PHY489/1489: LECTURE 20

SPONTANEOUS SYMMETRY BREAKING

LAST TIME:

- Last time, we introduced the Lagrangian formalism as an alternative to the equations of motion
- Local gauge symmetry works basically the same as before
  - e.g. for a U(1) gauge symmetry:

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi = 0 \qquad i\gamma^\mu\partial_\mu\psi - m\psi = 0$$

$$\mathcal{L} \to \mathcal{L} - q\bar{\psi}\gamma^\mu\psi\, A_\mu \qquad A_\mu \to A_\mu - \partial_\mu\theta$$

$$\partial_\mu \to D_\mu \equiv \partial_\mu + iqA_\mu$$

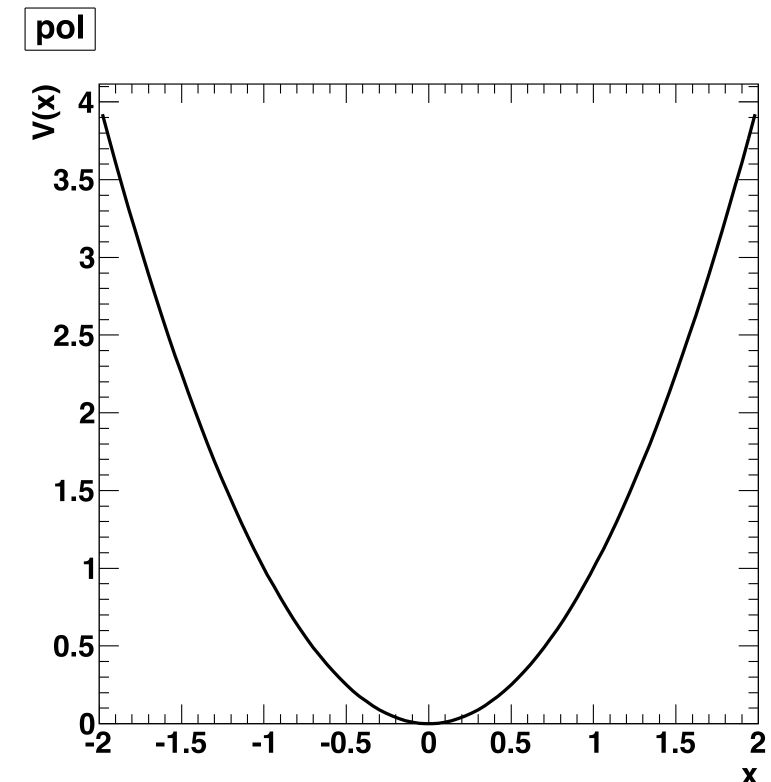
- We found that local gauge symmetry forbids the gauge bosons from having mass
- We also found that the $SU(2)_L \times U(1)_Y$ symmetry we introduced for the weak interaction also forbids fermion masses!

MORE ON THE MASS TERM

$$\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi = 0$$

$$\mathcal{L}_P = \frac{-1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{8\pi}m^2 A^\nu A_\nu - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

