PHY489/1489

LECTURE 12:
QUANTUM CHROMODYNAMICS

N.B.:

- We will qualitatively cover various important aspects of QCD in this lecture
- We will not do any detailed calculations, though we will show some elements of how things may go if we did . . . .
- These key aspects of QCD are an important part of particle physics and establishing gauge theories as the basis for our description of particle physics.

NON-ABELIAN GAUGE THEORY

- Last time we considered the "SU(2) gauge theory"
  - pair of Dirac fields with local gauge invariance under the group SU(2)

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow e^{i\frac{g}{2}\vec{\theta}\cdot\vec{\sigma}} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

  - elements of the group parameterized by Pauli matrices as "generators":

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- In order to maintain local gauge invariance
  - need to add an extra term:

$$(i\partial\!\!\!/ - \frac{g}{2}\vec{\sigma} \cdot \vec{A\!\!\!/} - m)\psi = 0$$

  - where the A^i transform as follows in the gauge transformation.

$$\vec{A}_\mu \rightarrow \vec{A}_\mu - \frac{1}{2}\partial_\mu \theta - \frac{g}{2}\epsilon_{ijk}\theta_i A^j_\mu$$

CONSEQUENCES

- We have three new fields:

$$\vec{A}_\mu \to \vec{A}_\mu - \frac{1}{2}\partial_\mu \theta - \frac{g}{2}\epsilon_{ijk}\theta_i A^j_\mu \qquad \vec{A}_\mu \Rightarrow A^i_\mu$$

  - they are Lorentz vector fields (like the photon)
  - last term from non-commutation of the generators (Pauli matrices)
    - the fields couple to themselves
  - we can give A life by introducing their own kinetic terms, etc.
    - the same gauge invariance that created them requires them to be massless

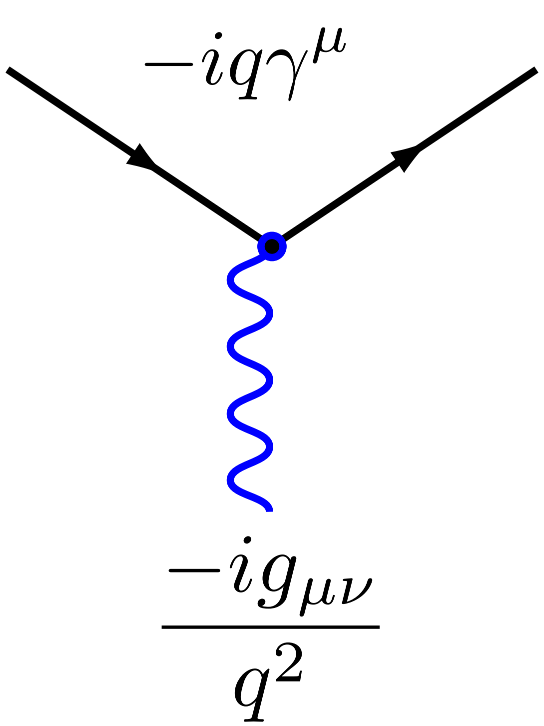
- We have a new interaction term:

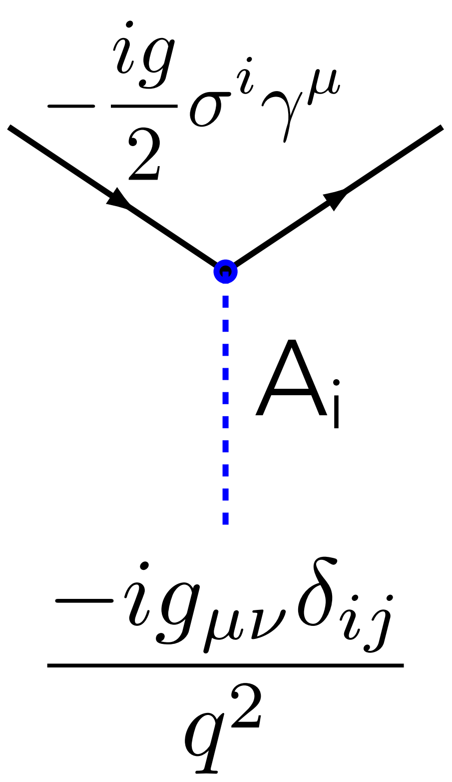
$$(i\not{\partial} - \frac{g}{2}\vec{\sigma}\cdot\vec{\not{A}} - m)\psi = 0$$

