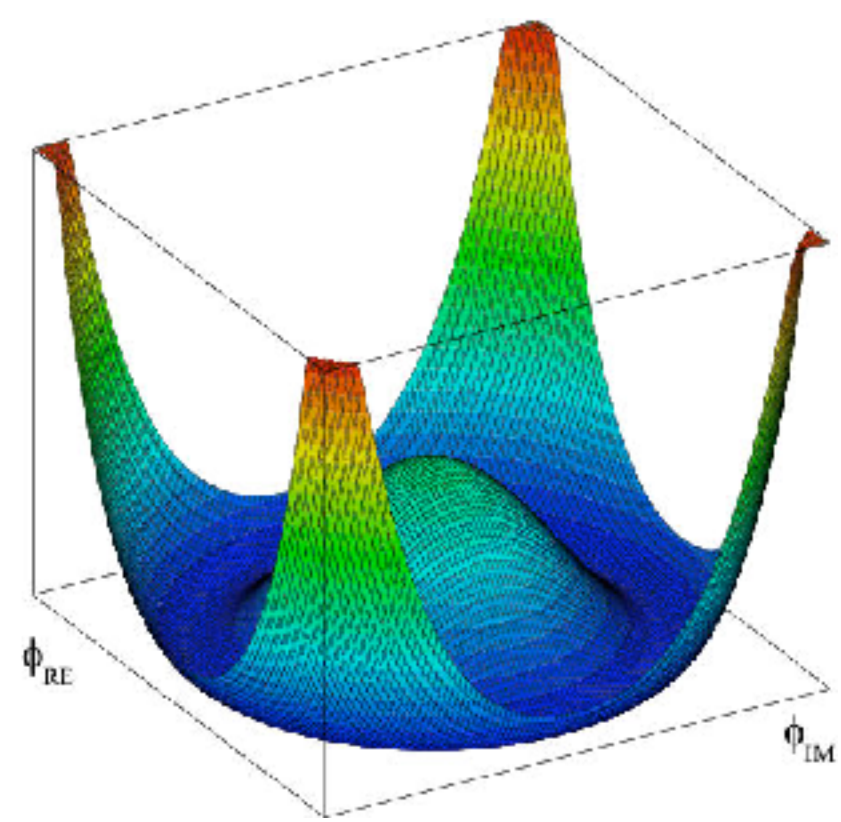
$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

- Note that the Lagrangian also includes a kinetic term for a massless field χ
  - this is called a "Nambu-Goldstone boson"
- Another term:

$$\frac{q\mu}{\lambda}(\partial_\mu \chi)A^\mu$$



- is problematic . . .
  - The A particle spontaneously turns into χ



ACCOUNTING ISSUE

$$(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi$$

$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

- We started with:
  - two scalar fields (ϕ_1, ϕ_2 or alternatively ϕ^*,ϕ)
  - a **massless** gauge boson (two polarizations)
- We end up with:
  - two scalar fields (η, χ)
  - a **massive** gauge boson (three polarizations)
- where did the extra degree of freedom come from?

GAUGE TRANSFORMATION

$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

- if we isolate the the terms related to χ and A

$$\frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) + \frac{q\mu}{\lambda}(\partial_\mu\chi)A^\mu + \frac{1}{2}\left(\frac{q\mu}{\lambda}\right)^2 A_\mu A^\mu$$

$$\frac{1}{2}\left(\frac{q\mu}{\lambda}\right)^2\left[A_\mu + \frac{\lambda}{q\mu}(\partial_\mu\chi)\right]\left[A^\mu + \frac{\lambda}{q\mu}(\partial^\mu\chi)\right]$$

