PHYSICS 489/1489: LECTURE 19

LAGRANGIANS AND SPONTANEOUS SYMMETRY BREAKING

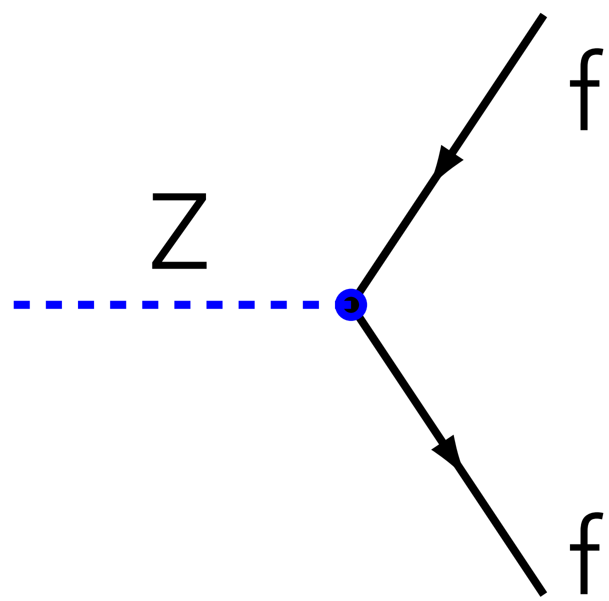
REVIEW

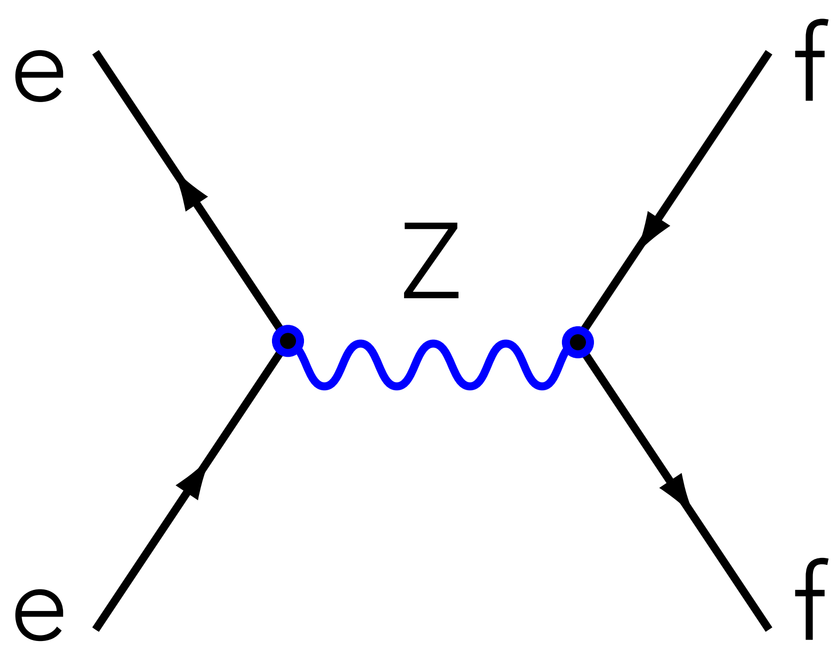
- gauge invariance:
  - global vs local
  - consequences of local gauge symmetry
  - “abelian” vs. non-abelian
- Weak interactions:
  - Chirality
    - helicity suppression
  - Flavor change
    - CKM matrix, PMNS matrix
    - GIM mechanism/suppression
- Electroweak mixing
  - θ_w

