PHYSICS 489/1489: LECTURE 18

TESTS OF EW MIXING

HOW DID IT HAPPEN?

- First, we considered the chiral states of each particle separately

  $$e \to e_L, e_R \qquad \nu \to \nu_L, \nu_R$$

  - We considered a SU(2) gauge theory that couples only to left chiral particles
    - This gave us $W^{\pm}$ (weak CC interaction) plus another “neutral current” interaction W_3
    - we know this can’t be the photon (nor the Z)
  - Introduced a new gauge field B with $U(1)_Y$ gauge symmetry (like EM) but where we can assign different “hypercharge” Y to each chiral state
  - postulate a $SU(2)_L \times U(1)_Y$ gauge symmetry
- Postulate that the A, Z are linear combinations of W_3 and B
- See what we have to do to:
  - get equal left/right chiral coupling to A (consistent with EM)
  - get appropriate electrons charges (-1 for electron, 0 for neutrino)
- Sets relations between the coupling constants and hypercharge of each state.
  - completely determines the properties of Z interactions with quarks, leptons

    $$e = g \sin\theta_W = g' \cos\theta_W = g_Z \cos\theta_W \sin\theta_W$$

STEPPING BACK

- We take:
    - a $SU(2)_L$ gauge group coupling only to left chiral fermions (**W**)
    - a $U(1)_Y$ gauge group with different couplings to left and right chiral fields (B)
- came together to form:
    - weak charged currents with only left chiral couplings
    - a neutral current with equal left/right coupling
    - a neutral current with imbalanced left/right coupling
    - a single parameter θ_W relates coupling constant and other parameters to electromagnetic coupling constant.
- We already studied the first two
- Let's explore the third a bit more

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

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Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

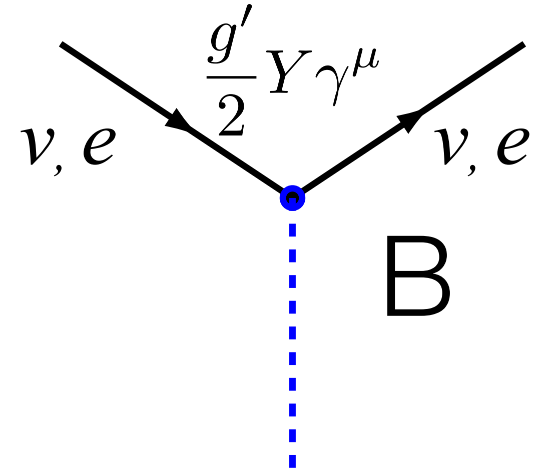
1. Introduction

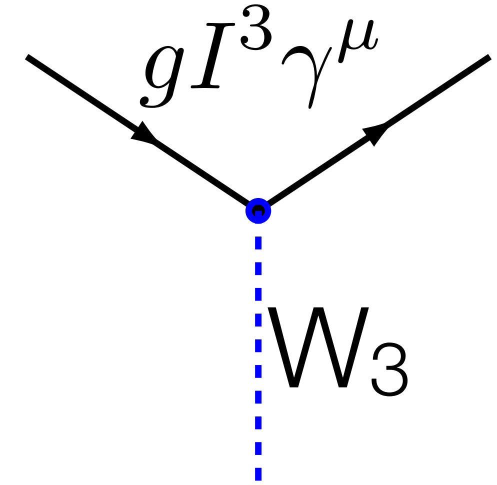
At first sight there may be little or no similarity between electromagnetic effects and the phenomena associated with weak interactions. Yet certain remarkable parallels emerge with the supposition that the weak interactions are mediated by unstable bosons. Both interactions are universal, for only a single coupling constant suffices to describe a wide class of phenomena: both interactions are generated by vectorial Yukawa couplings of spin-one fields ††. Schwinger first suggested the existence of an "isotopic" triplet of vector fields whose universal couplings would generate both the weak interactions and electromagnetism — the two oppositely charged fields mediate weak interactions and the neutral field is light [2]). A certain ambiguity beclouds the self-interactions among the three vector bosons; these can equivalently be interpreted as weak or electromagnetic couplings. The more recent accumulation of experimental evidence supporting the $\Delta I = \frac{1}{2}$ rule characterizing the non-leptonic decay modes of strange particles indicates a need for at least one additional neutral intermediary [3]).

The mass of the charged intermediaries must be greater than the K-meson mass, but the photon mass is zero — surely this is the principal stumbling block in any pursuit of the analogy between hypothetical vector mesons and photons. It is a stumbling block we must overlook. To say that the decay intermediaries

- Sometimes called "electroweak unification"
  - some people don't like this
  - usually means embedding two interactions into a larger gauge group with one coupling constant
  - "electroweak mixing" may be a better term
- But the consequences are profound
  - electromagnetic and weak interactions are inextricably linked by the fact that the photon and Z are chimeras contains bits of:
    - $SU(2)_L$ gauge group that governs the weak charge current
    - $U(1)_Y$ gauge group that also has right chiral couplings.
- The Z boson contains obvious hints of this mix
  - right chiral couplings
  - modified coupling constant
  - dependence of properties on electric charge
  - (different mass from W)