  - this last transformation effectively represents a gauge transformation

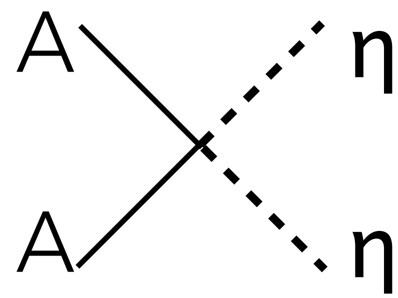
$$A_\mu \rightarrow A_\mu + \frac{\lambda}{q\mu}(\partial_\mu\chi)$$

- “gauge transform” to make χ disappear explicitly from the Lagrangian
- the χ field corresponds to the “new” longitudinal polarization of the A
- “The gauge boson ate the Goldstone boson”

GAUGE COUPLINGS

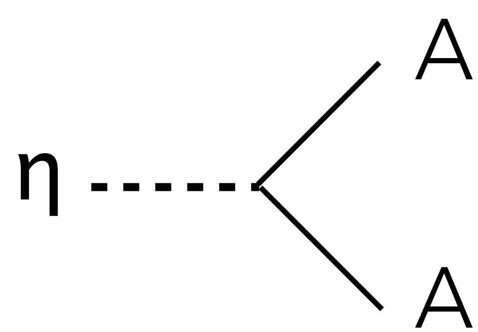
$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

$$q^2 A_\mu A^\mu \eta^2$$

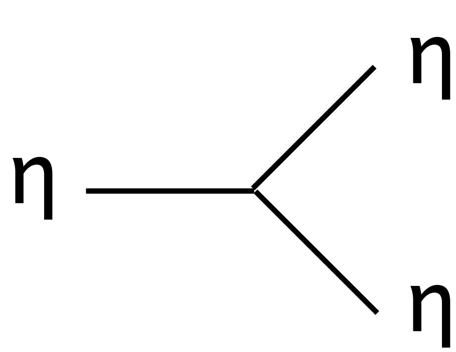


$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

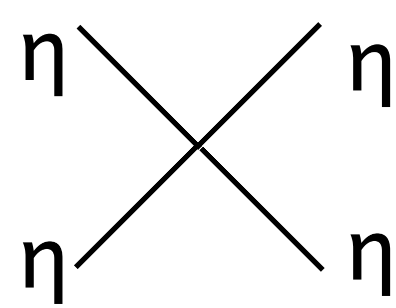
$$q^2 \frac{\mu}{\lambda} A_\mu A^\mu \eta$$



$$\lambda \frac{\mu}{\lambda} \eta^3$$



$$\frac{1}{4}\eta^4$$



CONCLUSIONS OF THE TOY STORY

- We made a theory of a "charged" scalar boson:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^*)(\partial^\mu \phi) + \frac{1}{2}\mu^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2$$

  - where the vacuum states have $\phi \neq 0$
- enforce U(1) gauge invariance

$$\mathcal{L} = \frac{1}{2}(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi + \frac{1}{2}\mu^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2 \quad -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

- expand about arbitrary vacuum point:

$$\eta = \phi_1 - \frac{\mu}{\lambda} \qquad\qquad \chi = \phi_2$$

- we end up with:
  - a mass term for the gauge boson
  - a massive scalar field
  - massless scalar field becomes extra polarization of the massive gauge boson.
  - various (self) interactions between the scalar field and gauge boson
    - note that these interactions are not optional. .. . they must be there!

ELECTROWEAK SYMMETRY BREAKING

- We consider a weak isodoublet of Higgs fields

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

  - with Y=1 = 2(Q-I_3): top/bottom component has Q=1, 0

- with the Lagrangian:

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial_\mu \phi) - \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

- $\boldsymbol{\phi}$ has a "ring" of degenerate vacuum states at

$$\phi^\dagger \phi = -\frac{\mu^2}{2\lambda} \qquad \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- Introduce gauge symmetry under $SU(2)_L x U(1)_Y$

$$\partial_\mu \to D_\mu = \partial_\mu + i g_W \, \vec{\tau} \cdot \mathbf{W}_\mu + i g' \frac{Y}{2} B_\mu$$