$$\frac{g}{2}\,\sigma_i \cdot A^i_\mu\,\gamma^\mu\psi$$

  - in analogy to the U(1) case:

$$(i\not{\partial} - q\not{A} - m)\psi = 0$$





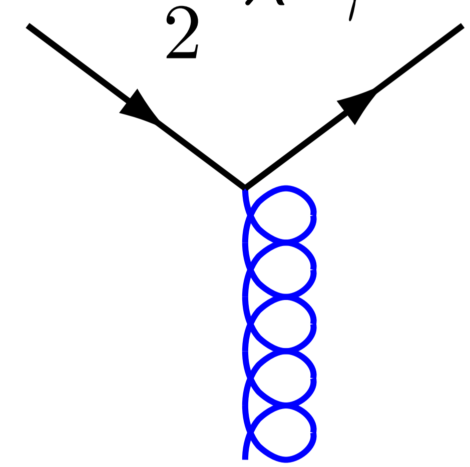
SU(3) GAUGE SYMMETRY

- Basically the same same story:

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \to e^{i\frac{g_s}{2}\vec{\alpha}\cdot\vec{T}} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

- where T_a are eight 3x3 matrices which parameterize the SU(3) transformation.
  - Conventionally, we can use the "Gell-Mann" matrices which are the analog of the Pauli matrices in SU(3)

$$T_a = \frac{1}{2}\lambda_a$$



- This time, we need to add eight fields

$$(i\not{\partial} - g_s T^a \not{G}^a - m)\psi = 0$$

$$G^a \to G^a_\mu - \frac{1}{2}\partial_\mu \alpha^a - g_s f_{ijk}\alpha_i G^j_\mu$$

"GELL-MANN" MATRICES:

- SU(3) has 8 real degrees of freedom
  - 18 complex numbers - 9 unitary relations - 1 for "special"
  - This basis is a convention

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

QUANTUM CHROMODYNAMICS

- This is the theory of QCD as we know it
  - local gauge theory under SU(3)
  - eight massless "gluons": bosons introduced for local gauge invariance
  - acts on triplets of Dirac fields

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \rightarrow e^{i\frac{g_s}{2}\vec{\alpha}\cdot\vec{T}} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

"red"
"green"
"blue"

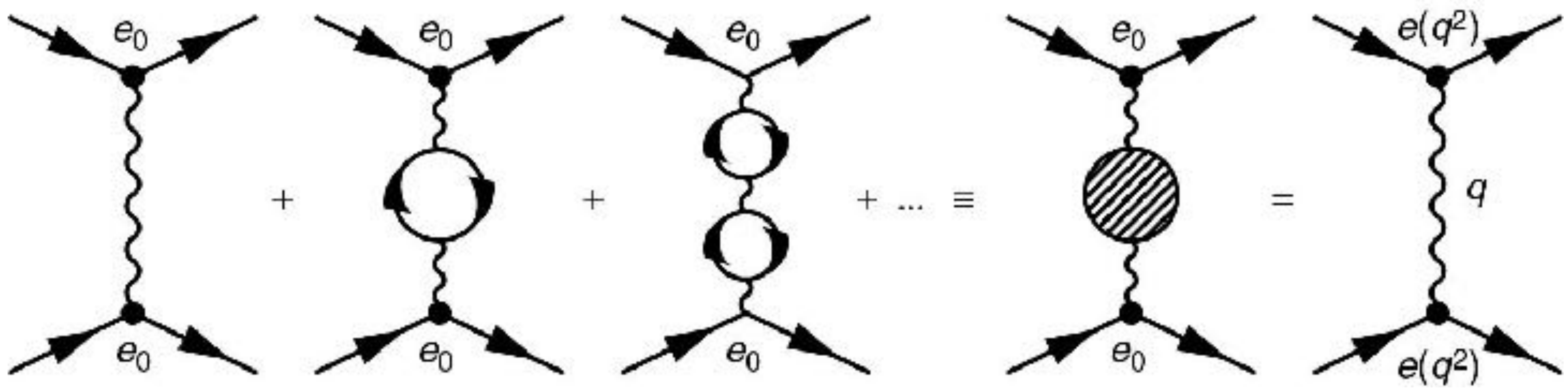
- Fundamental connection between "transformation" and "interaction"
  - fields that do not transform under the gauge transformation do not interact
  - objects that do not transform are called "singlets" and do not participate
    - quarks are "triplets" under SU(3) ("fundamental")
    - leptons and other "uncolored" objects are "singlets"
- similarity of QED, QCD arise because they are both (local) gauge theories

