PHYSICS 489/1489: LECTURE 19

TESTS OF EW MIXING AND LAGRANGIANS

MIDTERM:

- The last problem in the midterm asked you to consider:

$$\overline{(P_L v)}\gamma^\mu(P_L u) = \overline{(P_R v)}\gamma^\mu(P_R u) = 0$$

- As it turns out, this problem is not posed correctly and I also solved it incorrectly (SAD!).
- Please bring your exam to Randy for reevaluation on this problem
    - outcome can only be positive
    - we cannot adjust your grade unless you bring your midterm

FINAL EXAMINATION

- Will consist of:
  - ~4 short answer questions
  - 2 detailed calculations with Feynman rules, amplitudes, decay rates/ cross sections
- Formula sheet will be provided
  - relevant Feynman rules, helicity spinors, phase space expressions
    - I'll try to circulate before hand. . . .
  - you can additionally bring one page of equations and notes (feel free to use both sides) and a basic calculator
  - will cover material up to/including today's lecture
  - emphasis on material since midterm
- 7-10PM on Friday, 16 December
- TC 239 (Seeley Hall, Trinity College) , 6 Hoskin Avenue

ELECTROWEAK MIXING

- Two lectures ago, we saw how:
  - a $SU(2)_L$ gauge group coupling only to left chiral fermions (**W**)
  - a $U(1)_Y$ gauge group with both (but different) couplings to left and right chiral fields (B)
- came together to form:
  - weak charged currents with only left chiral couplings
  - a neutral current with equal left/right coupling
  - a neutral current with imbalanced left/right coupling
- We already studied the first two
- Let’s explore the third a bit more

Z COUPLINGS

- the Z couplings resulted from a mixing of W_3 and B

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

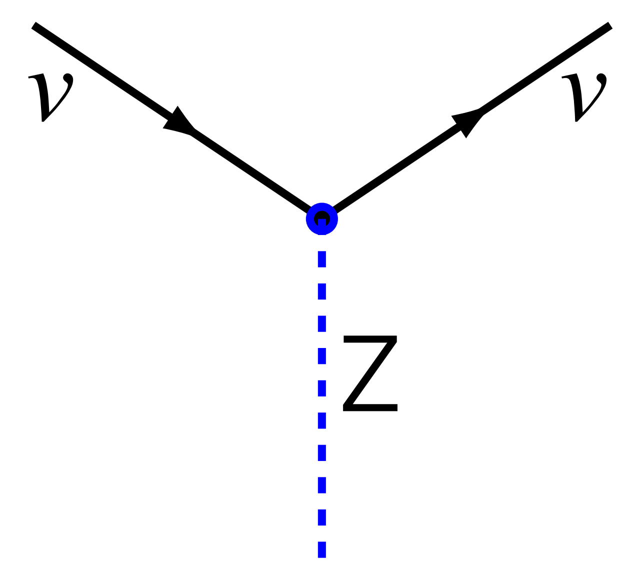
- Recovering the EM interaction as we know it introduced relations between the coupling constants and Y

$$g_Z \equiv \frac{g}{\cos\theta_W} = \frac{e}{\sin\theta_W \cos\theta_W} \qquad g' = g_Z \sin\theta_W \qquad Y = 2(Q - I^3_W)$$

- For the neutrino:

$$\bar{\nu}_L \gamma^\mu \nu_L \to -\frac{g'}{2} Y_{\nu_L} \sin\theta_W + \frac{1}{2} g \cos\theta_W \to \frac{g_Z}{2}$$

$$\bar{\nu}_R \gamma^\mu \nu_R \to -\frac{g'}{2} Y_{\nu_R} \sin\theta_W \to 0$$



$$\frac{-i g_Z}{2} \gamma^\mu (1 - \gamma^5)$$

  - which we can translate into a vertex factor
  - in this case the coupling is pure left chiral

