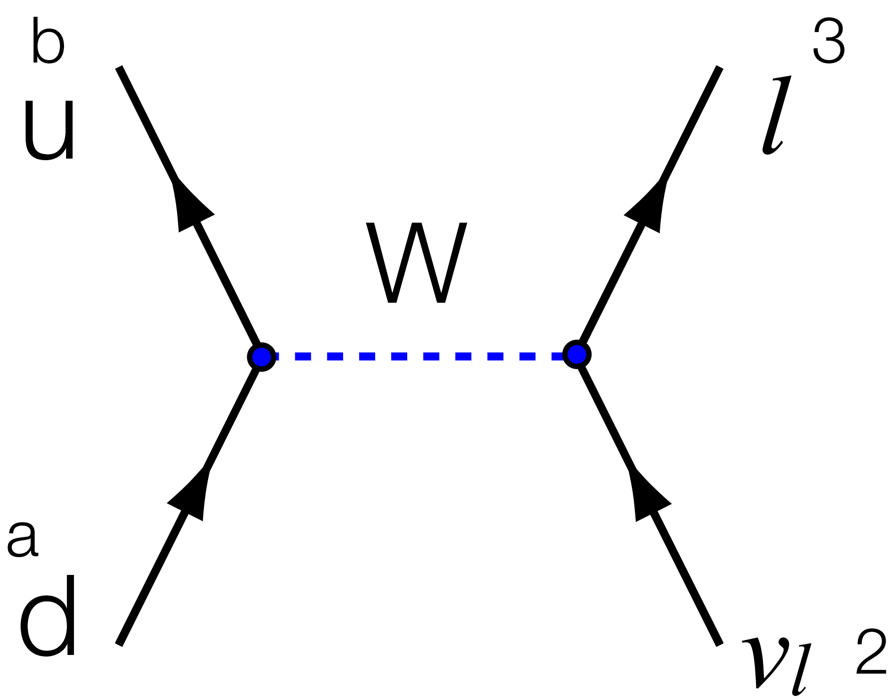
PHY489/1489

LECTURE 15:
WEAK INTERACTIONS CONTINUED

ODD FEATURES OF WEAK INTERACTION

- Heavy gauge bosons
- “Chirality” associated with γ^5
  - projects helicity states in the limit of massless particles
  - revisit today
- Flavor change
  - unitarity of “mixing”
  - introduce today

EXAMPLE: PION DECAY



- Lepton fermion leg

$$[\bar{u}_3 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) v_2]$$

- Quark Fermion leg

$$[\bar{v}_b \frac{-ig_w}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) u_a]$$

$$[\bar{v}_b \frac{-ig_w}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) u_a] \Rightarrow F^\nu = f_\pi p^\nu$$

- Propagator

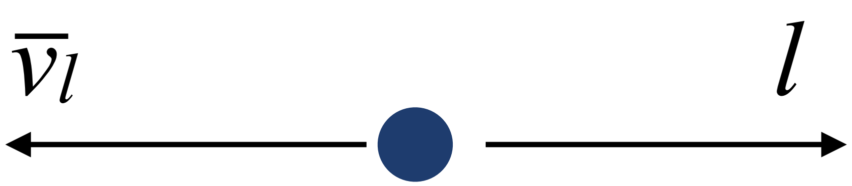
$$\int \frac{1}{(2\pi)^4} d^4 q \qquad \frac{ig_{\mu\nu}}{M_W^2 c^2}$$

$$\mathcal{M} = \frac{g_W^2}{8 M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) v_2\right] f_\pi p_\mu$$

CARRYING OUT THE CALCULATION

$$\mathcal{M} = \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\right] f_\pi p_\mu$$

$$\bar{u}_3\gamma^\mu(1-\gamma^5)v_2$$



- As before, we consider the various helicity combinations . . .
  - assume decay occurs along the z-axis
- but some interesting things happen

$$1-\gamma^5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$$v_{\uparrow 2} = \sqrt{E}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \qquad v_{\downarrow 2} = \sqrt{E}\begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}$$

"Chiral" projection operator

BACK TO MATRIX ELEMENT

$$\mathcal{M} = \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\right] f_\pi p_\mu$$

$$u_{\uparrow 3} = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{p}{E+m} \\ 0 \end{pmatrix}$$

- if pion is at rest, only p_0 matters(= m_π)
  - ignore $\mu \neq 0$
  - $\bar{u}\,\gamma^\mu = u^\dagger\gamma^0\gamma^\mu \rightarrow u^\dagger$

$$u_{\downarrow 3} = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{p}{E+m} \end{pmatrix}$$

- we only need to consider $v_{\uparrow 2}$

$$u^\dagger_{\uparrow 3} = \sqrt{E_3+m_3}\begin{pmatrix} 1, & 0, & \frac{p}{E_3+m_3}, & 0 \end{pmatrix}$$

$$u^\dagger_{\downarrow 3} = \sqrt{E_3+m_3}\begin{pmatrix} 0, & 1, & 0, & -\frac{p}{E_3+m_3} \end{pmatrix}$$

$$(1-\gamma^5)v_{\uparrow 2} = 2\sqrt{E_2}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\bar{u}_3\gamma^\mu(1-\gamma^5)f_\pi p_0 \rightarrow 2\sqrt{E_2}\sqrt{E_3+m_3}\left(1-\frac{p}{E_3+m_3}\right)f_\pi m_\pi$$

$$\mathcal{M} = \frac{g_W^2}{4M_W^2}f_\pi m_\pi\sqrt{E_2(E_3+m_3)}\left(1-\frac{p}{E_3+m_3}\right)$$

SOME KINEMATICS

$$E_3 = \frac{m_\pi^2 + m_\ell^2}{2m_\pi}$$

$$\mathcal{M} = \frac{g_W^2}{4M_W^2} f_\pi m_\pi \sqrt{E_2(E_3 + m_3)} \left(1 - \frac{p}{E_3 + m_3}\right)$$

$$p_2 = E_2 = \frac{m_\pi^2 - m_\ell^2}{2m_\pi} = p_3$$

$$\mathcal{M} = \frac{g_W^2}{4M_W^2} f_\pi m_\pi \sqrt{\frac{m_\pi^2 - m_\ell^2}{2m_\pi}\frac{m_\pi^2 + m_\ell^2 + 2m_\pi m_\ell}{2m_\pi}} \left(\frac{2m_\pi}{m_\pi^2 + m_\ell^2 + 2m_\pi m_\ell}\right)\left(\frac{m_\pi^2 + m_\ell^2 + 2m_\pi m_\ell - m_\pi^2 + m_\ell^2}{2m_\pi}\right)$$