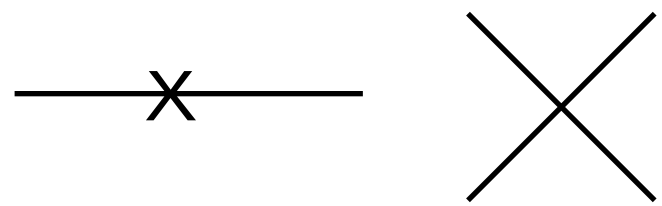


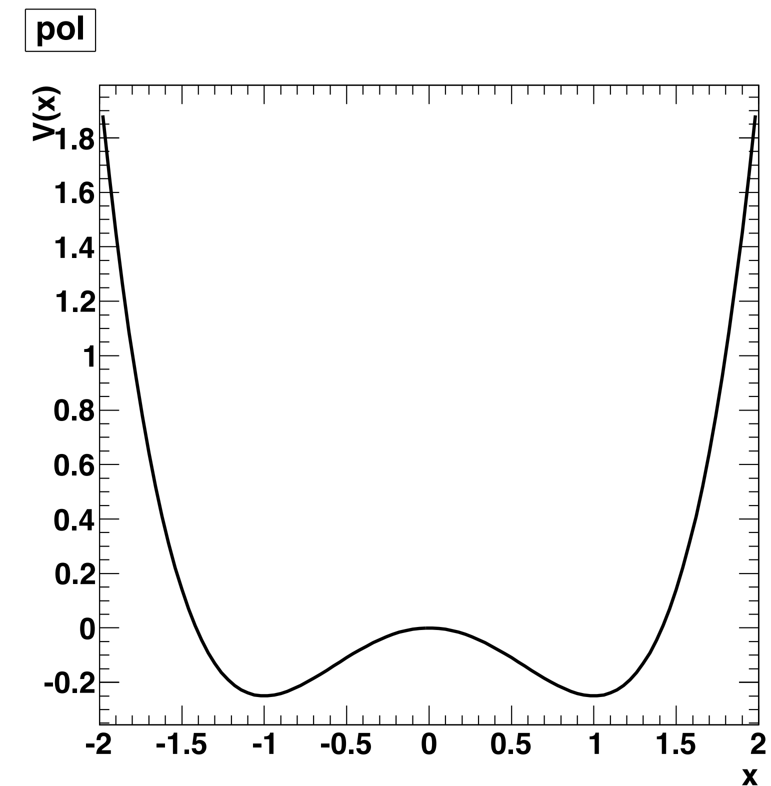
- mass terms are quadratic in the field
  - *i.e.* if there is a quadratic term in the lagrangian, it behaves as a mass.

“VACUUM EXPECTATION VALUE”

- Consider the Lagrangian

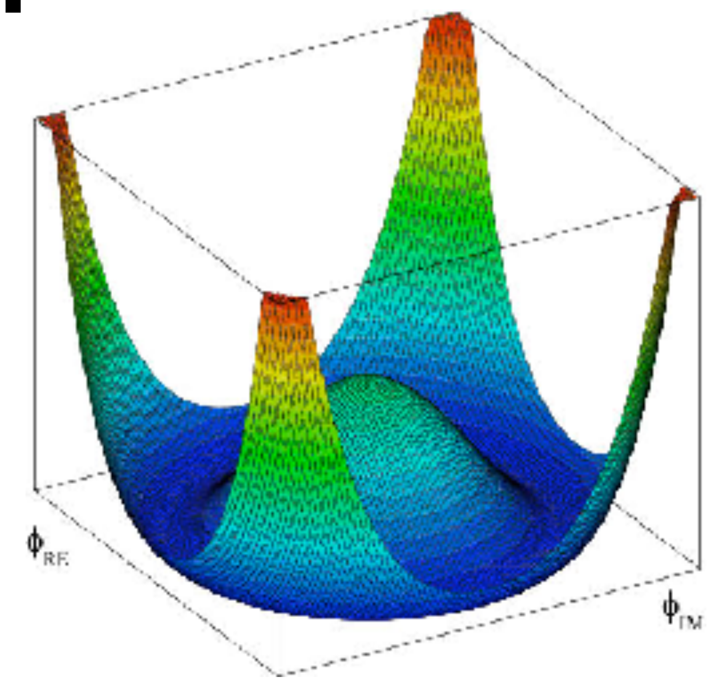
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$$





- Note that ϕ=0 is not a stable configuration
  - the vacuum (e.g. lowest energy state) actually happens when ϕ has some non-zero value
  - “vacuum expectation value” (VEV)
- Perturbation theory must start from a stable vacuum in order to work
  - choose a vacuum state
  - “spontaneous symmetry” breaking

CONTINUOUS SYMMETRY



- Consider a complex scalar Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^*)(\partial^\mu \phi) + \frac{1}{2}\mu^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2$$

$$\phi = \phi_1 + i\phi_2$$

- Instead of two potential vacuum configurations, we now have an infinite number of connected states

$$|\phi| = \frac{\mu}{\lambda}$$

- Expand about a vacuum point
  - let's also make it locally gauge invariant by introducing the "covariant derivative"
  - that means we get a gauge boson along for the ride

$$\partial_\mu \to D_\mu \equiv \partial_\mu + iqA_\mu$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi + \frac{1}{2}\mu^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2 - \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}$$