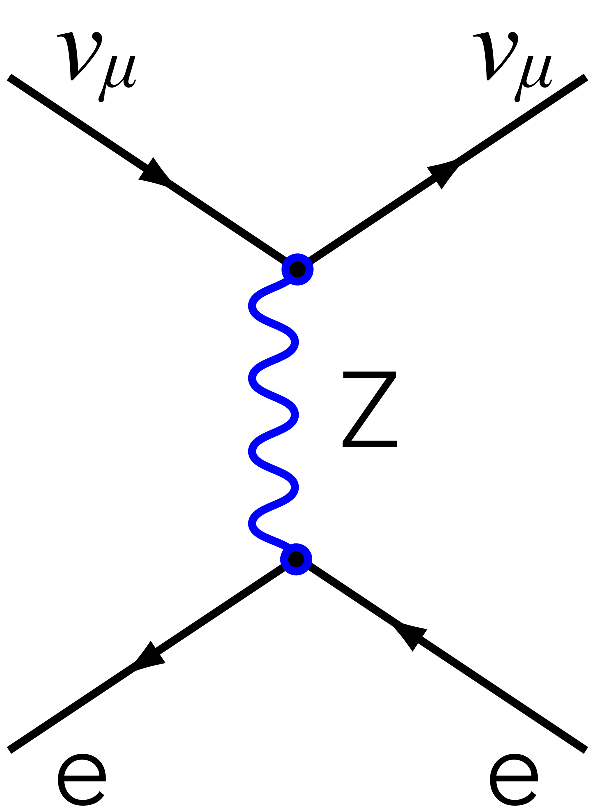
- Extra office hours next week?

MORE ELECTROWEAK TESTS

- $\sin^2\theta_W = 0.23146 \pm 0.00012$
- Branching fractions of the Z (last time)
- Neutral current scattering
- Left/right asymmetries in $e^+ + e^- \rightarrow f + \bar{f}$:
  - chiral dependence of coupling
- Mass of W and Z







LAGRANGIAN MECHANICS

- Describe a system with coordinates and its time derivatives:

$$L = L(q, \dot{q}) = T - U$$

- Equations of motion are obtained by minimizing the action

$$S = \int dt L(q_i, \dot{q}_i)$$

  - resulting in Euler-Largange equations $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$

- For a point particle in a potential with Cartesian coordinates:

$$L = \frac{1}{2} m \dot{q}_i^2 - U(q_i) \qquad \frac{d}{dt}(m\dot{q}_i) + \frac{\partial U}{\partial q_i} = 0 \qquad m\ddot{q}_i = -\frac{\partial U}{\partial q_i}$$

$$m\ddot{x} = -\frac{\partial U}{\partial x} \qquad m\ddot{y} = -\frac{\partial U}{\partial y} \qquad m\ddot{z} = -\frac{\partial U}{\partial z}$$

FOR "FIELDS:"

- Fields become the "coordinate" with space time as the "dynamical variable"
  - q(t) ➞ ϕ(x)
  - $L \Rightarrow \mathcal{L}(\phi(x), \partial_\mu \phi(x)) \quad L = \int d^3x\, \mathcal{L}(\phi, \partial_\mu \phi)$
  - The action is now defined as: $\int dt L = \int dt \int d^3x\, \mathcal{L}(\phi, \partial_\mu \phi) = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$
  - Euler-Lagrange Equatoins:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \longrightarrow \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = \frac{\partial \mathcal{L}}{\partial \phi}$$

- Examples of Lagrangians and their equations of motion

$$\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 \qquad \partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi = 0 \qquad i\gamma^\mu \partial_\mu \psi - m\psi = 0$$

$$\mathcal{L}_P = \frac{-1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{8\pi}m^2 A^\nu A_\nu \qquad \partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + m^2 A^\nu = 0$$

LOCAL GAUGE INVARIANCE

- We can recast our previous discussion about local gauge invariance in the Lagrangian framework
- Example: consider the Dirac Lagrangian with local gauge transformation

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi = 0$$

$$\psi \to e^{iq\theta(x)}\ \psi$$

$$\partial_\mu \psi \to e^{iq\theta}\partial_\mu \psi + iq\ \partial_\mu \theta\ e^{iq\theta}\ \partial_\mu \psi$$

$$\mathcal{L} \to \mathcal{L} - q\ \bar{\psi}\gamma^\mu\ (\partial_\mu \theta)\ \psi$$

- As before, need to add a new field and interaction

$$\mathcal{L} \to \mathcal{L} - q\bar{\psi}\gamma^\mu \psi\ A_\mu \qquad A_\mu \to A_\mu - \partial_\mu \theta$$