$$\frac{1}{2m_\pi}(m_\pi + m_\ell)\sqrt{m_\pi^2 - m_\ell^2}$$

$$2 \times \frac{m_\ell^2 + m_\pi m_\ell}{(m_\pi + m_\ell)^2}$$

$$\frac{m_\ell}{m_\pi}\sqrt{m_\pi^2 - m_\ell^2}$$

$$\mathcal{M} = \frac{g_W^2}{4M_W^2} f_\pi m_\ell \sqrt{m_\pi^2 - m_\ell^2}$$

DECAY RATE

$$\mathcal{M} = \frac{g_W^2}{4M_W^2} f_\pi m_\ell \sqrt{m_\pi^2 - m_\ell^2}$$

$$p_2 = E_2 = \frac{m_\pi^2 - m_\ell^2}{2m_\pi} = p_3$$

$$\Gamma = \frac{|\mathbf{p}^*|}{32\pi^2 M^2} \int |\mathcal{M}|^2 \, d\Omega = \frac{|\mathbf{p}^*|}{8\pi m_\pi^2} |\mathcal{M}|^2$$

$$= \frac{g_W^4}{8\pi m_\pi^2 \times 16 m_W^4} f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2) \frac{m_\pi^2 - m_\ell^2}{2m_\pi}$$

$$= \frac{g_W^4}{256\pi m_\pi^3 m_W^4} f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)^2$$

RATIOS:

- Consider the two decays:
  - $\pi^- \rightarrow e^- + \bar{\nu}_e$
  - $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

$$\Gamma = \frac{g_W^4}{256 M_W^4 m_\pi^3} f_\pi^2 m_3^2 (m_\pi^2 - m_3^2)^2$$

$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2 (m_\pi^2 - m_e^2)^2}{m_\mu^2 (m_\pi^2 - m_\mu^2)^2}$$

- Using the masses
  - m_π = 139.57 MeV
  - m_μ = 105.65 MeV
  - m_e = 0.511 MeV

$$\frac{\Gamma_e}{\Gamma_\mu} = 1.28 \times 10^{-4}$$

PIENU

$$\frac{\Gamma_e}{\Gamma_\mu} = 1.2344 \pm 0.0023(\mathrm{stat}) \pm 0.0019(\mathrm{syst.}) \times 10^{-4}$$

Improved Measurement of the $\pi \to e\nu$ Branching Ratio

A. Aguilar-Arevalo,[1] M. Aoki,[2] M. Blecher,[3] D. I. Britton,[4] D. A. Bryman,[5] D. vom Bruch,[5] S. Chen,[6] J. Comfort,[7] M. Ding,[6] L. Doria,[8] S. Cuen-Rochin,[5] P. Gumplinger,[8] A. Hussein,[9] Y. Igarashi,[10] S. Ito,[2] S. H. Kettell,[11] L. Kurchaninov,[8] L. S. Littenberg,[11] C. Malbrunot,[5,*] R. E. Mischke,[8] T. Numao,[8] D. Protopopescu,[4] A. Sher,[8] T. Sullivan,[5] D. Vavilov,[8] and K. Yamada[2]

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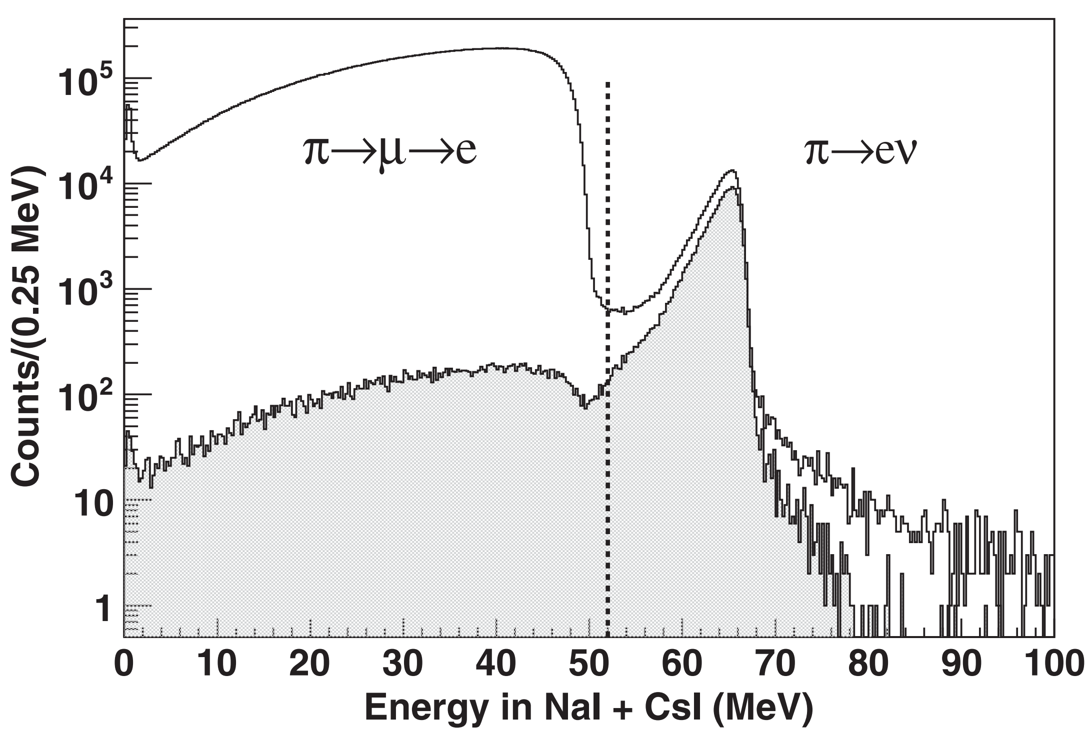
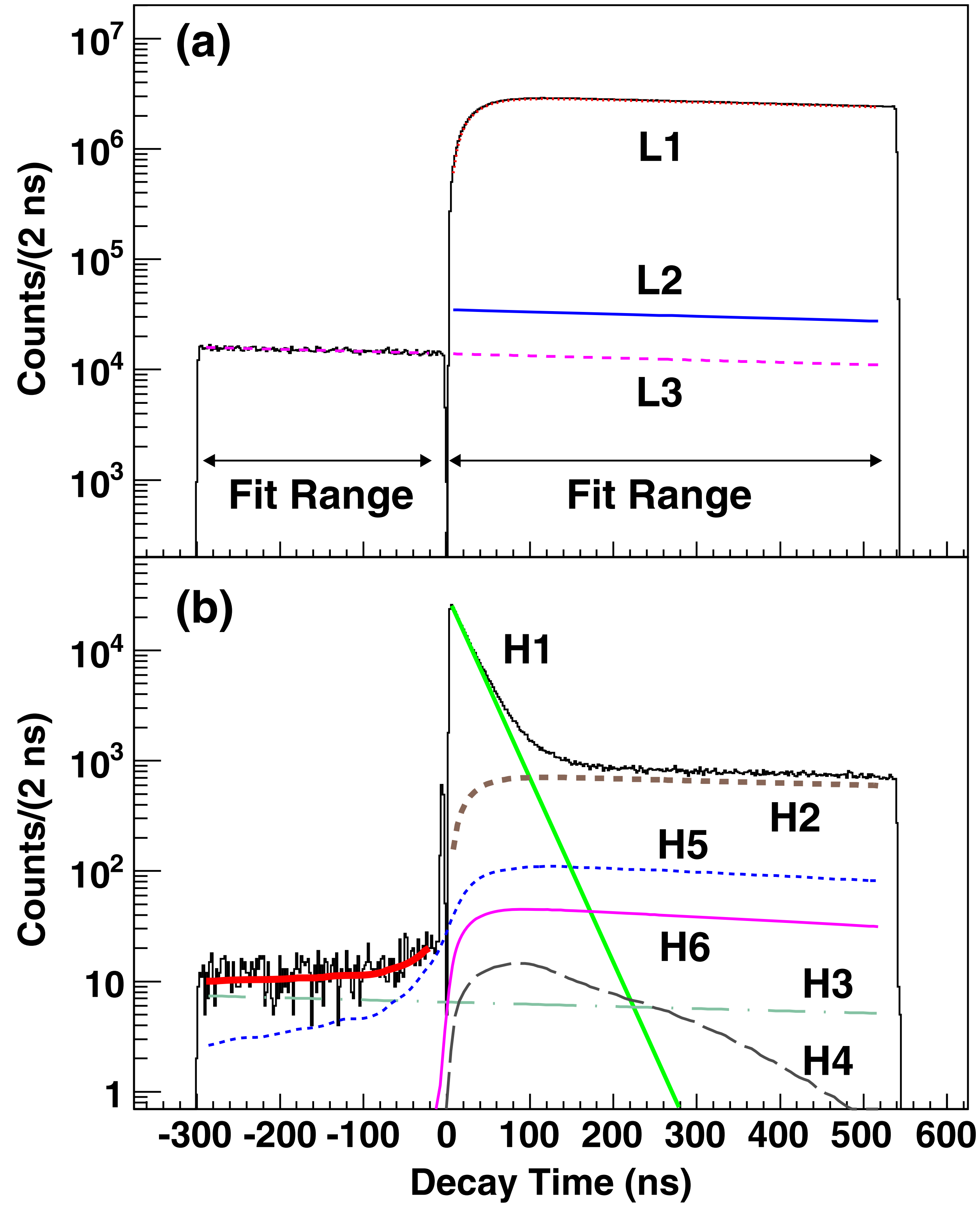
A new measurement of the branching ratio $R_{e/\mu} = \Gamma(\pi^+ \to e^+\nu + \pi^+ \to e^+\nu\gamma)/\Gamma(\pi^+ \to \mu^+\nu + \pi^+ \to \mu^+\nu\gamma)$ resulted in $R^{\mathrm{exp}}_{e/\mu} = [1.2344 \pm 0.0023(\mathrm{stat}) \pm 0.0019(\mathrm{syst})] \times 10^{-4}$. This is in agreement with the standard model prediction and improves the test of electron-muon universality to the level of 0.1%.