COLOR CONFINEMENT

- "colored" objects are not observed directly in nature
  - i.e. all objects we see take the form of objects with net zero color
  - "color singlet states"
  - in particular, bare gluons and quarks never appear to be seen
- Examples:
  - mesons $q\bar{q}$ objects with opposite color
  - baryons: qqq objects which combine to form a zero color state
  - and antibaryons
- There is no rigorous proof that this must be the case, but conceptually it can be understood in the context of the strength of the interaction.
  - "pulling apart" two coloured objects leads to additional gluons being produced which then form and "neutralize" them.

“RUNNING” COUPLING CONSTANT

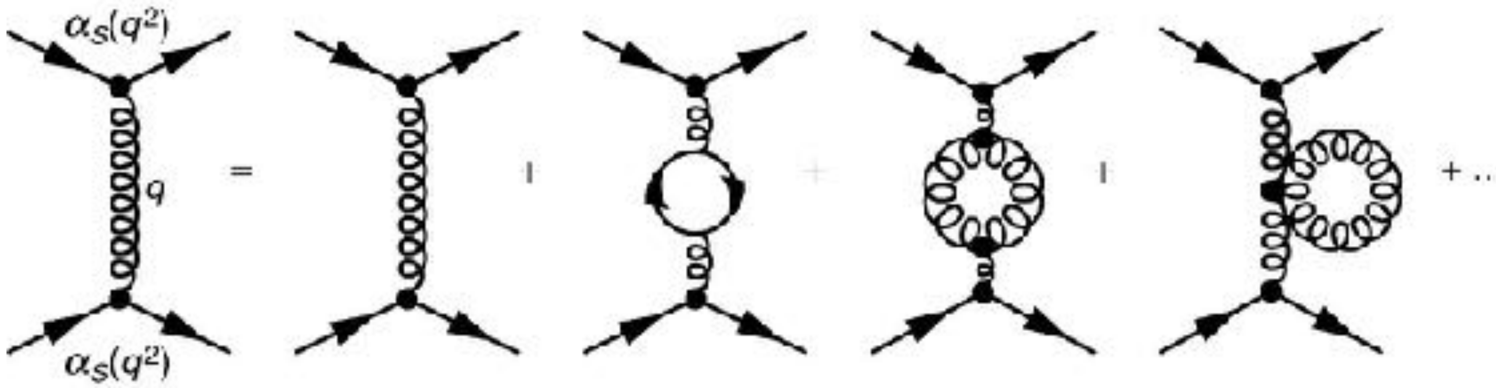
- We have talked about “coupling constants” in QED and QCD
- A major development was the acceptance/incorporation of the idea that these are not constants at all
  - higher order diagrams result in a q^2 dependence for a process

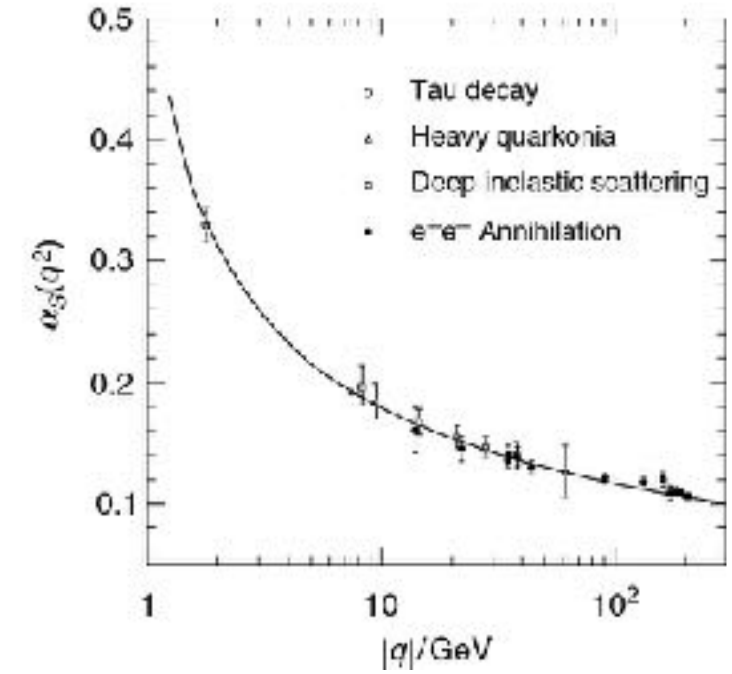


  - contribution from all the diagrams can be summarized as a coupling “constant” a a fixed q^2
    - for electromagnetism, this results in large coupling as q^2 increases.
  - “renormalization”