- Another way to summarize this is to convert the derivative to a "covariant derivative"

$$\partial_\mu \to D_\mu \equiv \partial_\mu + iqA_\mu$$

A FEW ENHANCEMENTS

- As it stands, the A field is static
- We can give it "life" by adding a kinematic term

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu \psi - q\bar{\psi}\gamma^\mu\psi\ A_\mu - \frac{1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{8\pi} m^2 A^\nu A_\nu$$

- but recalling the transformation: $A_\mu \to A_\mu - \partial_\mu \theta$
  - we find that the last term (the mass) is not gauge-invariant

- We can also extend to a "non-abelian" gauge symmetry:

$$\psi \to e^{ig\vec{\tau}\cdot\mathbf{a}(x)}\psi \equiv S\psi \qquad \partial_\mu\psi \to \partial_\mu(S\psi) = S(\partial_\mu\psi) + (\partial_\mu S)\psi$$

  - where as before we need to add another term and fields:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \boxed{-q(\bar{\psi}\gamma^\mu\vec{\tau}\psi)\cdot\mathbf{A}_\mu} \qquad \mathbf{A}^k_\mu \to \mathbf{A}^k_\mu - \partial_\mu \mathbf{a}_k - g f_{ijk}\,\mathbf{a}_i\,\mathbf{A}^j_\mu$$

  - and the mass term is once again forbidden
  - the gauge invariance can be restored by: $\partial_\mu \to D_\mu \equiv \partial_\mu + ig\vec{\tau}\cdot\mathbf{A}_\mu$

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

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Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

1. Introduction

At first sight there may be little or no similarity between electromagnetic effects and the phenomena associated with weak interactions. Yet certain remarkable parallels emerge with the supposition that the weak interactions are mediated by unstable bosons. Both interactions are universal, for only a single coupling constant suffices to describe a wide class of phenomena: both interactions are generated by vectorial Yukawa couplings of spin-one fields ††. Schwinger first suggested the existence of an "isotopic" triplet of vector fields whose universal couplings would generate both the weak interactions and electromagnetism — the two oppositely charged fields mediate weak interactions and the neutral field is light [2]). A certain ambiguity beclouds the self-interactions among the three vector bosons; these can equivalently be interpreted as weak or electromagnetic couplings. The more recent accumulation of experimental evidence supporting the $\Delta I = \frac{1}{2}$ rule characterizing the non-leptonic decay modes of strange particles indicates a need for at least one additional neutral intermediary [3]).

The mass of the charged intermediaries must be greater than the K-meson mass, but the photon mass is zero — surely this is the principal stumbling block in any pursuit of the analogy between hypothetical vector mesons and photons. It is a stumbling block we must overlook. To say that the decay intermediaries

- We now turn to this "stumbling block"

ONE MORE DILEMMA

- Consider the Dirac mass term:

$$m\bar{\psi}\psi$$

$$= \bar{\psi}_L\psi_L + \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L + \bar{\psi}_R\psi_R$$

$$= \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$

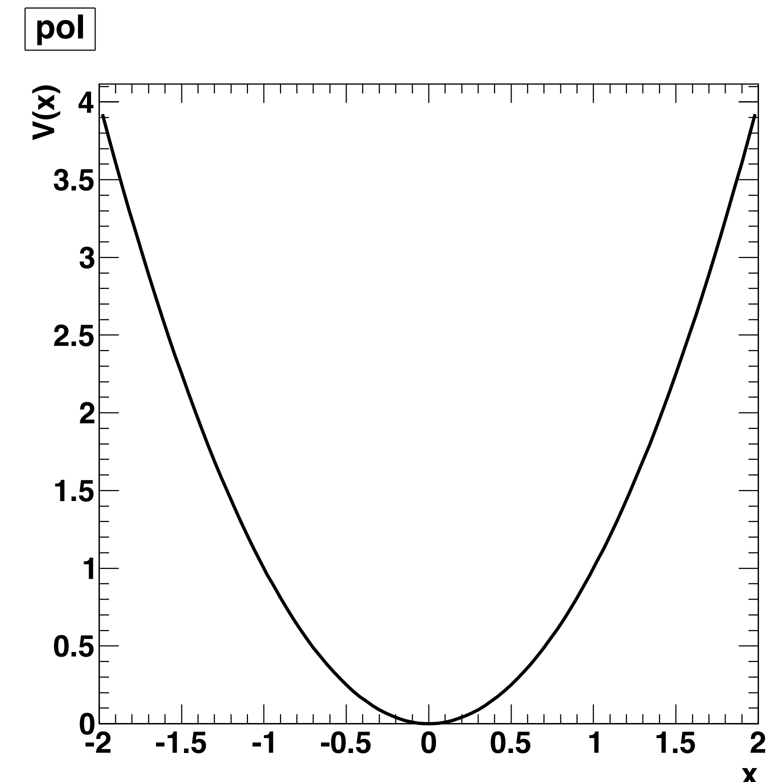
- mass terms result from the coupling of left and right chiral states of a particle
- violates gauge symmetry in the $SU(2)_L \times U(1)_Y$ model of weak interactions
- direct fermion mass terms (quarks, leptons) are also forbidden.