Z COUPLINGS





- the Z couplings resulted from

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

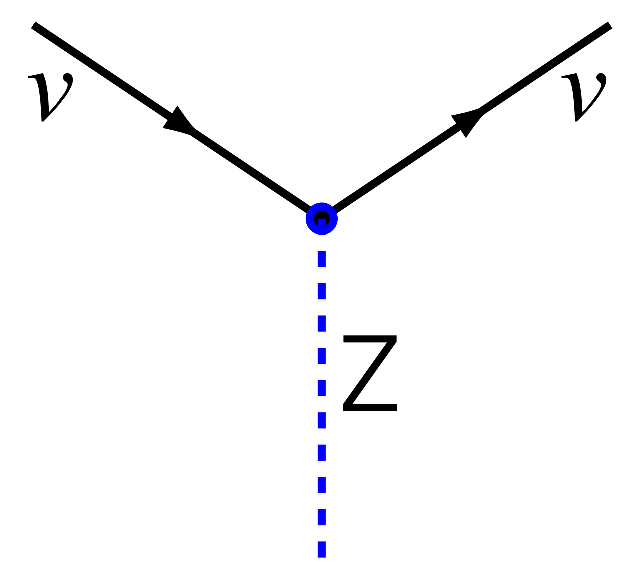
- Recovering the EM interaction as we know it introduced relations between the coupling constants and Y

$$g_Z \equiv \frac{g}{\cos\theta_W} = \frac{e}{\sin\theta_W \cos\theta_W} \qquad g' = g_Z \sin\theta_W \qquad Y = 2(Q - I^3_W)$$

- For the neutrino:

$$\bar{\nu}_L \gamma^\mu \nu_L \to -\frac{g'}{2} Y_{\nu_L} \sin\theta_W + \frac{1}{2} g \cos\theta_W \to \frac{g_Z}{2}$$

$$\bar{\nu}_R \gamma^\mu \nu_R \to -\frac{g'}{2} Y_{\nu_R} \sin\theta_W \qquad \to 0$$



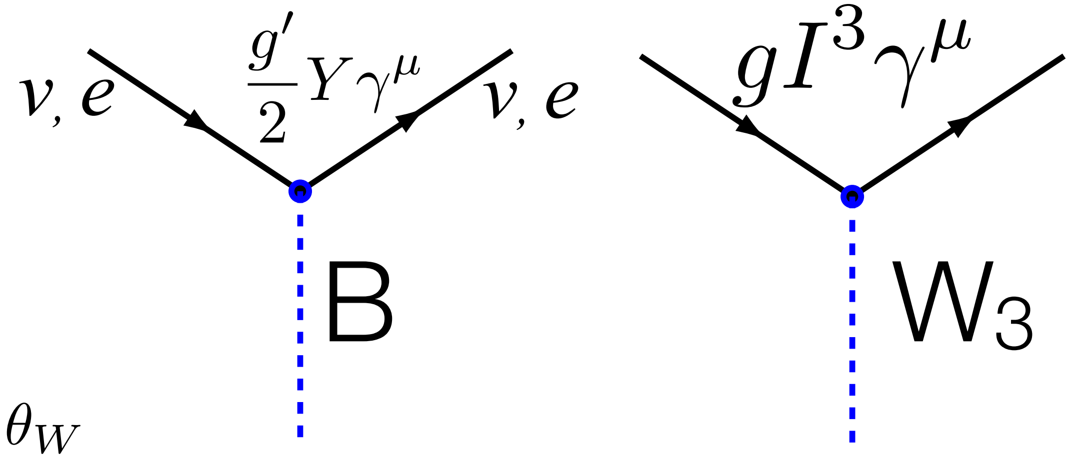
$$\frac{-ig_Z}{2}\gamma^\mu(1-\gamma^5)$$

  - which we can translate into a vertex factor
  - in this case the coupling is pure left chiral

$\boxed{g' = g_Z \sin\theta_W}$ $\boxed{g_Z \equiv \frac{g}{\cos\theta_W}} = \frac{e}{\sin\theta_W \cos\theta_W}$

GENERALLY:

$\boxed{Y = 2(Q - I_W^3)}$



- We got the following: $Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$
- for the left coupling we have:

$$\boxed{-\frac{g'}{2}\sin\theta_W} Y + \boxed{g\cos\theta_W} I_3$$

$$-\boxed{\frac{g_Z}{2}\sin^2\theta_W}\boxed{Y} + \boxed{g_Z\cos^2\theta_W} I_3$$

$$-\frac{g_Z}{2}\sin^2\theta_W \boxed{2(Q - I_3)} + g_Z\cos^2\theta_W I_3$$

$$c_L = I_3 - Q\sin^2\theta_W$$

- for the right coupling we have:

$$\boxed{-\frac{g'}{2}\sin\theta_W} Y$$

$$\boxed{-\frac{g_Z}{2}\sin^2\theta_W}\boxed{Y}$$

$$-\frac{g_Z}{2}\sin^2\theta_W \boxed{2(Q - I_3)}$$

$$c_R = -Q\sin^2\theta_W$$

- In general we can write the Z vertex in terms of:
  - left/right chiral couplings

$$c_L = I_3 - Q\sin^2\theta_W$$

$$-ig_Z\left[c_L\gamma^\mu(1-\gamma^5) + c_R\gamma^\mu(1+\gamma^5)\right]$$

$$c_R = -Q\sin^2\theta_W$$

  - vector/axial vector couplings:

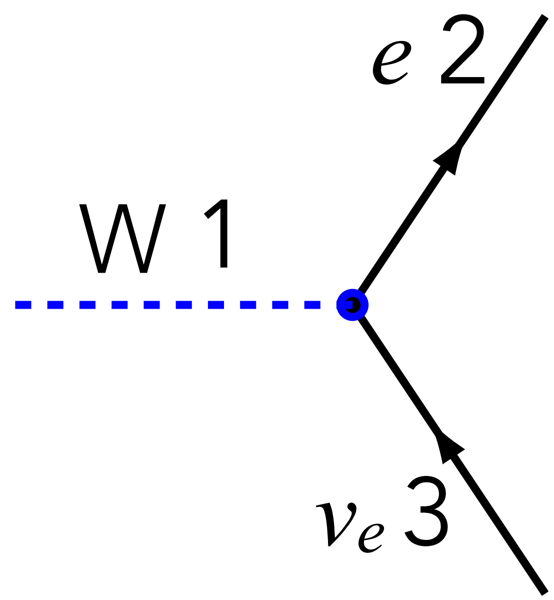
$$\frac{-ig_Z}{2}\gamma^\mu\left[c_V - c_A\gamma^5\right]$$

$$c_V = c_L + c_R \quad = I_3 - 2Q\sin^2\theta_W$$

$$c_A = c_L - c_R \quad = I_3$$

GAUGE BOSON FEYNMAN RULES

or Z → $\nu + \bar{\nu}$



- The Feynman rule for an incoming(outgoing) boson is its polarization vector:
  - ε_μ, ε_μ^*

$$\frac{-ig}{2\sqrt{2}}\ \left[\bar{u}_2\gamma^\mu(1-\gamma^5)v_3\right]\ \times (2\pi)^4(p_1-p_2-p_3)\ \epsilon_\mu(p_3)$$

$$\mathcal{M} = \frac{g}{2\sqrt{2}}\ \left[\bar{u}_2\gamma^\mu(1-\gamma^5)v_3\right]\ \epsilon_\mu(p_1)$$

- Relative to the z-axis, we can define

$$\epsilon_{+\mu} = \frac{1}{\sqrt{2}}(0,1,i,0)$$

$$\epsilon_{-\mu} = \frac{1}{\sqrt{2}}(0,-1,+i,0)$$

$$\epsilon_{L\mu} = (0,0,0,-1)$$

- for the rest of the amplitude, we know (in the massless limit):
  - e, ν_e come out with energy $M_W/2$
  - e with left helicity, $\bar{\nu}_e$ with right helicity

$$\bar{u}_2\gamma^\mu(1-\gamma^5)v_3 \Rightarrow \bar{u}_{2L}\gamma^\mu v_{3R}$$

- We evaluated this combination back in QED

$$\bar{u}_{2L}\gamma^\mu v_{3R} \Rightarrow 2E(0,-\cos\theta,-i,\sin\theta)$$

HELICITY COMBINATIONS

- Now we can consider any combinations of helicities by placing the appropriate spinors in the expression

$$\mathcal{M} = -\frac{g_e^2}{(p_1+p_2)^2}\,[\bar{u}(3)\ \overset{j_\mu}{\gamma^\mu}\ v(4)]\,[\bar{v}(2)\ \overset{j_e}{\gamma_\mu}\ u(1)]$$

- We will consider products like

$$\bar{\psi}\gamma^\mu\phi = \psi^\dagger\gamma^0\gamma^\mu\phi$$

$$\bar{\psi}\gamma^0\phi = \psi^\dagger\gamma^0\gamma^0\phi \quad = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \qquad \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\bar{\psi}\gamma^1\phi = \psi^\dagger\gamma^0\gamma^1\phi \quad = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{\psi}\gamma^2\phi = \psi^\dagger\gamma^0\gamma^2\phi \quad = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) \qquad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{\psi}\gamma^3\phi = \psi^\dagger\gamma^0\gamma^3\phi \quad = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2 \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$\mathbf{j}_\mu$

- The "↑↓" or "RL" combination

$$u_R = \sqrt{E}\begin{pmatrix} c_3 \\ s_3 \\ c_3 \\ s_3 \end{pmatrix} \qquad v_L = \sqrt{E}\begin{pmatrix} s_3 \\ -c_3 \\ s_3 \\ -c_3 \end{pmatrix}$$

$$\bar{u}_R\gamma^\mu v_L$$

$$\bar{u}_R\gamma^0 v_L = E \times (cs - cs + cs - cs) = 0$$
$$= \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4$$