BREAK THE SYMMETRY

$$\mathcal{L} = \frac{1}{2}(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi + \frac{1}{2}\mu^2\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2 \quad -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

- choose a vacuum point: $\phi_0 = \frac{\mu}{\lambda}$
- and reparameterize the fields as:

$$\eta = \phi_1 - \frac{\mu}{\lambda} \qquad \chi = \phi_2$$

- and rewrite the Lagrangian focussing on the kinetic part

$$(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi$$

$$\Rightarrow \left[(\partial_\mu \boxed{- iqA_\mu})(\eta + \boxed{\frac{\mu}{\lambda}} - i\chi)\right]\left[(\partial^\mu + \boxed{iqA^\mu})(\eta + \boxed{\frac{\mu}{\lambda}} + i\chi)\right]$$

$$\longrightarrow \frac{1}{2}\left(\frac{\mu q}{\lambda}\right)^2 A_\mu A^\mu$$

this is a mass term for the vector particle $m_A = 2\sqrt{\pi}\frac{q\mu}{\lambda}$

HOW DID THIS HAPPEN:

- Recall that our gauge invariant Lagrangian

$$(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi$$

- has a term $q^2 A_\mu A^\mu \phi^* \phi$

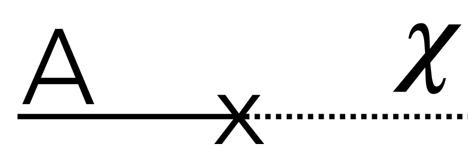
- Normally, ϕ is just a normal field
  - but the potential gives it a vacuum expectation (e.g. non-zero) base value that turns this into a mass term for A
  - we chose a particular vacuum configuration but the result is independent of our choice
  - the symmetry isn't "really" broken, just hidden by our choice

OTHER TERMS

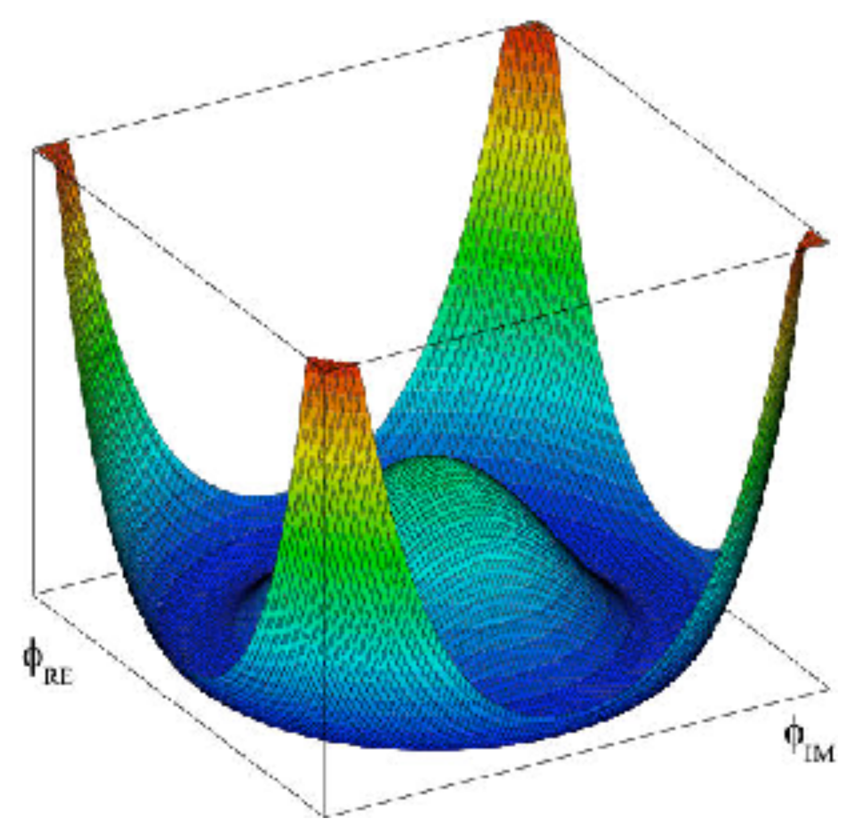
$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