QCD

- The first higher order correction to the gluon propagator





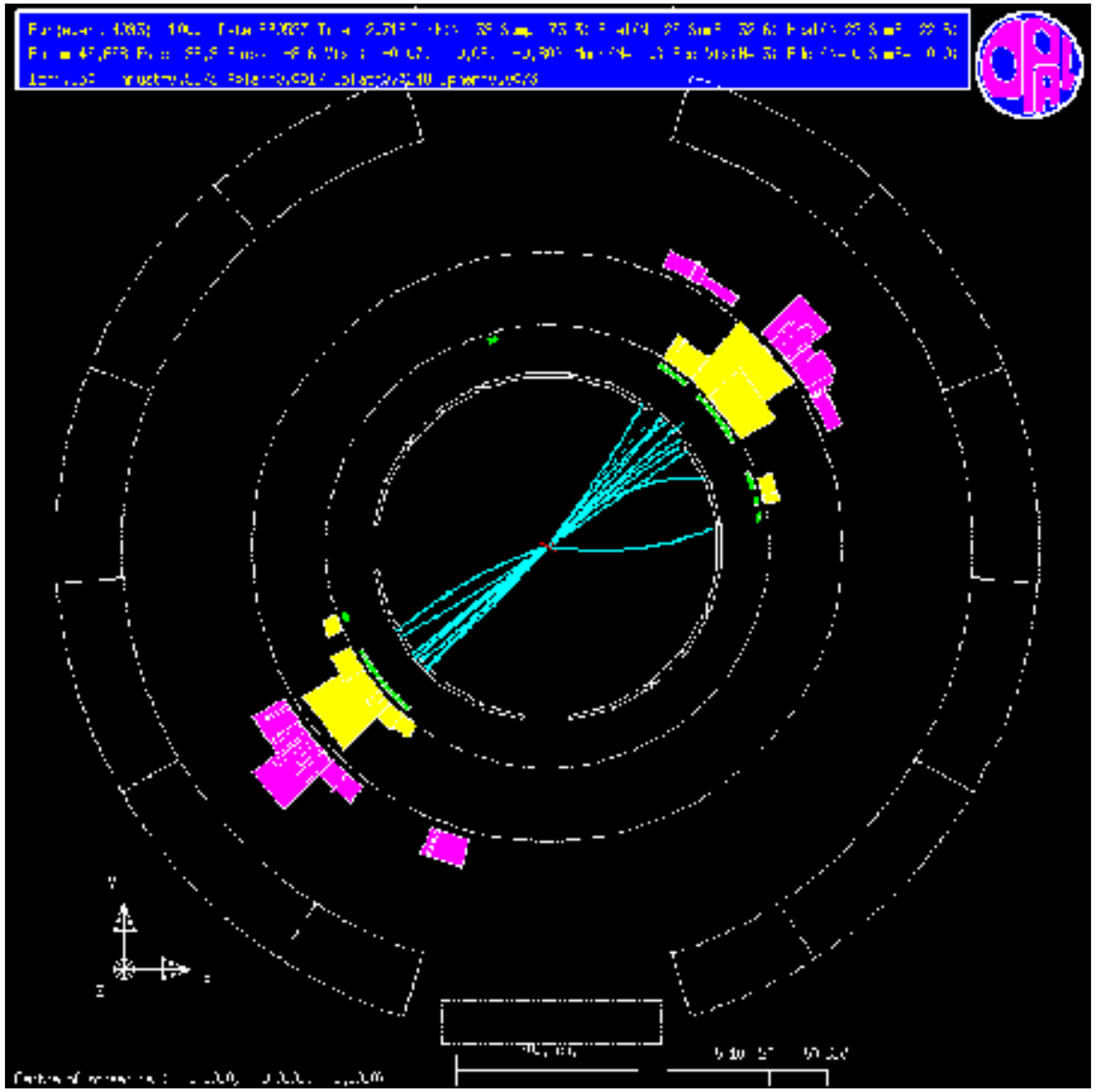
  - this results in:

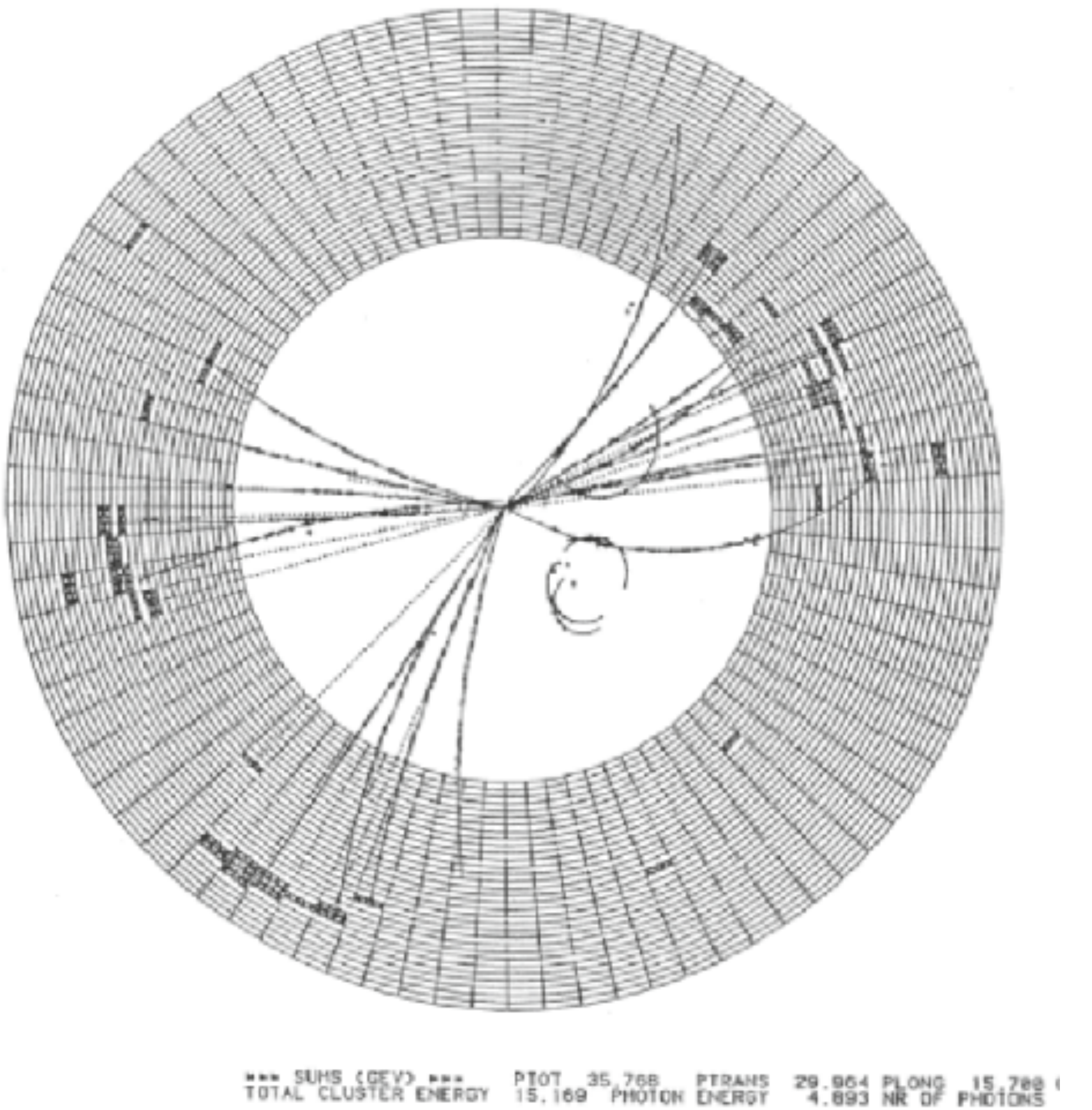
$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + B\alpha_s(\mu^2)\log\frac{q^2}{\mu^2}} \qquad B = \frac{11N_c - 2N_f}{12\pi}$$

  - note that it depends on the number of quark flavours
  - with six quarks, we find that B < 0
  - coupling constant *decreases* with q^2
  - Λ_{QCD}: where α_s ~1: 220 MeV