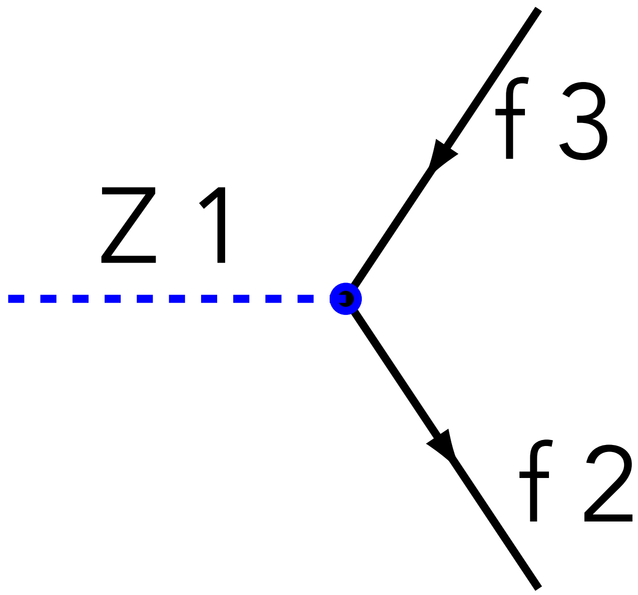
$$\bar{u}_R\gamma^1 v_L = E \times (-c^2 + s^2 - c^2 + s^2) = 2E(s^2 - c^2) = -2E\cos\theta$$
$$= \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1$$

$$\bar{u}_R\gamma^2 v_L = -iE \times (-c^2 - s^2 - c^2 - s^2) = 2ie(c^2 + s^2) = 2iE$$
$$= -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1)$$

$$\bar{u}_R\gamma^3 v_L = E \times (cs + sc + cs + sc = 4Esc = 2E\sin\theta$$
$$= \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2$$

$$\bar{u}_R\gamma^\mu v_L = 2E(0, -\cos\theta, i, \sin\theta)$$
$$\bar{u}_L\gamma^\mu v_R = 2E(0, -\cos\theta, -i, \sin\theta)$$
$$\bar{u}_R\gamma^\mu v_R = 2E(0, 0, 0, 0)$$
$$\bar{u}_L\gamma^\mu v_L = 2E(0, 0, 0, 0)$$

Z DECAYS:



$$\mathcal{M} = \frac{g_Z}{2}\bar{u}_2\gamma^\mu\left[c_V - c_A\gamma^5\right]v_3\epsilon_{1\mu}$$

- As usual, we will consider helicity/chiral states in the massless limit.
- Using the relation $\gamma^\mu(c_V - c_A\gamma^5)\frac{1 \pm \gamma_5}{2} \to \frac{1 \mp \gamma^5}{2}\gamma^\mu(c_V - c_A\gamma^5)$
- we can show:

$$\bar{u}_{2L}\gamma^\mu\left[c_V - c_A\gamma^5\right]v_{3L} = \bar{u}_{2R}\gamma^\mu\left[c_V - c_A\gamma^5\right]v_{3R} = 0$$

- so that we need only consider

$$\bar{u}_{2L}\gamma^\mu\left[c_V - c_A\gamma^5\right]v_{3R} \qquad \bar{u}_{2R}\gamma^\mu\left[c_V - c_A\gamma^5\right]v_{3L}$$

- to consider this in terms of c_L and c_R

$$c_V - c_A\gamma^5 \to (c_L + c_R) - (c_L - c_R)\gamma^5$$

- so that $c_L\bar{u}_{2L}\gamma^\mu\left[1 - \gamma^5\right]v_{3R} \qquad c_R\bar{u}_{2R}\gamma^\mu\left[1 + \gamma^5\right]v_{3L}$

Z DECAYS CONTINUED

$$\mathcal{M} = \frac{g_Z}{2}\bar{u}_2\gamma^\mu\left[c_V - c_A\gamma^5\right]v_3\epsilon_{1\mu}$$

- Use the previously calculated helicity combinations:

$$\frac{1}{2}\bar{u}_{2L}\gamma^\mu\left[1-\gamma^5\right]v_{3R} \to \bar{u}_{2L}\gamma^\mu v_{3R} = 2E(0, -\cos\theta, -i, \sin\theta)$$

$$\frac{1}{2}\bar{u}_{2R}\gamma^\mu\left[1+\gamma^5\right]v_{3L} \to \bar{u}_{2R}\gamma^\mu v_{3L} = 2E(0, -\cos\theta, i, \sin\theta)$$

  - where E = $m_Z/2$

- contract this with our Z polarization vectors

$$\epsilon_{+\mu} = \frac{1}{\sqrt{2}}(0,1,i,0) \quad \epsilon_{-\mu} = \frac{1}{\sqrt{2}}(0,-1,+i,0) \quad \epsilon_{L\mu} = (0,0,0,-1)$$

- to get six Z polarization/outgoing helicity combinations
- stick this with the other factors

$$g_z m_Z \times (c_L/c_R)$$

	+	-	L
LR	(1-cos θ)/√2	(1+cosθ)/√2	-sin θ
RL	(-1-cos θ)/√2	(-1+cosθ)/√2	-sin θ

FINAL STEPS:

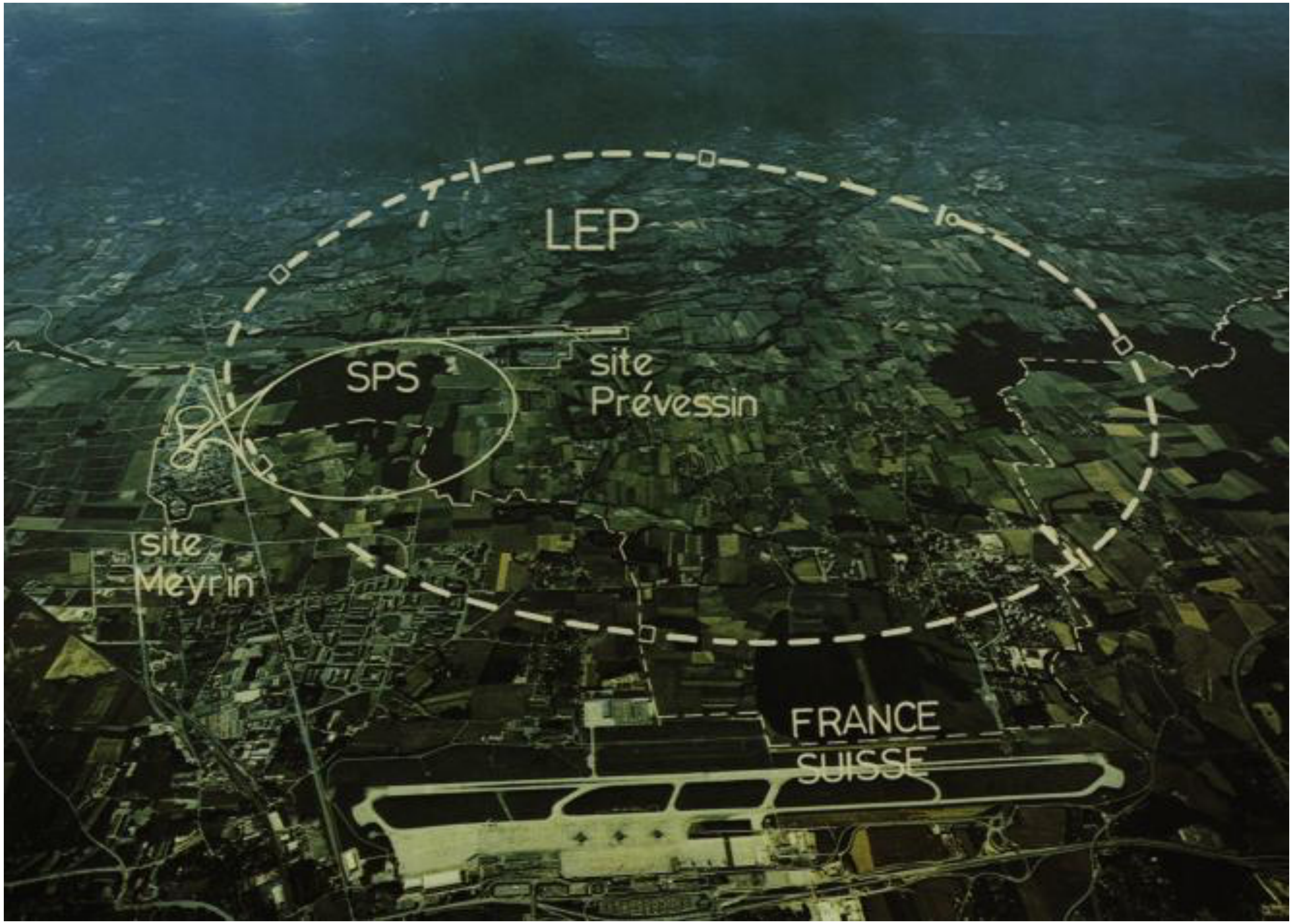
- We can square all the matrix elements and add them together to get the spin-summed amplitude

$$\sum |\mathcal{M}|^2 = 2g_Z^2 m_Z^2 (c_L^2 + c_R^2) \rightarrow g_Z^2 m_Z^2 (c_V^2 + c_A^2)$$

- Divide by the initial polarization states to average
- Putting it into our decay phase space formula

$$\Gamma = \frac{|\mathbf{p}|}{32\pi^2 m_Z^2} \int d\Omega |\mathcal{M}|^2 = \frac{g_Z^2 m_Z}{48\pi} (c_V^2 + c_A^2)$$