FORM OF INTERACTION

- We said this is a “vector - axial vector” interaction

$$\mathcal{M} = \frac{g_W^2}{8M_W^2 c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\right] f_\pi p_\mu$$

- What would a:
  - vector interaction look like?
  - scalar interaction?
- The form of the interaction affects the decay rate
  - if we assume a scalar interaction we would get

$$\frac{\Gamma_e}{\Gamma_\mu} =\sim 5$$

HELICITY SUPPRESSION

- For **massless** particles, chiral states are also helicity states, i.e.
  - we concluded $\frac{1}{2}(1-\gamma^5)v_2 \Rightarrow v_{2\uparrow}$
  - but we also concluded

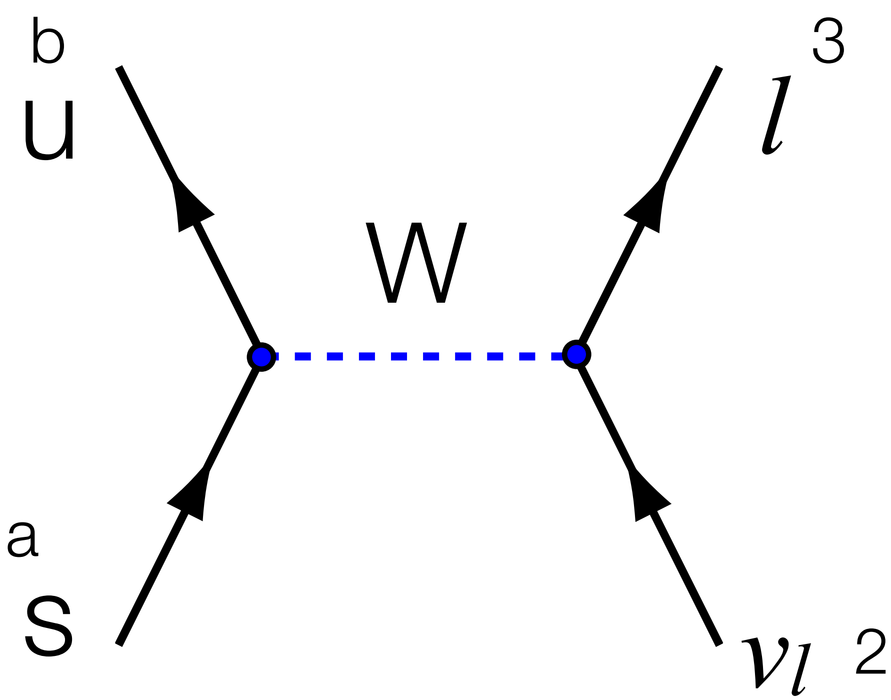
$$\bar{u}_3\frac{1}{2}\gamma^\mu(1-\gamma^5) \ \Rightarrow \bar{u}_3\frac{1}{2}(1+\gamma^5)\gamma^\mu \Rightarrow u_3^\dagger\gamma^0\frac{1}{2}(1+\gamma^5)\gamma^\mu \Rightarrow \left(\frac{1}{2}(1-\gamma^5)u_3\right)^\dagger\gamma^0\gamma^\mu \Rightarrow \bar{u}_{3L}\gamma^\mu$$

- i.e. antineutrino is right helicity, muon is left helicity
  - impossible to conserve angular momentum so the decay will not happen!
  - this is apparent also from the matrix element

$$\mathcal{M} = \frac{g_W^2 f_\pi m_\pi}{4M_W^2}\sqrt{E_3+m_3}\sqrt{|\mathbf{p_3}|}\left(1-\frac{|\mathbf{p_3}|}{E_3+m_3}\right)$$

- The decay only happens to the extent that the chiral projection of one of the particles departs from a helicity state due to its mass.
  - the closer the particles are to the massless limit, the more suppression