- Note that the Lagrangian also includes a kinetic term for a massless field χ
  - this is called a "Nambu-Goldstone boson"
- Another term:

$$\frac{q\mu}{\lambda}(\partial_\mu \chi)A^\mu$$



- is a bit problematic . . .
  - The A particle spontaneously turns into χ



ACCOUNTING ISSUE

$$(\partial_\mu - iqA_\mu)\phi^*(\partial^\mu + iqA^\mu)\phi$$

$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

- We started with:
  - two scalar fields (ϕ_1, ϕ_2 or alternatively ϕ^*,ϕ)
  - a massless gauge boson (two polarizations)
- We end up with:
  - two scalar fields (η, χ)
  - a massive gauge boson (three polarizations)
- where did the extra degree of freedom come from?

GAUGE TRANSFORMATION

$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

- if we isolate the the terms related to χ and A

$$\frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) + \frac{q\mu}{\lambda}(\partial_\mu\chi)A^\mu + \frac{1}{2}\left(\frac{q\mu}{\lambda}\right)^2 A_\mu A^\mu$$

$$\frac{1}{2}\left(\frac{q\mu}{\lambda}\right)^2\left[A_\mu + \frac{\lambda}{q\mu}(\partial_\mu\chi)\right]\left[A^\mu + \frac{\lambda}{q\mu}(\partial^\mu\chi)\right]$$

  - this last transformation effectively represents a gauge transformation

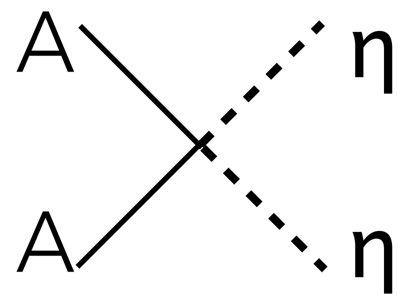
$$A_\mu \rightarrow A_\mu + \frac{\lambda}{q\mu}(\partial_\mu\chi)$$

- We can "gauge transform" the χ field to disappear explicitly from the Lagrangian
- the χ field corresponds to the "new" longitudinal polarization of the A
- "The gauge boson ate the goldstone boson"

GAUGE COUPLINGS

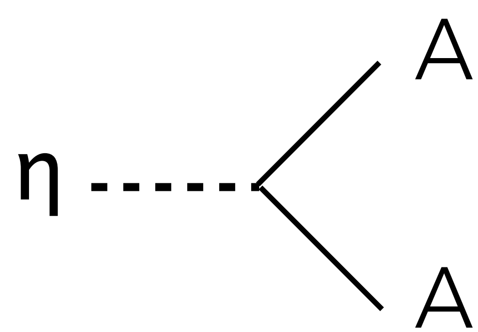
$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

$$q^2 A_\mu A^\mu \eta^2$$

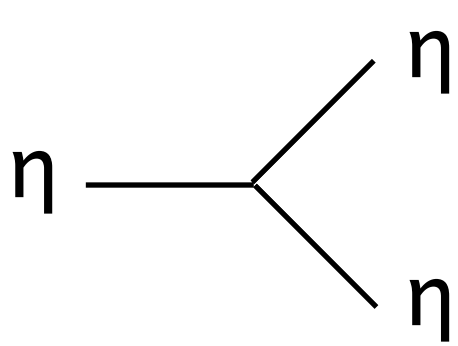


$$\Rightarrow \left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\lambda} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\lambda} + i\chi)\right]$$

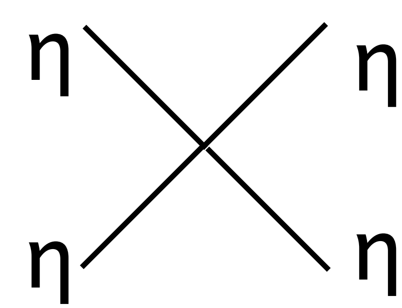
$$q^2 \frac{\mu}{\lambda} A_\mu A^\mu \eta$$



$$\lambda \frac{\mu}{\lambda} \eta^3$$



$$\frac{1}{4}\eta^4$$



ELECTROWEAK MODEL

- The features for the $SU(2)_L xU(1)_Y$ spontaneous symmetry breaking have the same basic concepts:
- We introduce a $SU(2)xU(1)_Y$ doublet of scalar fields a quartic potential