	C_V	C_A	$C_V^2+C_A^2$	REL.	FRAC.
ν_e, ν_μ, ν_τ	1/2	1/2	0.50	1.5	0.23
e, μ, t	$-1/2 + 2\sin^2\theta_W$	-1/2	0.251	0.753	0.12
u, c, t	$+1/2 - 4/3\sin^2\theta_W$	1/2	0.286	1.72	0.26
d, s, b	$-1/2 + 2/3\sin^2\theta_W$	-1/2	0.373	2.57	0.39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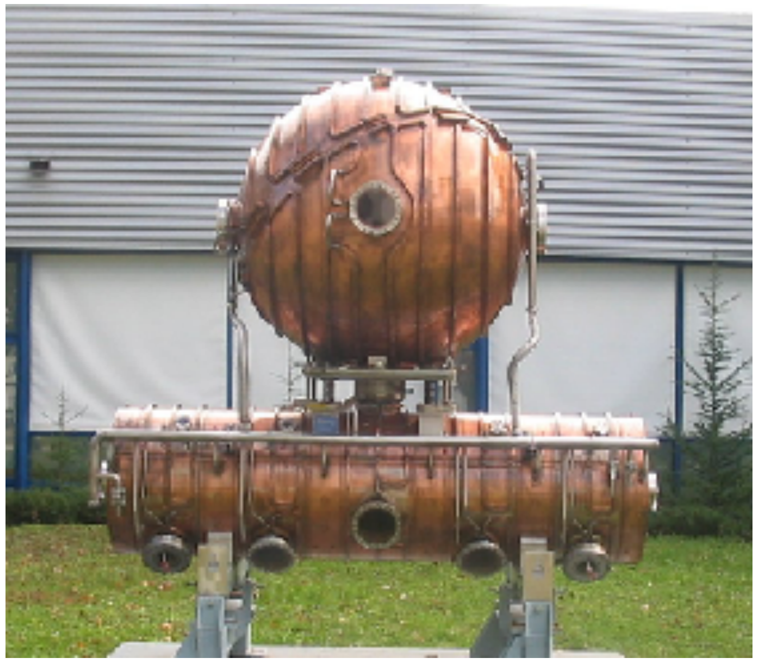
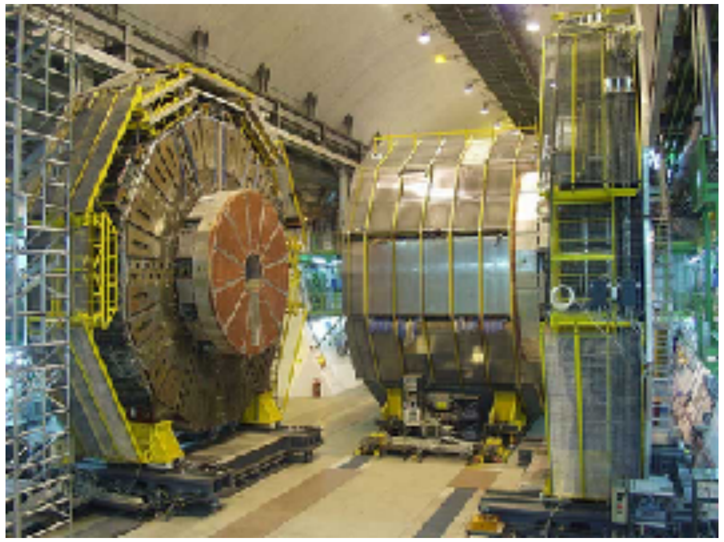
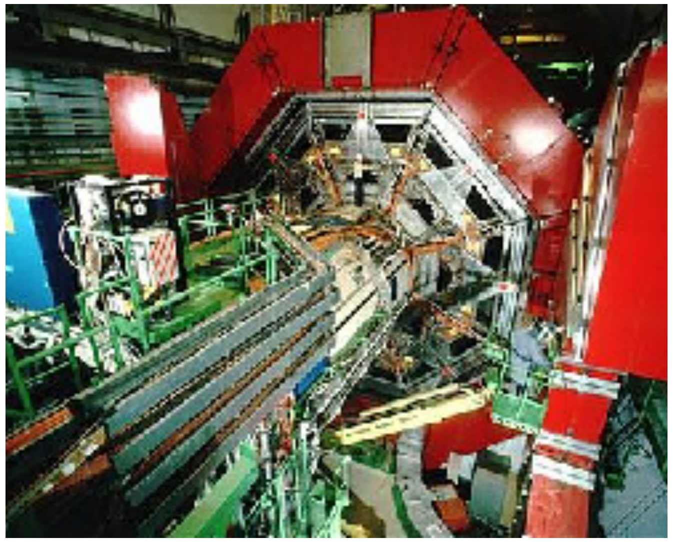
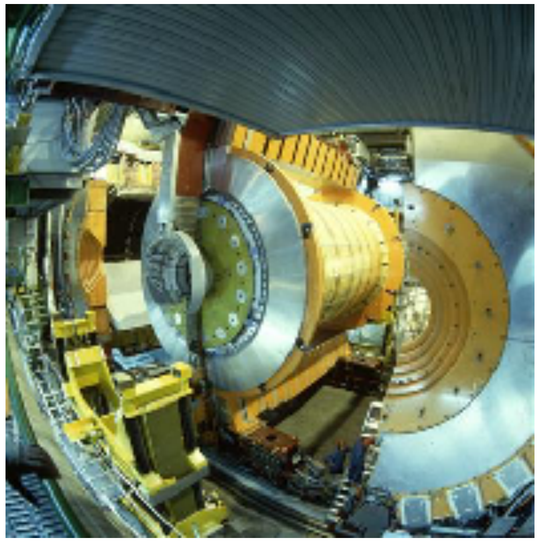
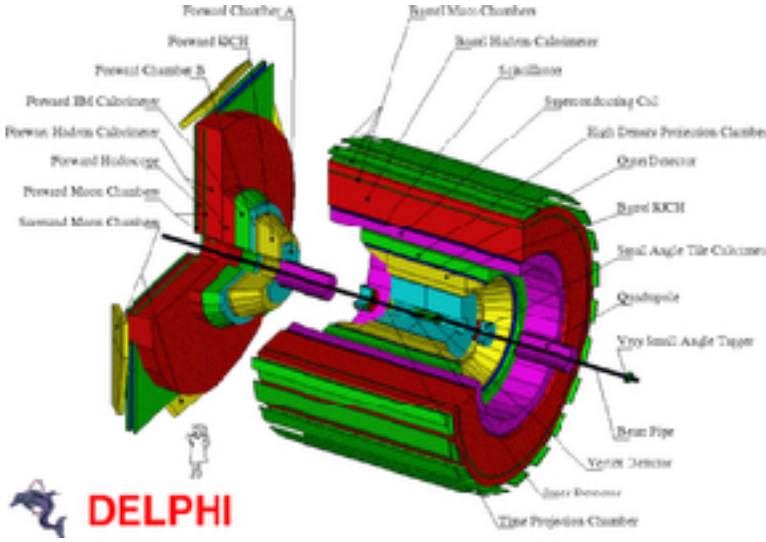