MORE ON THE MASS TERM

$$\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi = 0$$

$$\mathcal{L}_P = \frac{-1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{8\pi}m^2 A^\nu A_\nu \\ -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

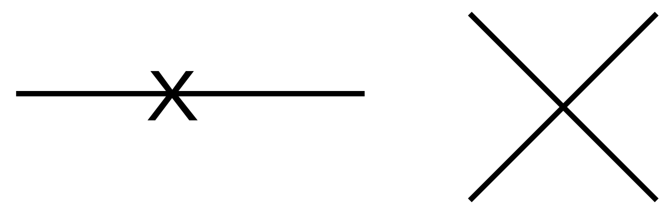


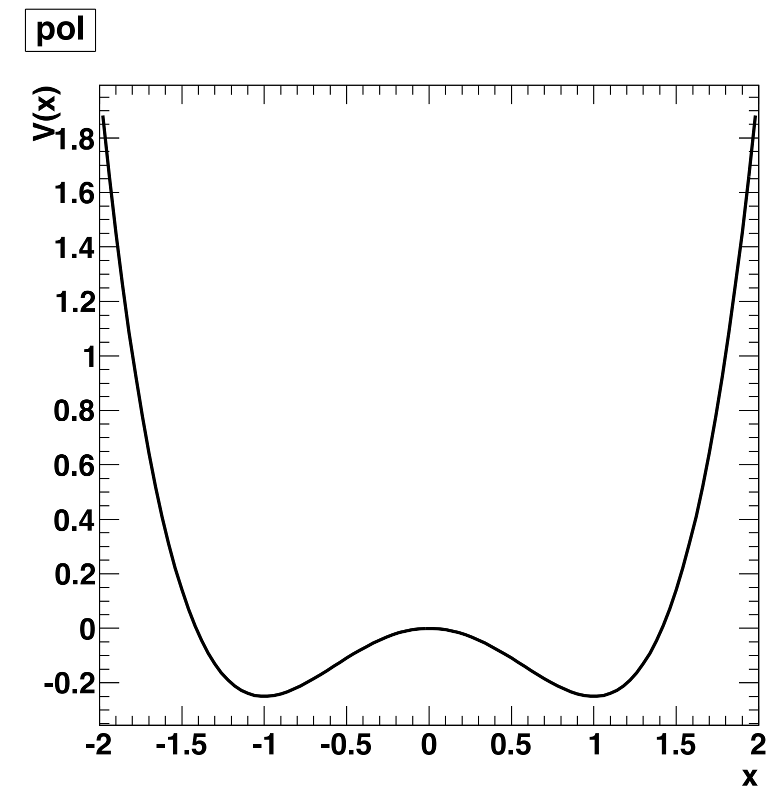
- mass terms are quadratic in the field
  - *i.e.* if a field has quadratic term in the Lagrangian, it behaves as the mass for that field

"VACUUM EXPECTATION VALUE"