THE COVARIANT DERIVATIVE

$$\partial_\mu \to D_\mu = \partial_\mu + ig_W\ \vec{\tau}\cdot\mathbf{W}_\mu + ig'\frac{Y}{2}B_\mu$$

$$\begin{pmatrix} \partial_\mu & 0 \\ 0 & \partial_\mu \end{pmatrix} + i\frac{g_W}{2}\left[\begin{pmatrix} W_\mu^3 & 0 \\ 0 & -W_\mu^3 \end{pmatrix} + \begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & 0 \end{pmatrix}\right] + i\frac{g'}{2}\begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 2\partial_\mu + ig_W W_\mu^3 + ig' B_\mu & ig_w(W_\mu^1 - iW_\mu^2) \\ ig_w(W_\mu^1 + iW_\mu^2) & 2\partial_\mu - ig_W W_\mu^3 + ig' B_\mu \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\frac{1}{2\sqrt{2}}\begin{pmatrix} ig_w(W_\mu^1 - iW_\mu^2) \\ 2\partial_\mu - ig_W W_\mu^3 + ig' B_\mu \end{pmatrix}(v+h)$$

THE KINEMATIC TERM:

$$\mathcal{L}_K = (\partial_\mu \phi)^\dagger (\partial_\mu \phi) \rightarrow (D_\mu \phi)^\dagger (D^\mu \phi)$$

$$D_\mu \phi = \frac{1}{2\sqrt{2}} \begin{pmatrix} i g_w (W_\mu^1 - i W_\mu^2) \\ 2\partial_\mu - i g_W W_\mu^3 + i g' B_\mu \end{pmatrix} (v + h)$$

$$\frac{1}{2}(\partial_\mu h)(\partial^\mu h)$$

$$\frac{g_W^2}{8}(W_\mu^1 + i W_\mu^2)(W^{1\mu} - i W^{2\mu})(v + h)^2$$

$$\frac{1}{8}(i g_W W_\mu^3 - i g' B_\mu)(-i g_w W^{3\mu} + i g' B^\mu)(v + h)^2$$

MASS TERMS

- quadratic in fields with a constant

$$\frac{g_W^2}{8}(W_\mu^1 + iW_\mu^2)(W^{1\mu} - iW^{2\mu})(v+h)^2$$

$$\frac{g_W^2 v^2}{8}(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}) = \frac{m_W^2}{2}(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu})$$

$$m_W = \frac{1}{2} g_W v$$

$$\frac{1}{8}(ig_W W_\mu^3 - ig' B_\mu)(-ig_W W^{3\mu} + ig' B^\mu)(v+h)^2$$

$$\frac{1}{8}(g_W W_\mu^3 - g' B_\mu)(g_W W^{3\mu} - g' B^\mu)(v+h)^2$$

$$\frac{v^2}{8}\left(W_\mu^3, B_\mu\right)\begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

DIAGONALIZE MATRIX

$$\frac{v^2}{8}\,(W_\mu^3, B_\mu)\left(\begin{array}{cc} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{array}\right)\left(\begin{array}{c} W^{3\mu} \\ B^\mu \end{array}\right)$$

$$\left|\left(\begin{array}{cc} g_W^2-\lambda & -g_W g' \\ -g_W g' & g'^2-\lambda \end{array}\right)\right| = 0 \quad (g_W^2-\lambda)(g'^2-\lambda) - g_W^2 g'^2 = 0$$

$$\lambda^2 - (g_W^2 + g'^2)\lambda = 0 \qquad \lambda = 0,\ g_W^2 + g'^2$$

$$\frac{v^2}{8}\,(A_\mu, Z_\mu)\left(\begin{array}{cc} 0 & 0 \\ 0 & g_W^2+g'^2 \end{array}\right)\left(\begin{array}{c} A^\mu \\ Z^\mu \end{array}\right)$$