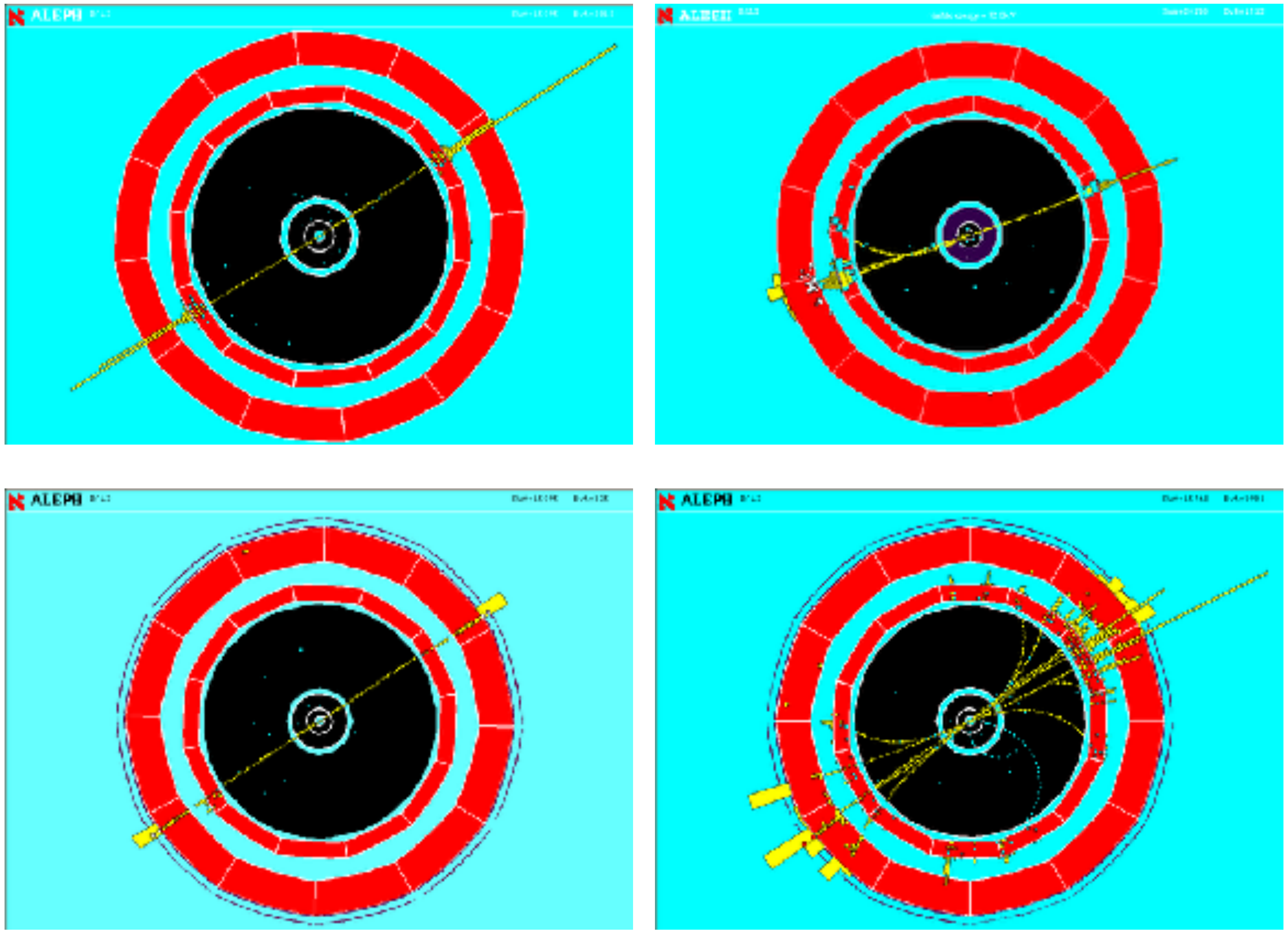


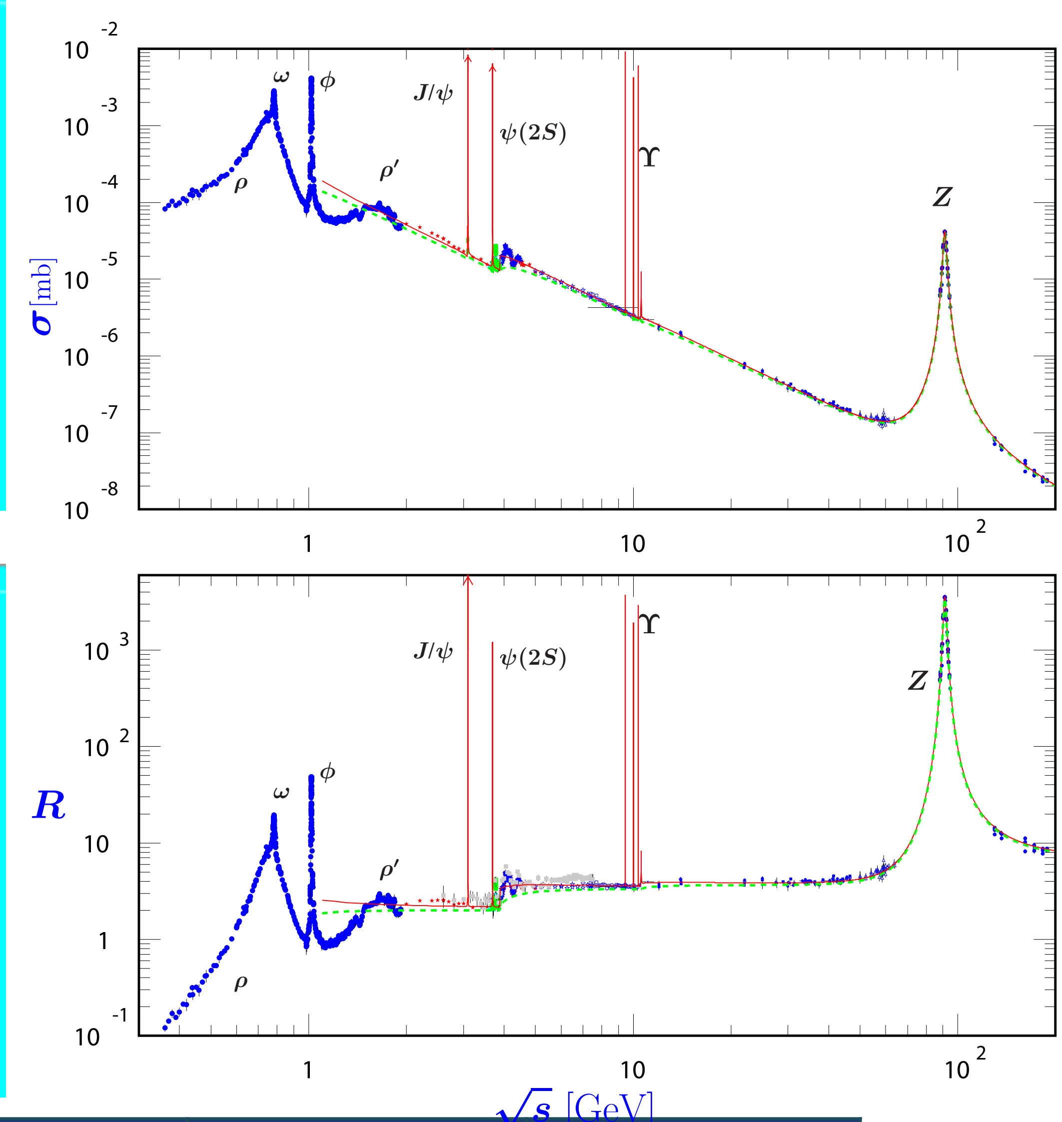





Forward Chamber A
Forward RICH
Forward Chamber B
Forward EM Calorimeter
Forward Hadron Calorimeter
Forward Hodoscope
Forward Muon Chambers
Surround Muon Chambers
Barrel Muon Chambers
Barrel Hadron Calorimeter
Scintillators
Superconducting Coil
High Density Projection Chamber
Outer Detector
Barrel RICH
Small Angle Tile Calorimeter
Quadrupole
Very Small Angle Tagger
Beam Pipe
Vertex Detector
Time Projection Chamber
DELPHI

MEAUREMENTS AT LEP





	C_V	C_A	$C_V^2+C_A^2$	REL.	FRAC.	MEAS.
ν_e, ν_μ, ν_τ	1/2	+1/2	0.50	1.50	0.20	0.20
e, μ, t	+ 2 $\sin^2\theta_W$ -1/2	-1/2	0.251	0.753	0.10	0.10
u, c, t	- 4/3 $\sin^2\theta_W$ + 1/2	+1/2	0.286	1.716	0.23	0.23
d, s, b	+ 2/3 $\sin^2\theta_W$ - 1/2	-1/2	0.373	3.357	0.46	0.47

SUMMARY:

- Electroweak mixing makes predictions about c_V, c_A (alternatively c_L, c_R) couplings of the Z boson that can be tested
  - different particle species have different couplings
- Please read chapters 17.1-17.3