EXAMPLE: KAON DECAY



- Lepton fermion leg

$$[\bar{u}_3 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) v_2]$$

- Quark Fermion leg

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- Propagator

$$\int \frac{1}{(2\pi)^4} d^4 q \qquad \frac{i g_{\mu\nu}}{M_W^2 c^2}$$

$$\mathcal{M} = \frac{g_W^2}{8 M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) v_2 \right] f_K p_\mu$$

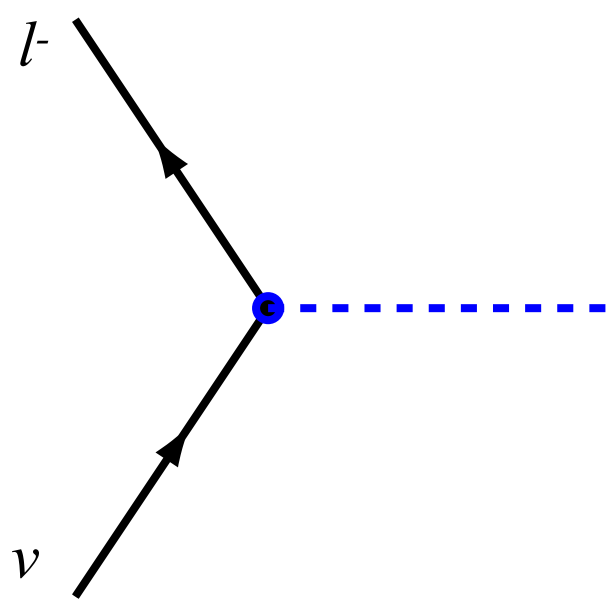
$m_\pi \sim 140$ MeV

$m_K \sim 494$ MeV

$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2 (m_K^2 - m_e^2)^2}{m_\mu^2 (m_K^2 - m_\mu^2)^2} = 2.57 \times 10^{-5}$$

$$\frac{\mathcal{B}[K \to e + \nu_e]}{\mathcal{B}[K \to \mu + \nu_\mu]} = \frac{1.582 \times 10^{-5}}{0.6356} = 2.49 \times 10^{-5}$$

WEAK CC FOR LEPTONS

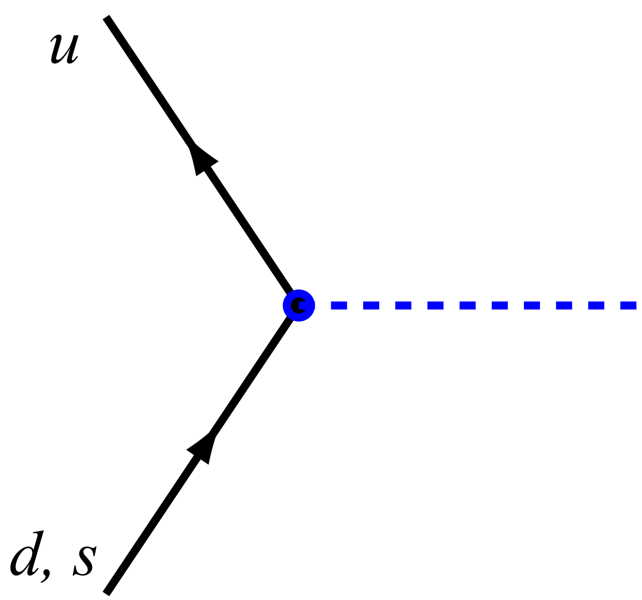


$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

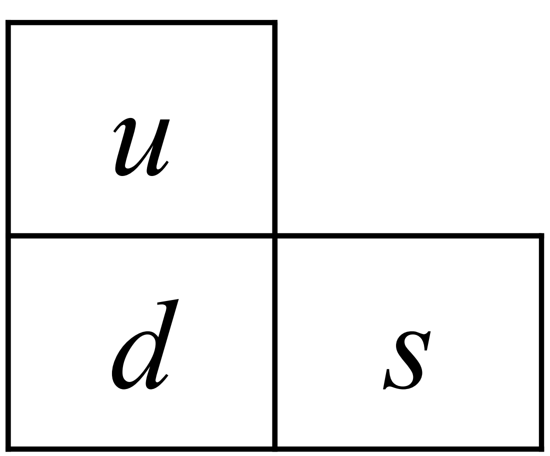
ν_e	ν_μ	ν_τ
e	μ	τ

- Reasonably simple
    - charged lepton connects to corresponding neutrino

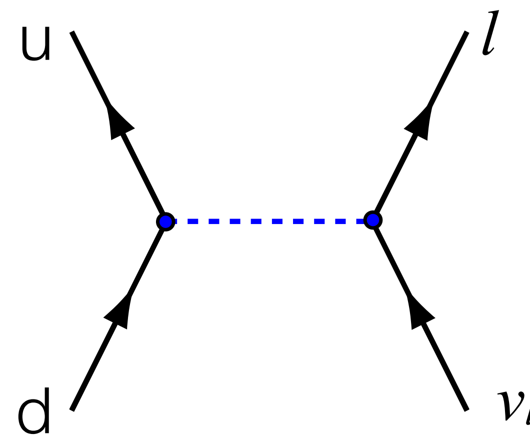
WEAK INTERACTION OF QUARKS

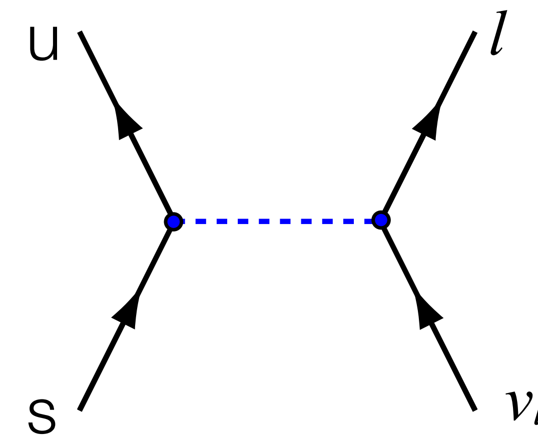


$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



- Step back to 1960s when we "knew" of three quarks
- Noticed that decays of "strange' particles was much slower than expected
- We can compare pion/kaon decays