GENERALLY:

$$g_Z \equiv \frac{g}{\cos\theta_W} = \frac{e}{\sin\theta_W \cos\theta_W}$$

$$g' = g_Z \sin\theta_W$$

- We got the following: $Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$
- for the left coupling we have:

$$-\frac{g'}{2}\sin\theta_W Y + g\cos\theta_W I_3$$

$$-\frac{g_Z}{2}\sin^2\theta_W Y + g\cos^2\theta_W I_3$$

$$-\frac{g_Z}{2}\sin^2\theta_W (2Q - I_3) + g\cos^2\theta_W I_3$$

$$c_L = g_Z(I_3 - Q\sin^2\theta_W)$$

- for the right coupling we have:

$$\frac{-g'}{2}\sin\theta_W Y$$

$$\frac{-g_Z}{2}\sin^2\theta_W Y$$

$$c_R = -Q\sin^2\theta_W$$

- In general we can write the Z vertex in terms of:
  - left/right chiral couplings

$$-ig_Z\left[c_L\gamma^\mu(1-\gamma^5) + c_R\gamma^\mu(1+\gamma^5)\right]$$

$$c_L = I_3 - Q\sin^2\theta_W$$

$$c_R = -Q\sin^2\theta_W$$

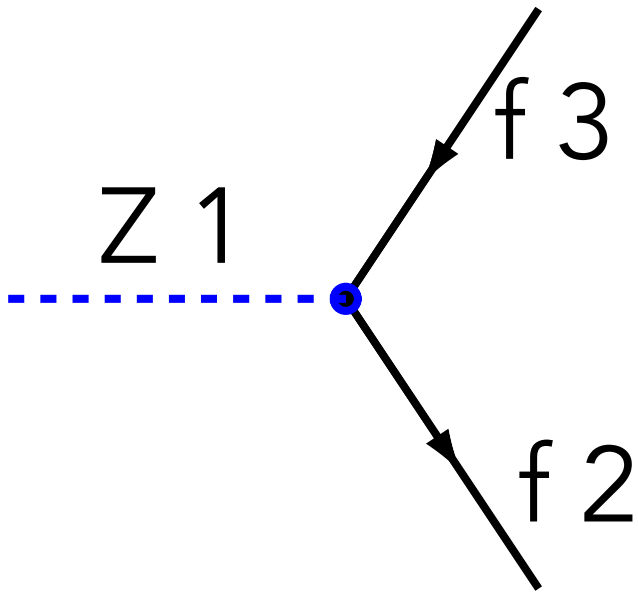
  - vector/axial vector couplings:

$$\frac{-ig_Z}{2}\gamma^\mu\left[c_V - c_A\gamma^5\right]$$

$$c_V = c_L + c_R \quad = I_3 - 2Q\sin^2\theta_W$$

$$c_A = c_L - c_R \quad = I_3$$

Z DECAYS:



$$\mathcal{M} = \frac{g_Z}{2}\bar{u}_2\gamma^\mu\left[c_V - c_A\gamma^5\right]v_3\epsilon_{1\mu}$$

- As usual, we will consider helicity/chiral states in the massless limit.
- Using the relation $\gamma^\mu(c_V - c_A\gamma^5)\frac{1\pm\gamma_5}{2} \rightarrow \frac{1\mp\gamma^5}{2}\gamma^\mu(c_V - c_A\gamma^5)$
- we can show:

$$\bar{u}_{2L}\gamma^\mu\left[c_V - c_A\gamma^5\right]v_{3L} = \bar{u}_{2R}\gamma^\mu\left[c_V - c_A\gamma^5\right]v_{3R} = 0$$

- so that we need only consider

$$\bar{u}_{2L}\gamma^\mu\left[c_V - c_A\gamma^5\right]v_{3R} \qquad \bar{u}_{2R}\gamma^\mu\left[c_V - c_A\gamma^5\right]v_{3L}$$

- to consider this in terms of c_L and c_R

$$c_V - c_A\gamma^5 \rightarrow (c_L + c_R) - (c_L - c_R)\gamma^5$$

- so that $c_L\bar{u}_{2L}\gamma^\mu\left[1-\gamma^5\right]v_{3R} \qquad c_R\bar{u}_{2R}\gamma^\mu\left[1+\gamma^5\right]v_{3L}$