$$m_A = 0 \qquad m_Z = \frac{v}{2}\sqrt{g_W^2 + g'^2}$$

COMPARE W/Z MASSES

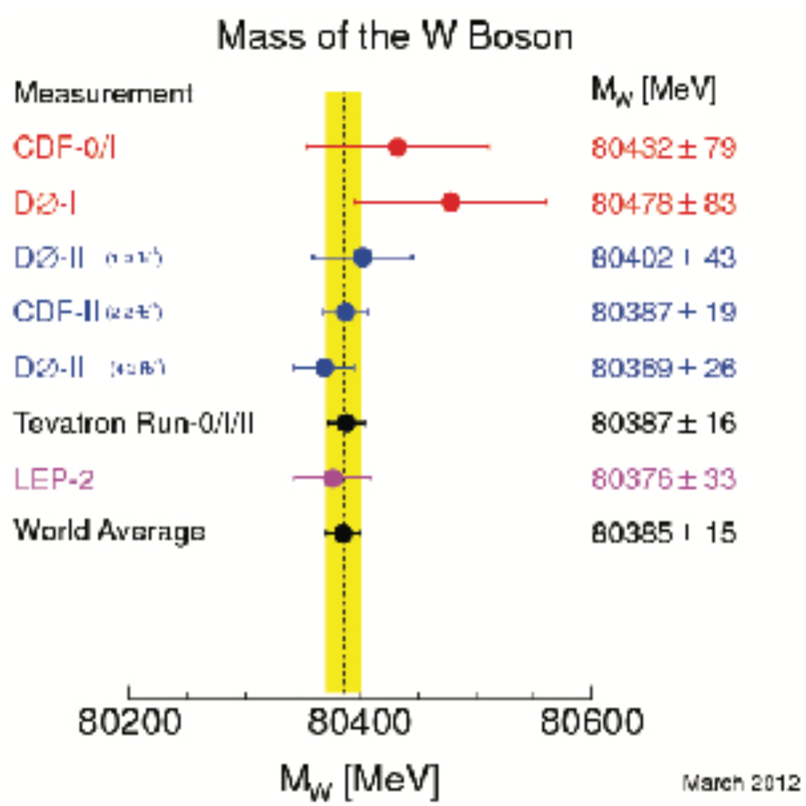
$$m_W = \frac{1}{2} g_W v \quad m_Z = \frac{v}{2}\sqrt{g_W^2 + g'^2}$$

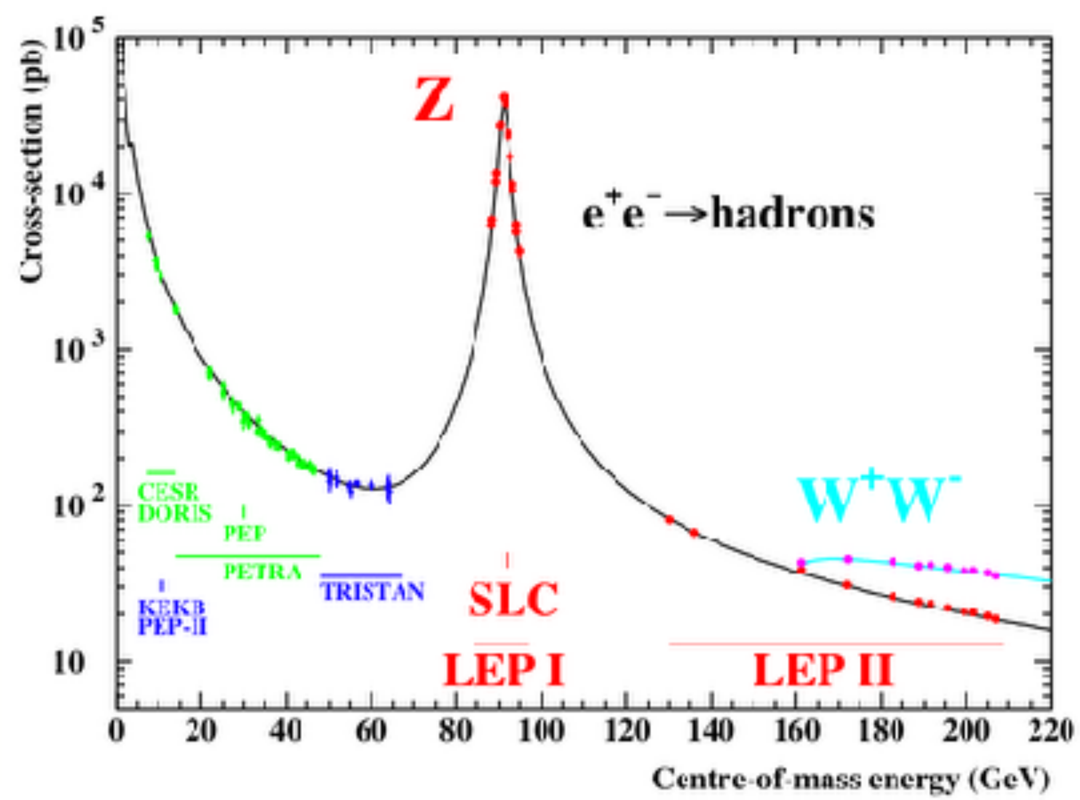
$$\frac{m_W}{m_Z} = \frac{g_W}{\sqrt{g_W^2 + g'^2}}$$