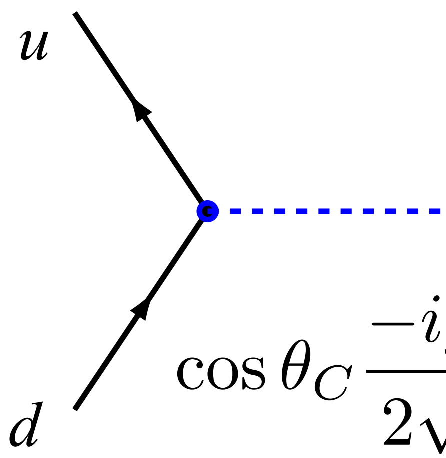


m_π = 139.57 MeV
m_K = 493.68 MeV

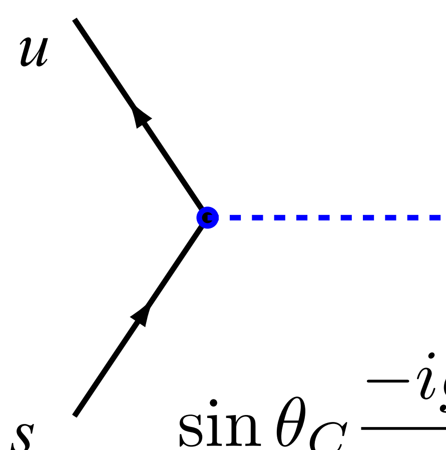
$$\Gamma = \frac{f_\pi^2}{\pi\hbar m_\pi^3}\left(\frac{g_w}{4M_W}\right)^4 m_l^2(m_\pi^2 - m_l^2)^2$$

$$\frac{\Gamma(K^- \to \mu^- + \nu_\mu)}{\Gamma(\pi^- \to \mu^- + \nu_\mu)} = \left(\frac{m_\pi}{m_K}\right)^3 \left(\frac{m_K^2 - m_\mu^2}{m_\pi^2 - m_\mu^2}\right)^2 \sim 18$$

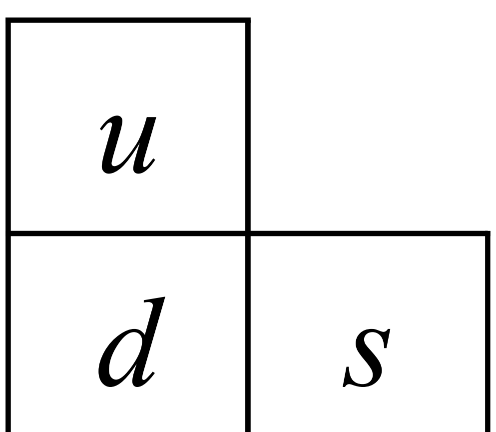
CABIBBO ANGLE



$$\cos\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

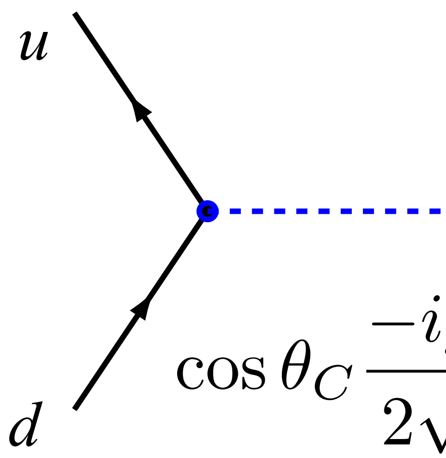


$$\sin\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

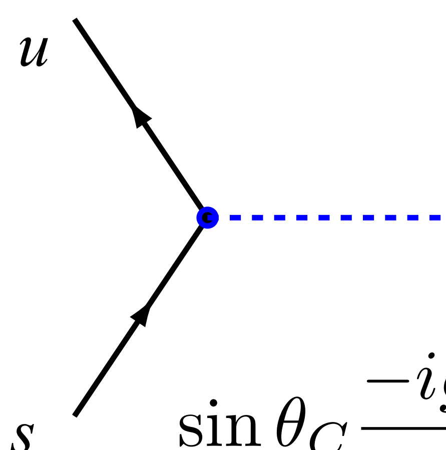


- Experiments find that this ratio is more like 1.3, indicating that something is wrong with our picture
- Cabbibo postulated that:
  - d ↔ u transitions scaled by factor of cos θ_c
  - s ↔ u transitions scaled by factor of sin θ_c
  - experimentally θ_c ~ 13°
- Cabibbo was able to relate a host of decay rates for strange and non-strange particles with a single parameter
  - "Cabibbo favored": decays with cos θ_c factor
  - "Cabibbo suppressed": decays with sin θ_c factor

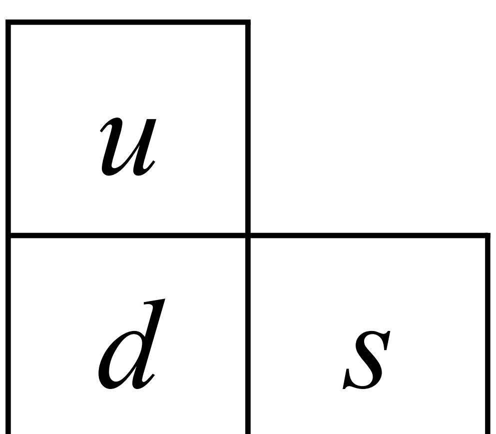
STILL A PROBLEM

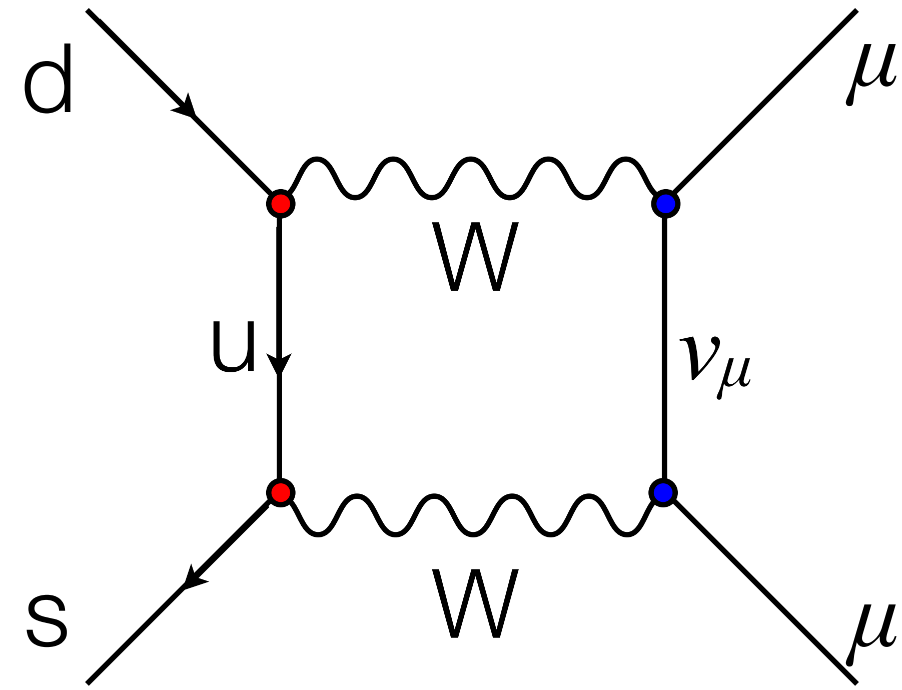


$$\cos\theta_C \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5)$$



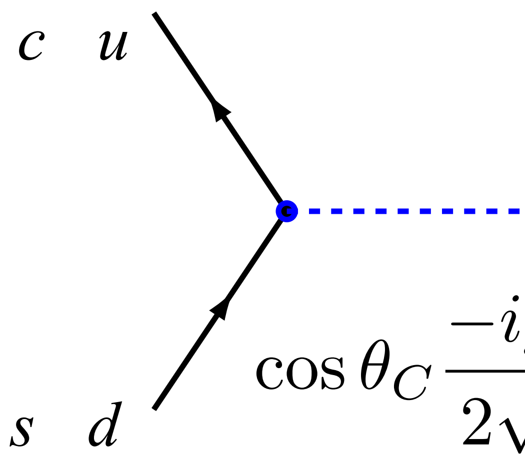
$$\sin\theta_C \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5)$$