Z DECAYS CONTINUED

$$\mathcal{M} = \frac{g_Z}{2}\bar{u}_2\gamma^\mu\left[c_V - c_A\gamma^5\right]v_3\epsilon_{1\mu}$$

- Use the previously calculated helicity combinations:

$$\frac{1}{2}\bar{u}_{2L}\gamma^\mu\left[1-\gamma^5\right]v_{3R} \to \bar{u}_{2L}\gamma^\mu v_{3R} = 2E(0, -\cos\theta, -i, \sin\theta)$$

$$\frac{1}{2}\bar{u}_{2R}\gamma^\mu\left[1+\gamma^5\right]v_{3L} \to \bar{u}_{2R}\gamma^\mu v_{3L} = 2E(0, -\cos\theta, i, \sin\theta)$$

  - where E = $m_Z/2$

- contract this with our Z polarization vectors

$$\epsilon_{+\mu} = \frac{1}{\sqrt{2}}(0,1,i,0) \quad \epsilon_{-\mu} = \frac{1}{\sqrt{2}}(0,-1,+i,0) \quad \epsilon_{L\mu} = (0,0,0,-1)$$

- to get six Z polarization/outgoing helicity combinations

- stick this with the other factors

$$\frac{g_Z m_Z}{\sqrt{2}} c_L/c_R$$

	+	-	L
L R	1 - cos θ	1+cos θ	-sin θ
R L	-1 - cos θ	-1+cos θ	-sin θ

FINAL STEPS:

- We can square all the matrix elements and add them together to get the spin-summed amplitude

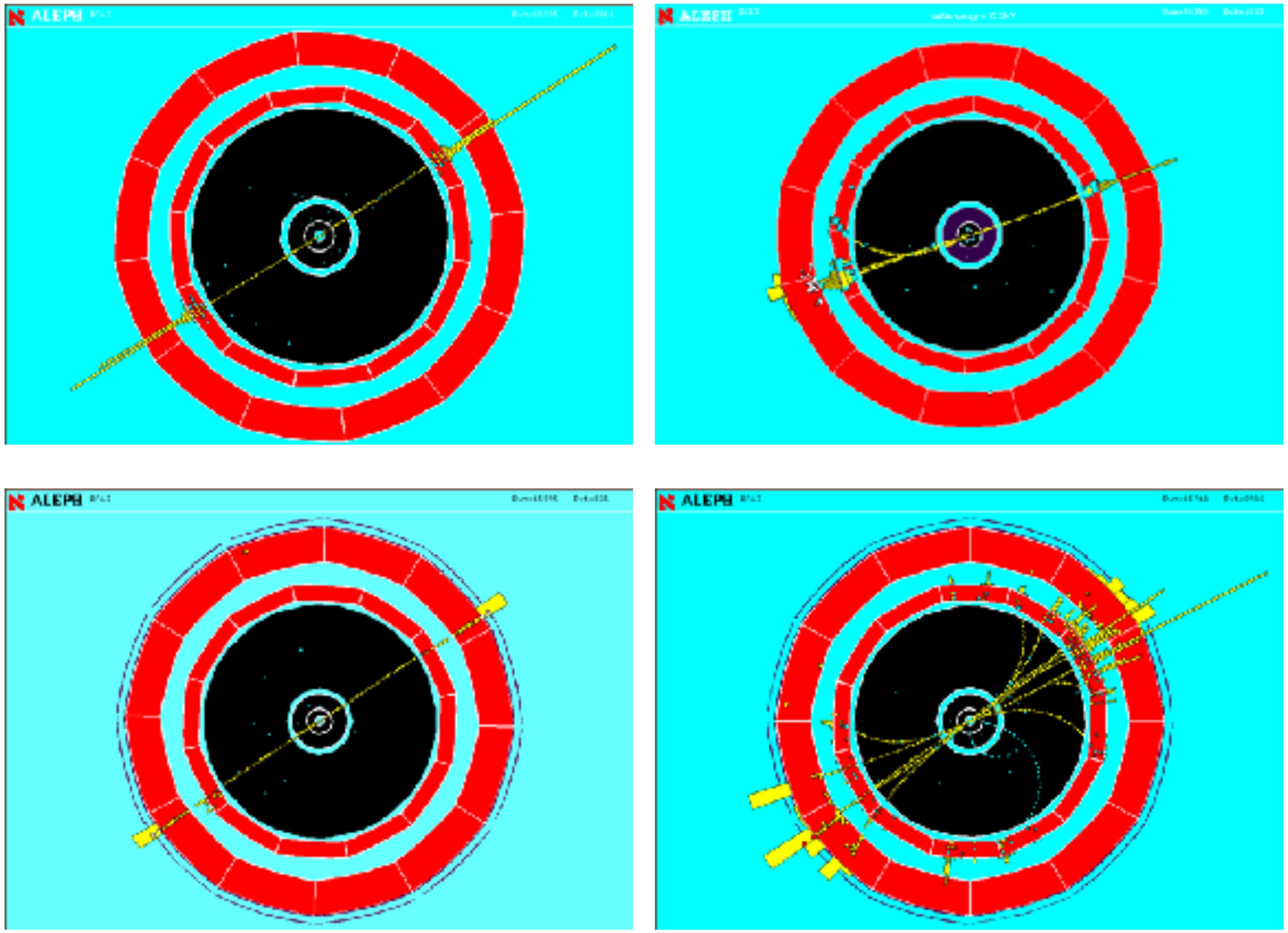
$$\sum |\mathcal{M}|^2 = 2g_Z^2 m_Z^2 (c_L^2 + c_R^2) \to g_Z^2 m_Z^2 (c_V^2 + c_A^2)$$