$$g_Z \equiv \frac{g}{\cos\theta_W} = \frac{e}{\sin\theta_W \cos\theta_W}$$

$$g' = g_Z \sin\theta_W$$

$$\frac{m_W}{m_Z} = \frac{g_W}{\sqrt{g_W^2 + g'^2}} = \frac{g_Z \cos\theta_W}{\sqrt{g_Z^2 \cos^2\theta + g_Z^2 \sin^2\theta_W}} = \cos\theta_W$$





$$\frac{m_W}{m_Z} = \frac{80.385 \text{ GeV}}{91.188 \text{ GeV}} = 0.882$$

$$\sin^2\theta_W = 0.224$$

ONE REMAINING ISSUE

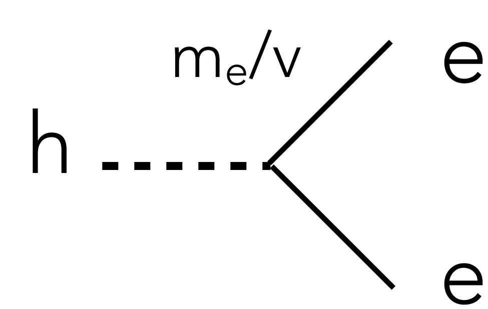
- The fermion masses!

$$\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$$

- we found this breaks gauge symmetry because it couples an $SU(2)_L$ doublet to a $SU(2)_L$ singlet

$$\mathcal{L}_Y = -g_e \left[(\bar{\nu}_L, \bar{e}_L) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + h.c. \right]$$

- by coupling the left chiral fields to the Higgs field, we can generate an overall singlet.
- by acquiring a vacuum expectation value, this becomes the mass of the electron.
    - $g_e v \sim m_e$



CONCLUSIONS:

- Gauge invariance forces us to make back doors for introducing mass into our theory
- Spontaneous symmetry breaking gives us a way of introducing a "constant" background with gauge quantum numbers to produce mass terms that preserve gauge symmetry
- As a consequence there is a tight interconnection between
  - the vacuum expectation value
  - gauge couplings
  - gauge boson masses
  - effectively fixed by the model and tested and can be tested.
- In the electroweak model, the fermion masses can be generated in the same way.
  - Fixes relation between vev, fermion mass, and Higgs coupling to the fermion

LAST LECTURE WITH "NEW CONTENT"

- Next (last) lecture:
    - contemporary issues/problems
    - Higgs bosons
    - work a problem or two
    - let me know if you have questions or particular things you want me to review.