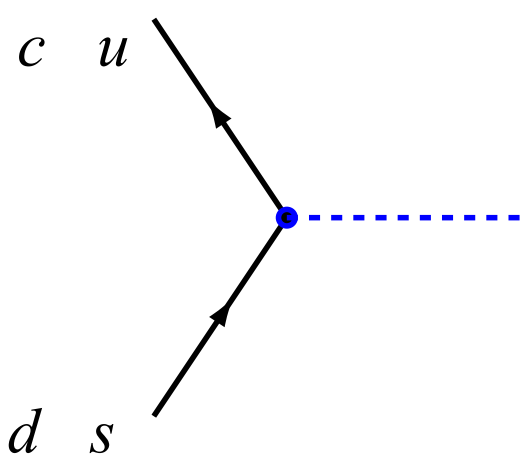


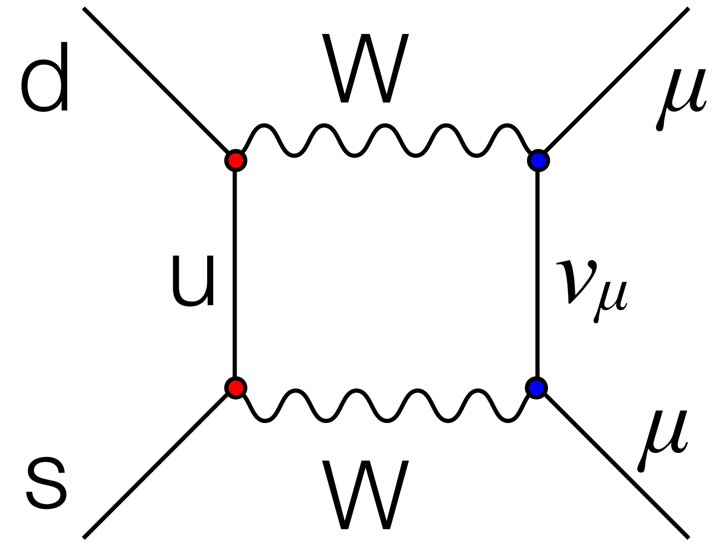
- Above process should happen as $K^0 \rightarrow \mu^+ + \mu^-$
  - but its branching fraction ($<10^{-8}$) is much lower than expected, even after considering Cabibbo factors

GIM MECHANISM

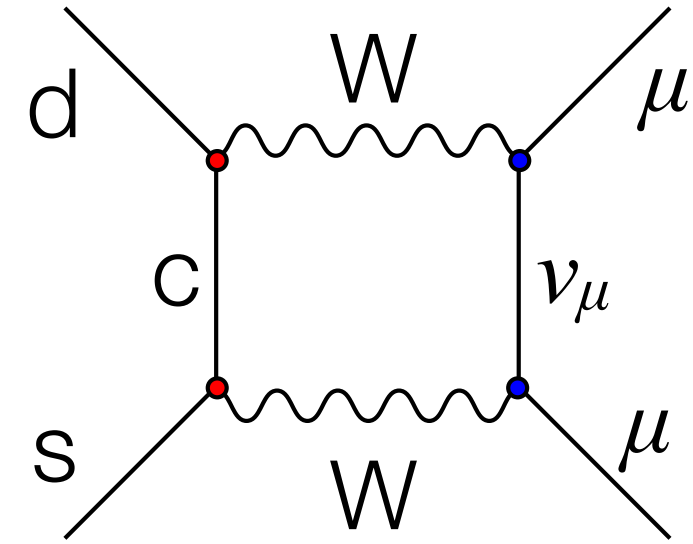


$$\cos\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$





$A_1 \sim \sin\theta_C \cos\theta_C$



$A_2 \sim -\sin\theta_C \cos\theta_C$

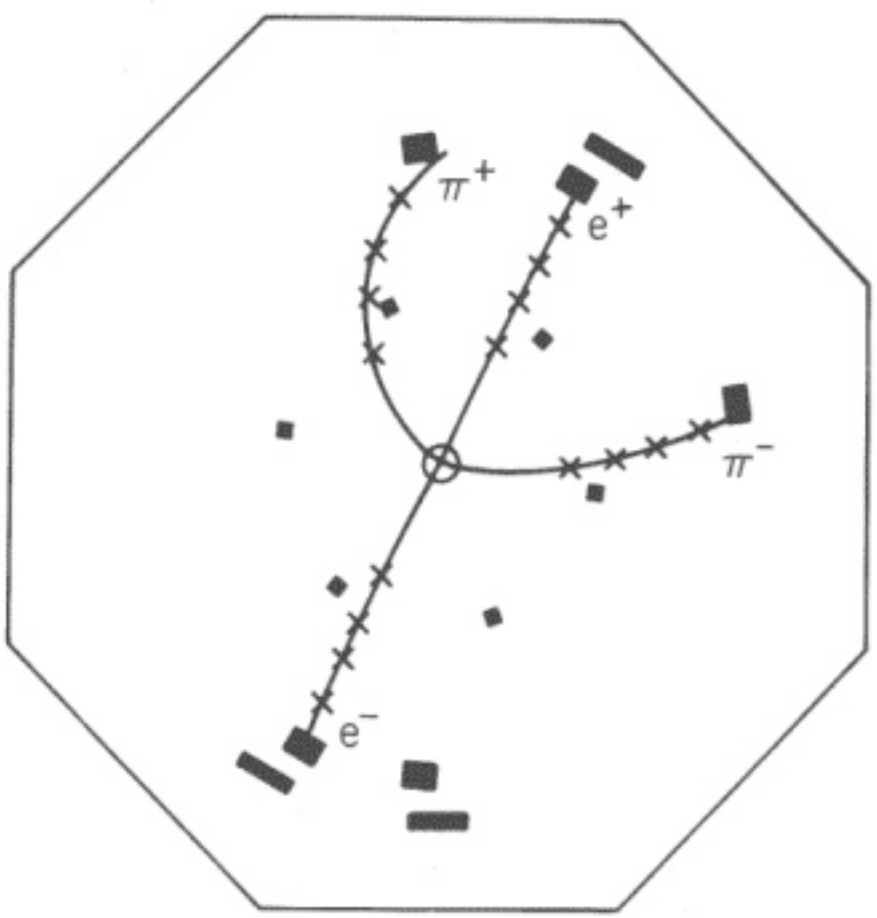
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