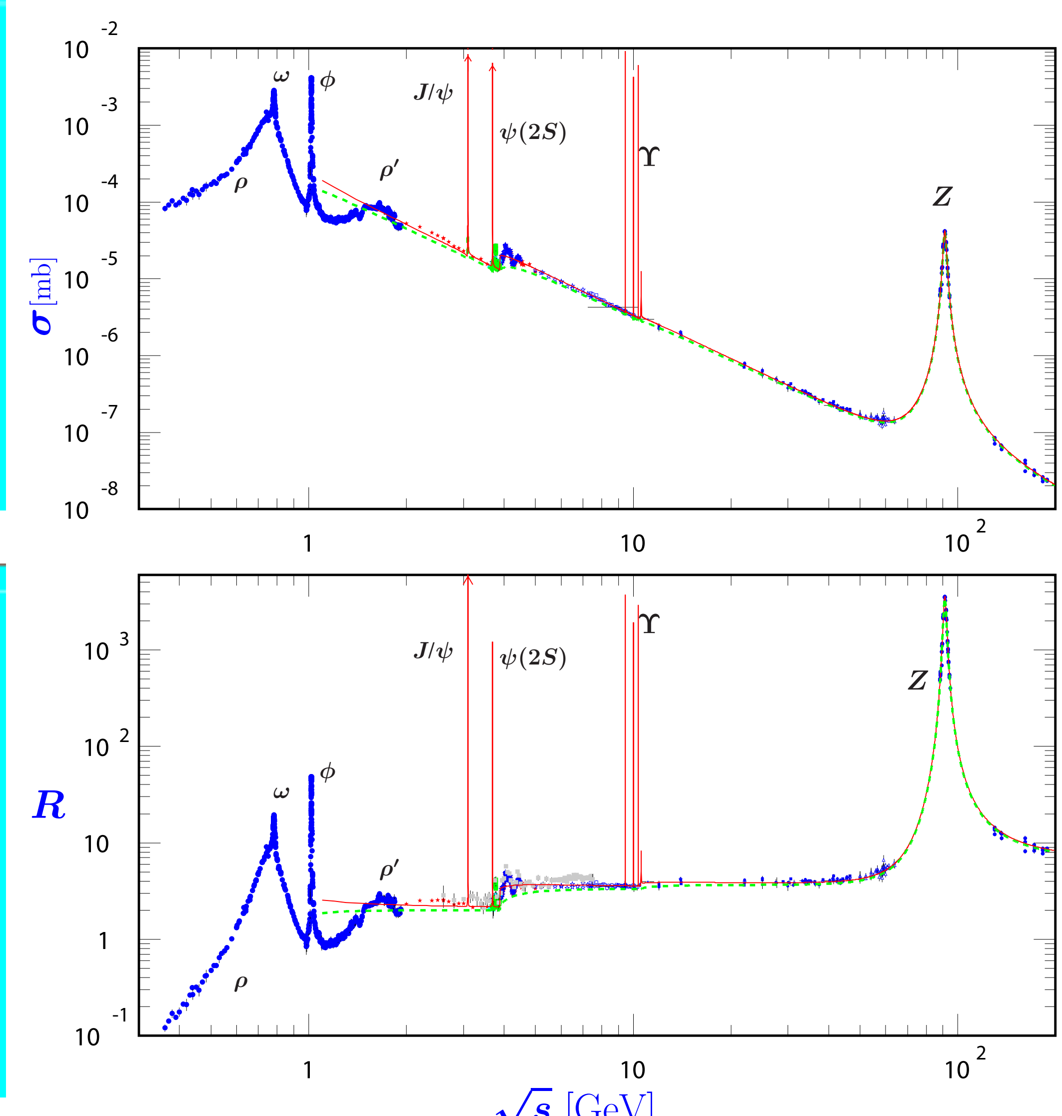
- Divide by the initial polarization states to average
- Putting it into our decay phase space formula