- Consider the Lagrangian

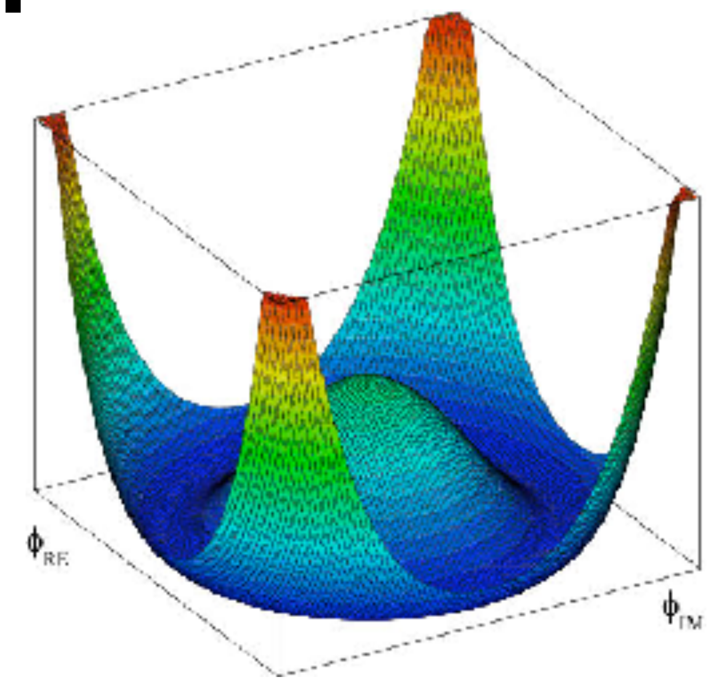
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$$





- Note that ϕ=0 is not a stable configuration
  - the vacuum (e.g. lowest energy state) actually happens when ϕ has some non-zero value
  - "vacuum expectation value" (VEV)
- Perturbation theory must start from a stable vacuum in order to work
  - choose a vacuum state
  - "spontaneous symmetry" breaking

CONTINUOUS SYMMETRY



- Consider a complex scalar Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^*)(\partial^\mu \phi) + \frac{1}{2}\mu^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2$$

$$\phi = \phi_1 + i\phi_2$$

- Instead of two potential vacuum configurations, we now have an infinite number of connected states

$$|\phi| = \frac{\mu}{\lambda}$$

- Expand about a vacuum point
  - let's also make it locally gauge invariant by introducing the "covariant derivative"
  - that means we get a gauge boson along for the ride

$$\partial_\mu \to D_\mu \equiv \partial_\mu + iqA_\mu$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi + \frac{1}{2}\mu^2\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2 - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

BREAK THE SYMMETRY

$$\mathcal{L} = \frac{1}{2}(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi + \frac{1}{2}\mu^2\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2 \quad -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

- choose a vacuum point: $\phi_0 = \dfrac{\mu}{\lambda}$
- and reparametrize the fields as:

$$\eta = \phi_1 - \frac{\mu}{\lambda} \qquad \chi = \phi_2$$

- and rewrite the Lagrangian focussing on the kinetic part

$$(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi$$

$$\Rightarrow \left[(\partial_\mu \boxed{- iqA_\mu})(\eta + \boxed{\frac{\mu}{\lambda}} - i\chi)\right]\left[(\partial^\mu + \boxed{iqA^\mu})(\eta + \boxed{\frac{\mu}{\lambda}} + i\chi)\right]$$

$$\longrightarrow \frac{1}{2}\left(\frac{\mu q}{\lambda}\right)^2 A_\mu A^\mu$$

this is a mass term for the vector particle $m_A = 2\sqrt{\pi}\dfrac{q\mu}{\lambda}$

HOW DID THIS HAPPEN:

- Recall that our gauge invariant Lagrangian

$$(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi$$

- has a term $q^2 A_\mu A^\mu \phi^* \phi$

- Normally, ϕ is just a normal field
  - but the potential gives it a vacuum expectation (e.g. non-zero) base value that turns this into a mass term for A
  - we chose a particular vacuum configuration but the result is independent of our choice
  - the symmetry isn't "really" broken, just hidden by our choice

NEXT TIME:

- Please read Chapters 17.4-17.7