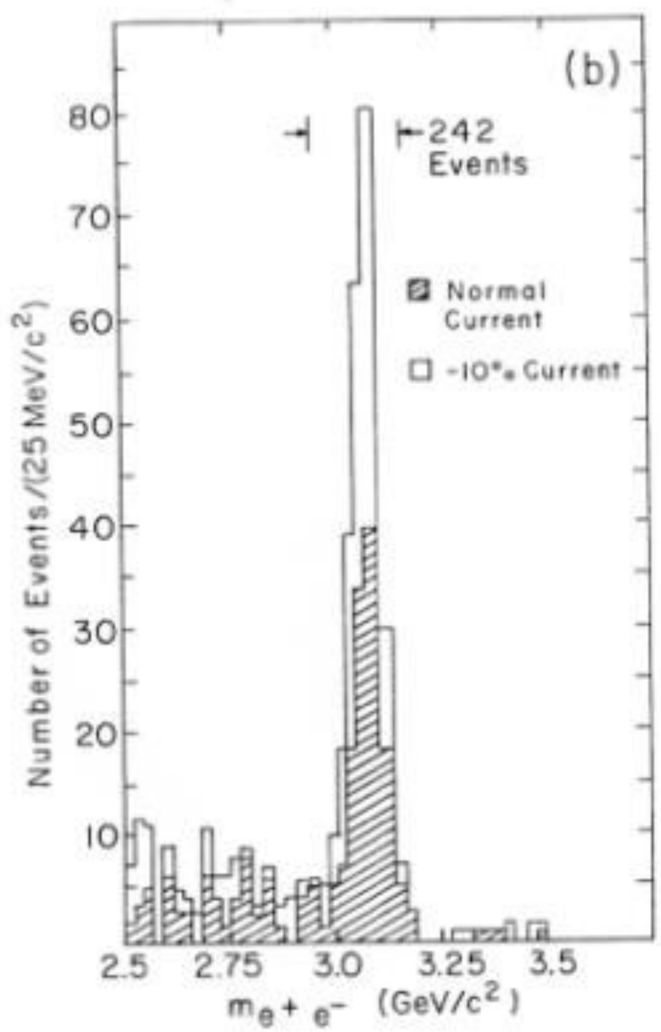
u	c
d	s

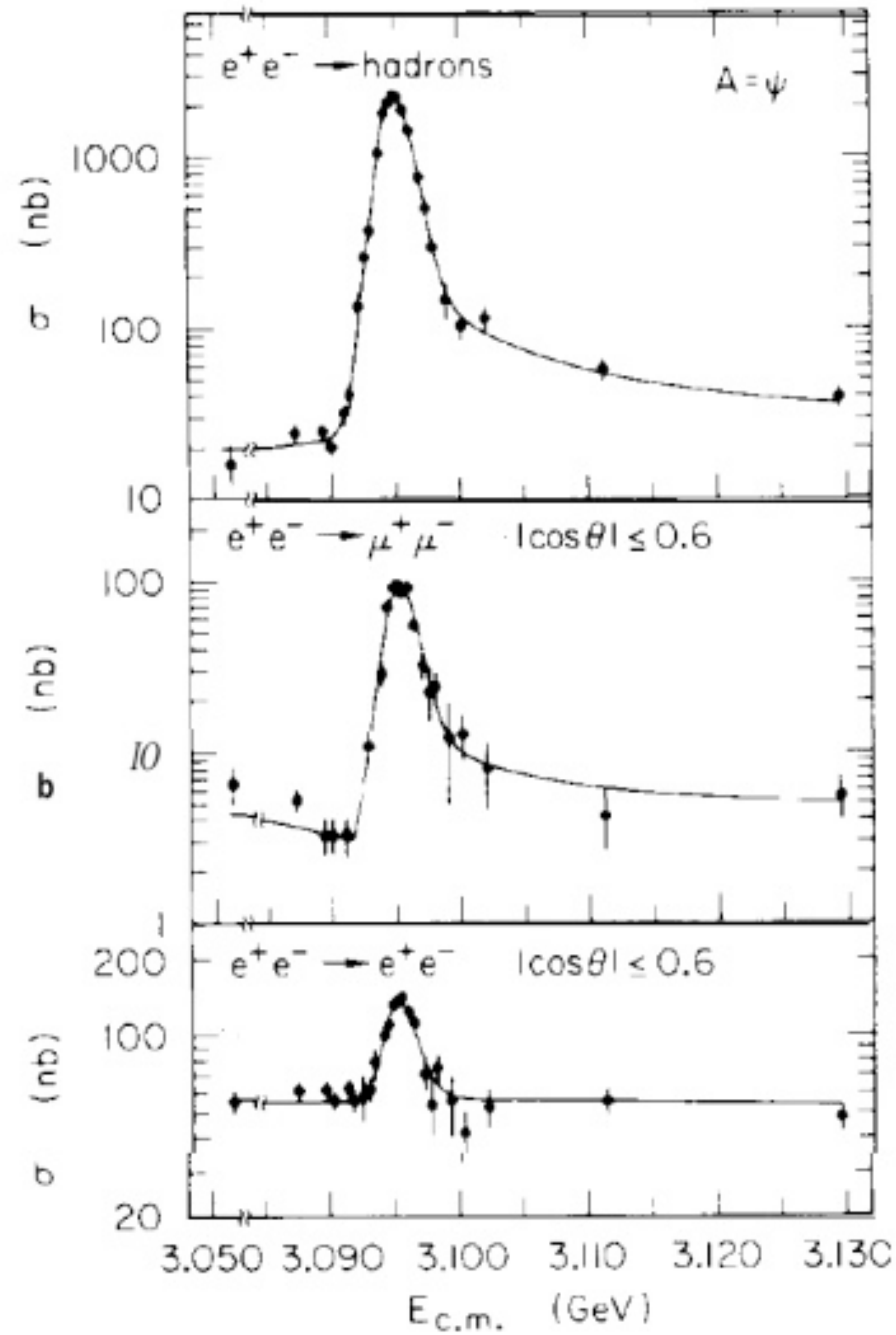
- Introduce a fourth quark
  - “charm” that cancels contribution from u quark
- “Mixing”
  - mass eigenstates (conventional name for quarks) are linear combination of “flavor eigenstates” as indicated above
  - d′ is defined as state that couples to u via the W boson
  - s′ is defined as state that couples to c via W boson

THE NOVEMBER REVOLUTION

- 1974: Discovery of the J/ψ particle
  - evidence of a bound state with a heavy quark
  - Brought together many elements of what we call the standard model
    - quarks, gauge theory, etc