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

- make it gauge invariant by introducing a covariant $SU(2)_L xU(1)_Y$ derivative
- we spontaneously break the symmetry
- ϕ acquires a vacuum expectation value that gives W and Z mass
  - W mass governed completely by the $SU(2)_L$ gauge coupling constant and the vacuum expectation value
  - the A, Z mass, however, will involve both the $SU(2)_L$ and $U(1)_Y$ gauge couplings
    - diagonalization of the mass matrix will result in a massless A boson and modified mass for the Z boson.

$$\frac{m_W}{m_Z} = \cos\theta_W$$

ONE REMAINING ISSUE

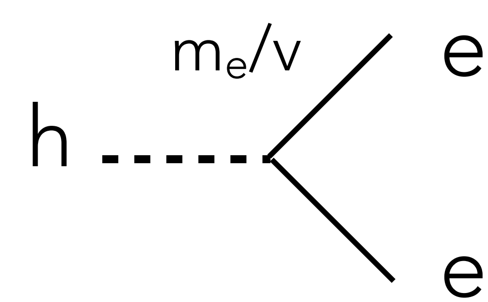
- The fermion masses!

$$\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$$

- we found this breaks gauge symmetry because it couples an $SU(2)_L$ doublet to a $SU(2)_L$ singlet

$$\mathcal{L}_Y = -g_e \left[(\bar{\nu}_L, \bar{e}_L) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + h.c. \right]$$

- by coupling the left chiral fields to the Higgs field, we can generate an overall singlet.
- by acquiring a vacuum expectation value, this becomes the mass of the electron.
    - $g_e\, v \sim m_e$



CONCLUSIONS:

- Gauge invariance forces us to make back doors for introducing mass into our theory
- Spontaneous symmetry breaking gives us a way of introducing a "constant" background with gauge quantum numbers to produce mass terms that preserve gauge symmetry
- As a consequence there is a tight interconnection between
  - the vacuum expectation value
  - gauge couplings
  - gauge boson masses
  - effectively fixed by the model and tested and can be tested.
- In the electroweak model, the fermion masses can be generated in the same way.
  - Fixes relation between vev, fermion mass, and Higgs coupling to the fermion

NEXT TUESDAY

- Pierre Savard: discussion on Higgs physics
  - how did we discover the Higgs boson?
  - what do we know about it now?
  - how do we know that it is “the” Higgs boson?
- Thank you for all your hard work in the class
- I hope you have learned something and obtained an appreciation for how spectacular and mysterious particle physics is.
- Reminder:
  - Midterm grading
  - Additional office hours
    - please let me know if there is anything that was particular unclear, etc.

Tues 13 Sep	**Introduction**	
Thurs 15 Sep	Review of Special Relativity	Vancouver
Tues 20 Sep	Quantum Mechanics	
Thurs 22 Sep	Golden Rule, decays, cross sections	
Tues 27 Sep	Relativistic wave equations	
Thurs 29 Sep	Solutions of the Dirac Equation	
Tues 4 Oct	Feynman rules for QED	
Thurs 6 Oct	Electron-positron annihilation	Tokai
Tues 11 Oct	Electron-positron annihilation	
Thurs 13 Oct	Electron-proton scattering and form factors	
Tues 18 Oct	Deep inelastic scattering	Vancouver
Thurs 20 Oct	Scaling	Los Angeles
Tues 25 Oct	Weak interaction	
Thurs 27 Oct	Weak interactions, continued	Mainz
Tues 1 Nov	Lepton universality and neutrino scattering	
Thurs 3 Nov	Midterm	Beijing
Tues 8 Nov	No Class	
Thurs 10 Nov	Weak interaction of quarks	
Tues 15 Nov	CP Violation	Fermilab
Thurs 17 Nov	Electroweak Unification	Michigan
Tues 22 Nov	No class	Seoul
Thurs 24 Nov	More on electroweak unification	
Tues 29 Nov	Electroweak physics	
Thurs 1 Dec	Local gauge invariance and Lagrangians	San Francisco
Tues 6 Dec	Higgs mechanism	