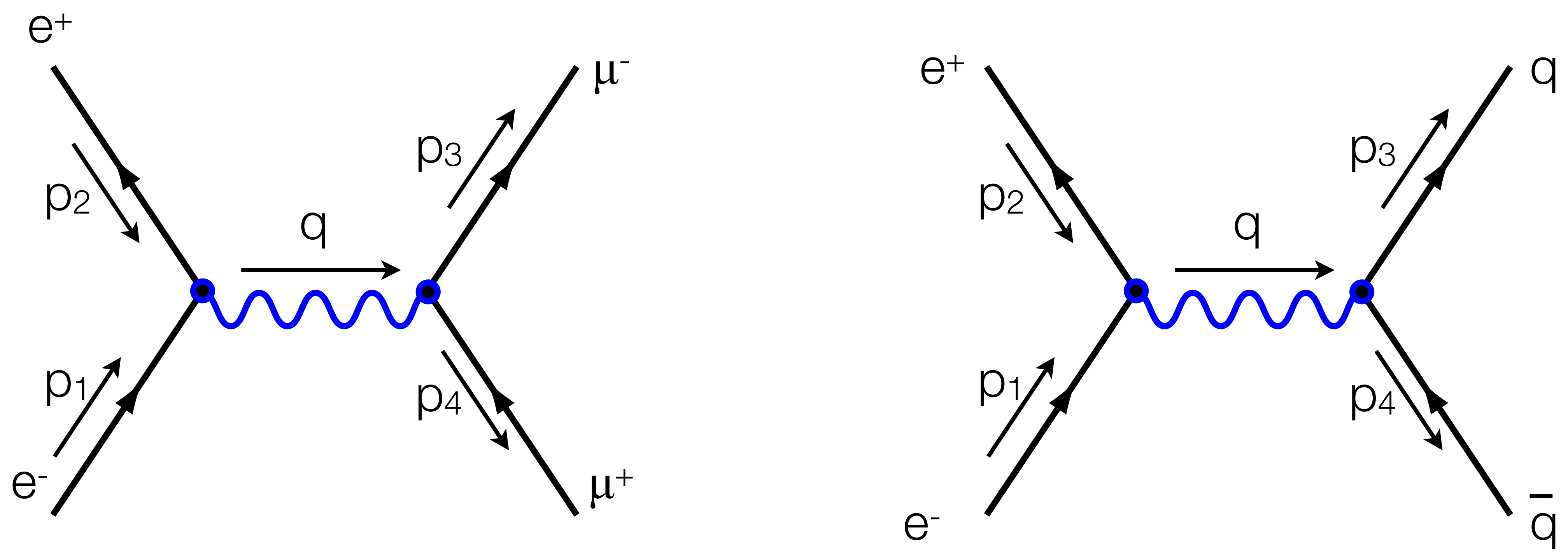


$q\bar{q}$ PRODUCTION:





$e^+ + e^- \rightarrow q + \bar{q}$



- All the calculation steps are the same, except . . .
  - quarks do not have unit charge
  - quarks can be emitted in three possible color combinations
  - different quarks have different masses

CROSS SECTION

- Recall total cross section for μ pair production

$$\sigma = \frac{\pi}{3}\left(\frac{\hbar c\alpha}{E}\right)^2 \qquad \sigma_{\mu^+\mu^-} = \frac{\pi}{3}\left(\frac{\hbar c\alpha}{E}\right)^2 \Rightarrow \sigma_{q_i\bar{q}_i} = 3Q_i^2 \times \sigma_{\mu^+\mu^-}$$

- now take the ratio

$$R = \frac{\sum_i \sigma_{q\bar{q}_i}}{\sigma_{\mu^+\mu^-}} = 3 \times \sum Q_i^2$$