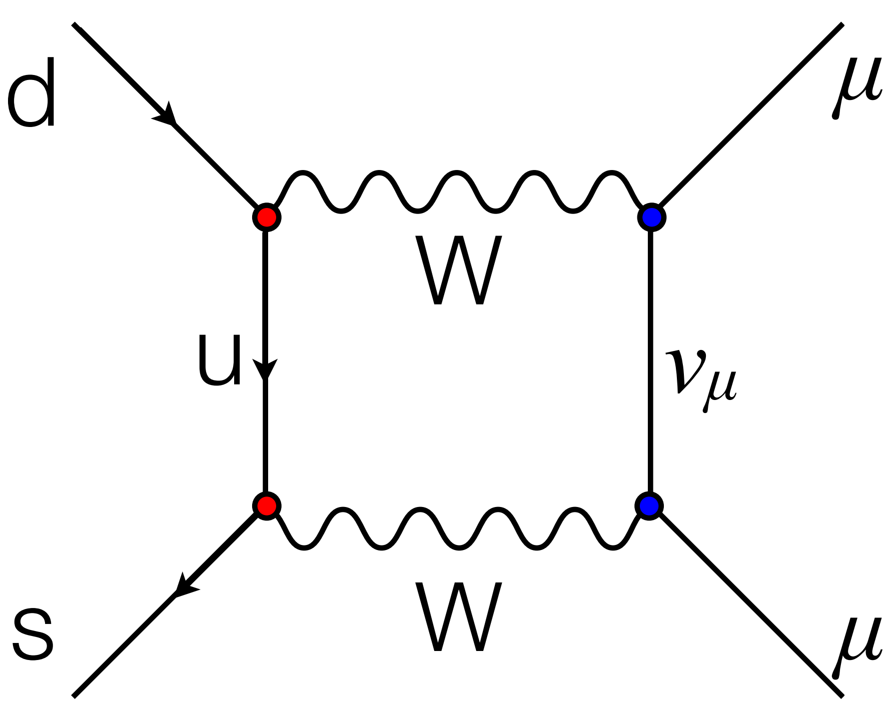




TOWARDS THREE GENERATIONS

ν_e	ν_μ	ν_τ
e	μ	τ

u	c	t
d	s	b



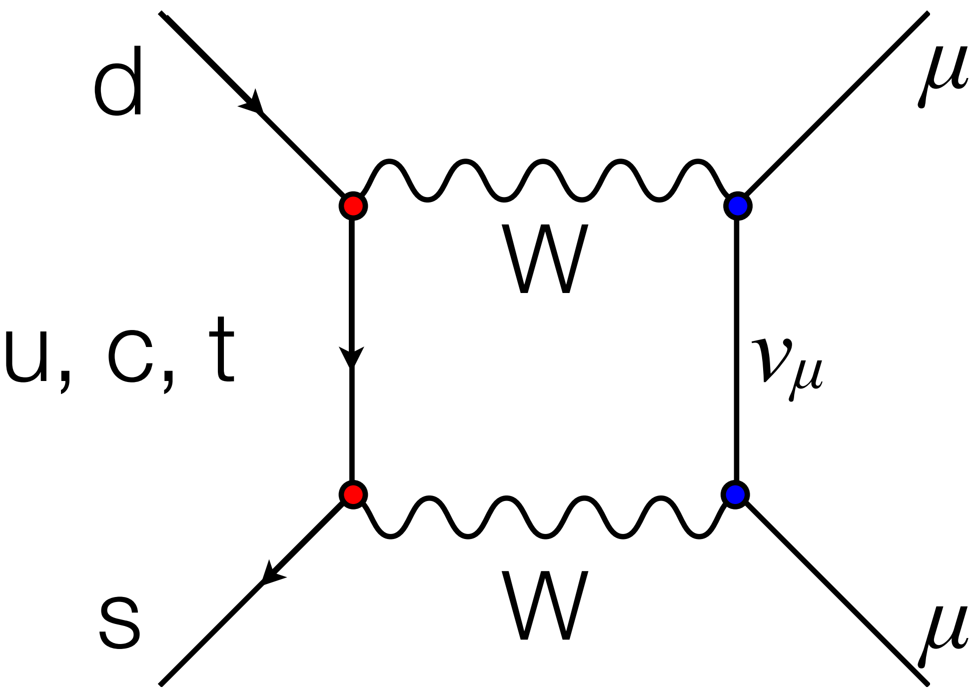
- Prior to the discovery of the Charm quark, Kobayashi and Maskawa contemplated the possibility of six quarks (three generations) in 1964
- Generalize Cabibbo angle to 3x3 matrix relating mass/flavor states

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Apply
  - factor of V_{ab}^* for a→b transition
  - factor of V_{ab} for b→a transition
    - note that antiquark transitions are complex conjugated relative to quark transitions
    - “just follow the arrows”

$$V_{ud}\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma^5) \qquad V_{us}^*\frac{-ig_W}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)$$

GIM MECHANISM IN CKM



$$V_{ud}\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma^5) \qquad V_{us}^*\frac{-ig_W}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)$$

$$V_{cd}\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma^5) \qquad V_{cs}^*\frac{-ig_W}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)$$

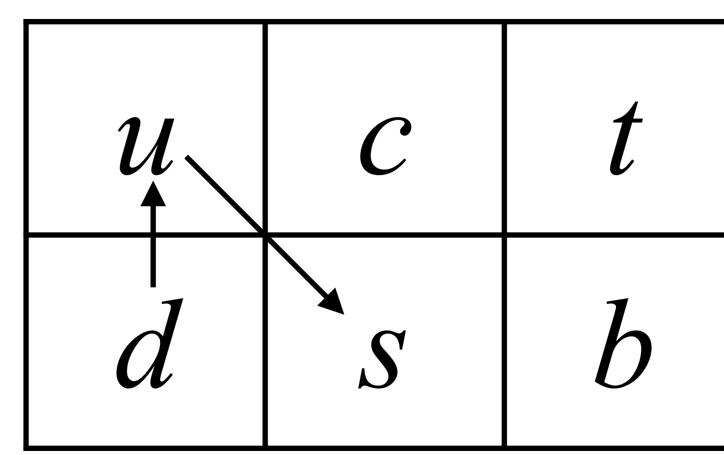
$$V_{td}\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma^5) \qquad V_{ts}^*\frac{-ig_W}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^*$$

$$\begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- In general, "flavor changing neutral currents" that proceed via a loop and two CC transitions will have this suppression
- "Nature abhors flavour changing neutral currents"

$$|U_{CKM}| \sim \begin{pmatrix} 0.9738 & 0.2272 & 0.0040 \\ 0.2271 & 0.9730 & 0.0422 \\ 0.0081 & 0.0416 & 0.9991 \end{pmatrix}$$



THREE SUPPRESSION MECHANISMS

- (coupling constant)
- Propagator
- Helicity Suppression
- GIM suppression
- When are they (not) in effect?

CONCLUSIONS

- Please read 12.1, 12.2, 14.1-3, 14.7