$$\Gamma = \frac{|\mathbf{p}|}{32\pi^2 m_Z^2} \int d\Omega |\mathcal{M}|^2 \quad = \frac{g_Z^2 m_Z^2}{48\pi} (c_V^2 + c_A^2)$$

	C_V	C_A	$C_V^2 + C_A^2$	REL.	FRAC.
ν_e, ν_μ, ν_τ	1/2	1/2	0.50	1.5	0.23
e, μ, t	$-1/2 + 2 \sin^2 \theta_W$	-1/2	0.251	0.753	0.12
u, c, t	$+1/2 - 4/3 \sin^2 \theta_W$	1/2	0.286	1.72	0.26
d, s, b	$-1/2 + 2/3 \sin^2 \theta_W$	-1/2	0.373	2.57	0.39

MEAUREMENTS AT LEP





	C_V	C_A	$C_V^2+C_A^2$	REL.	FRAC.	MEAS.
ν_e, ν_μ, ν_τ	1/2	1/2	0.50	1.50	0.20	0.20
e, μ, t	$-1/2 + 2\sin^2\theta_W$	-1/2	0.251	0.753	0.10	0.10
u, c, t	$+1/2 - 4/3\sin^2\theta_W$	1/2	0.286	1.716	0.23	0.23
d, s, b	$-1/2 + 2/3\sin^2\theta_W$	-1/2	0.373	3.357	0.46	0.47

LAGRANGIAN MECHANICS

- Describe a system with coordinates and its time derivatives:

$$L = L(q, \dot{q}) = T - U$$

- Equations of motion are obtained by minimizing the action

$$S = \int dt L(q_i, \dot{q}_i)$$

  - resulting in Euler-Largange equations $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$

- For a point particle in a potential with Cartesian coordinates:

$$L = \frac{1}{2} m \dot{q}_i^2 - U(q_i) \qquad \frac{d}{dt}(m\dot{q}_i) + \frac{\partial U}{\partial q_i} = 0 \qquad m\ddot{q}_i = -\frac{\partial U}{\partial q_i}$$

$$m\ddot{x} = -\frac{\partial U}{\partial x} \qquad m\ddot{y} = -\frac{\partial U}{\partial y} \qquad m\ddot{z} = -\frac{\partial U}{\partial z}$$

FOR "FIELDS:"