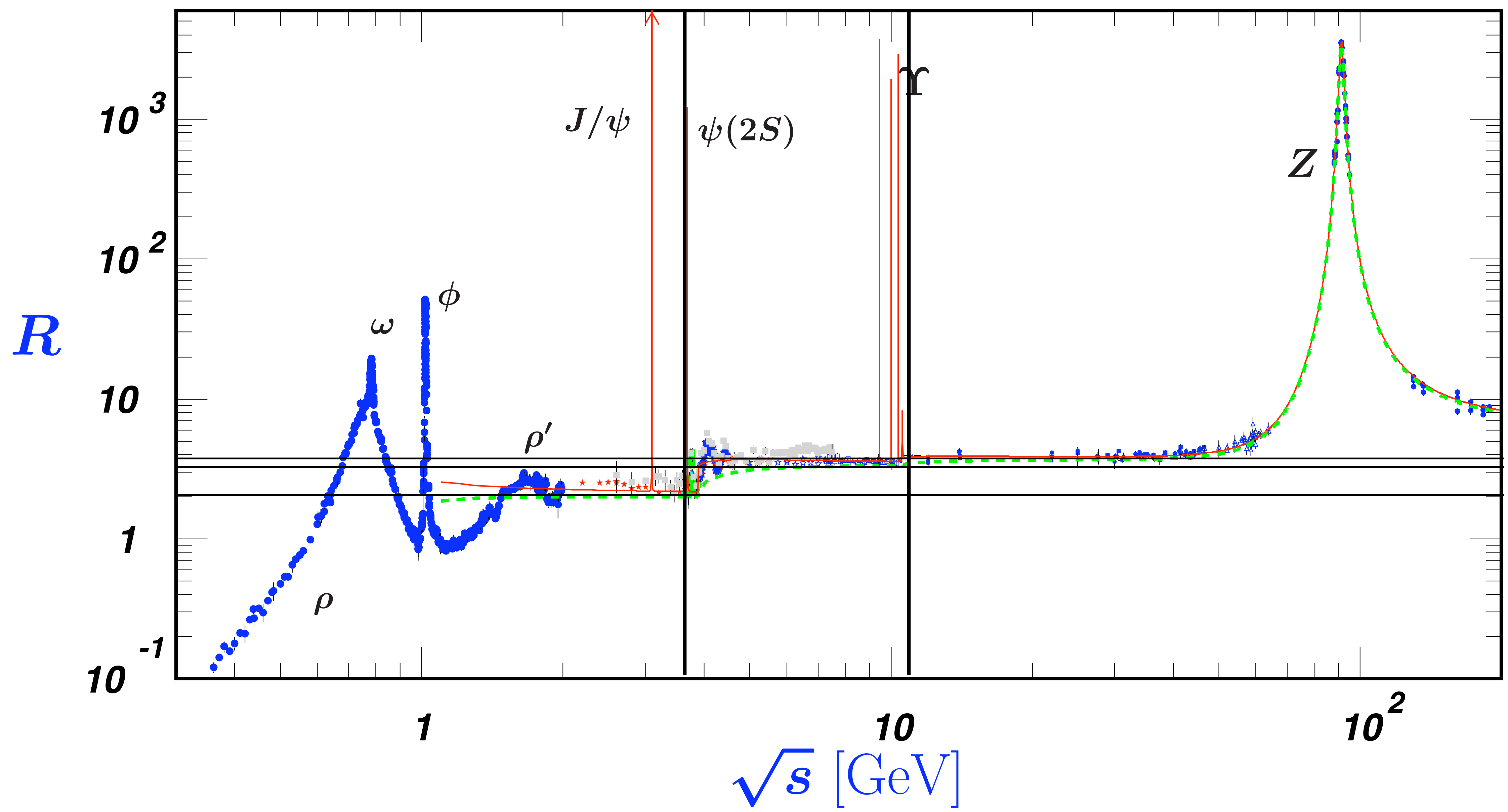
- Which quarks can be produced depends on energy

$$\sqrt{s} < 2m_c \sim 3.7\text{GeV} \qquad R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2) = 2$$

$$\sqrt{s} < 2m_b \Rightarrow 10.6\text{GeV} \qquad R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2 + (2/3)^2) = 10/3$$

$$\sqrt{s} < 2m_t \sim 330\text{GeV} \qquad R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2 + (2/3)^2 + (1/3)^2) = 11/3$$

MEAUREMENTS



SUMMARY:

- QED and QCD have a common origin in local gauge theories
  - QED is a U(1) gauge theory, 1 gauge boson
  - QCD is a SU(3) gauge theory, 8 gauge bosons
- The triplet of states associated with the SU(3) transformations is the "color" space
  - objects must transform under SU(3) to feel the QCD interaction
- Confinement:
  - only colorless/"singlet" states are observed
- Asymptotic Freedom:
  - strength of QCD (α_s) decreases as q^2 rises
  - "Λ_{QCD}": q^2 region where QCD is "strong" ($\alpha_s \sim 1$)

NEXT TIME:

- We'll move onto weak interactions
  - Please read chapter 11
- Problem Set 2 due today in Box K
  - Problem set 3 will be posted soon.