- Fields become the "coordinate" with space time as the "dynamical variable"
  - q(t) → f(x)
  - $L \Rightarrow \mathcal{L}(\phi(x), \partial_\mu \phi(x)) \quad L = \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi)$
  - The action is now defined as: $\int dt L = \int dt \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi) = \int d^4x \, \mathcal{L}(\phi, \partial_\mu \phi)$
  - Euler-Lagrange Equatoins:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \longrightarrow \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = \frac{\partial \mathcal{L}}{\partial \phi}$$

- Examples of Lagrangians and their equations of motion

$$\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 \qquad \partial_\mu \partial^\mu \phi + m^2 \phi^2 = 0$$

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi = 0 \qquad i\gamma^\mu \partial_\mu \psi - m\psi = 0$$

$$\mathcal{L}_P = \frac{-1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{8\pi}m^2 A^\nu A_\nu \qquad \partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + m^2 A^\nu = 0$$

LOCAL GAUGE INVARIANCE

- We can recast our previous discussion about local gauge invariance in the Lagrangian framework
- Example: consider the Dirac Lagrangian with local gauge transformation

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi = 0$$

$$\psi \to e^{iq\theta(x)}\ \psi$$

$$\partial_\mu\psi \to e^{iq\theta}\partial_\mu\psi + iq\partial_\mu\theta\ e^{iq\theta}\partial_\mu\psi$$

$$\mathcal{L} \to \mathcal{L} - q\ \bar{\psi}\gamma^\mu\ (\partial_\mu\theta)\ \psi$$

- As before, need to add a new field and interaction

$$\mathcal{L} \to \mathcal{L} - q\bar{\psi}\gamma^\mu\psi\ A_\mu \qquad A_\mu \to A_\mu - \partial_\mu\theta$$

- Another way to summarize this is to convert the derivative to a "covarinan derivative"

$$\partial_\mu \to D_\mu \equiv \partial_\mu + iqA_\mu$$

A FEW ENHANCEMENTS

- As it stands, the A field is static
- We can give it "life" by adding a kinematic term

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu \psi - q\bar{\psi}\gamma^\mu\psi \; A_\mu - \frac{1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{8\pi} m^2 A^\nu A_\nu$$

- but recalling the transformation: $A_\mu \to A_\mu - \partial_\mu \theta$
  - we find that the last term (the mass) is not gauge-invariant

- We can also extend to a "non-abelian" gauge symmetry:

$$\psi \to e^{ig\vec{\tau}\cdot\mathbf{a}(x)}\psi \equiv S\psi \qquad \partial_\mu\psi \to \partial_\mu(S\psi) = S(\partial_\mu\psi) + (\partial_\mu S)\psi$$

  - where as before we need to add another term and fields:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \; \boxed{-q(\bar{\psi}\gamma^\mu\vec{\tau}\psi)\cdot\mathbf{A}_\mu} \quad \vec{\tau}\cdot\mathbf{A}_\mu \Rightarrow S(\vec{\tau}\cdot\mathbf{A}_\mu)S^{-1} + i\left(\frac{\hbar}{q}\right)(\partial_\mu S)S^{-1}$$

  - and the mass term is once again forbidden
  - the gauge invariance can be restored by: $\partial_\mu \to D_\mu \equiv \partial_\mu + ig\vec{\tau}\cdot\mathbf{A}_\mu$

ONE MORE DILEMMA

- Consider the Dirac mass term:

$$m\bar{\psi}\psi$$

$$= \bar{\psi}_L\psi_L + \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L + \bar{\psi}_R\psi_R$$

$$= \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$

- mass terms result form the coupling of left and right chiral states of a particle
- this violates gauge symmetry in the $SU(2)_L x U(1)_Y$ model of weak interactions
- thus direct fermion mass terms (quarks, leptons) are also forbidden.

SUMMARY:

- Electroweak mixing makes predictions about c_V, c_A (alternatively c_L, c_R) couplings of the Z boson that can be tested
  - different particle species have different couplings
- We can recast the equations of motion in terms of Lagrangians and reintroduce gauge symmetry
- We find that gauge symmetry really doesn't like masses
- Please read chapters 17.4-17.5