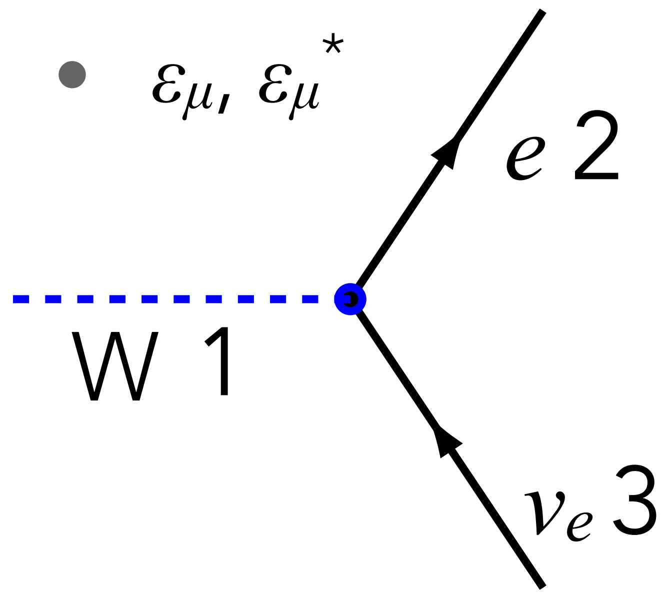
PHY489/1489: LECTURE 18

NEUTRINO OSCILLATIONS

GAUGE BOSON FEYNMAN RULES

- The Feynman rule for an incoming(outgoing) vector boson is its polarization vector:
  - ε_μ, $\varepsilon_\mu{}^*$



$$\frac{-ig}{2\sqrt{2}}\ \left[\bar{u}_2\gamma^\mu(1-\gamma^5)v_3\right]\ \times(2\pi)^4(p_1-p_2-p_3)\ \ \epsilon_\mu(p_1)$$

$$\mathcal{M}=\frac{g}{2\sqrt{2}}\ \left[\bar{u}_2\gamma^\mu(1-\gamma^5)v_3\right]\ \epsilon_\mu(p_1)$$

- Relative to the z-axis, we can define

$$\epsilon_{+\mu}=\frac{1}{\sqrt{2}}(0,1,i,0)$$

$$\epsilon_{-\mu}=\frac{1}{\sqrt{2}}(0,-1,+i,0)$$

$$\epsilon_{L\mu}=\frac{1}{m}(p_z,0,0,E)$$

- for the fermion, we know (in the massless limit):
  - e, ν_e come out with energy $M_W/2$
  - e with left helicity, $\bar{\nu}_e$ with right helicity

$$\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\rightarrow\bar{u}_{2\downarrow}\gamma^\mu v_{3\uparrow}$$

- We evaluated this combination back in QED

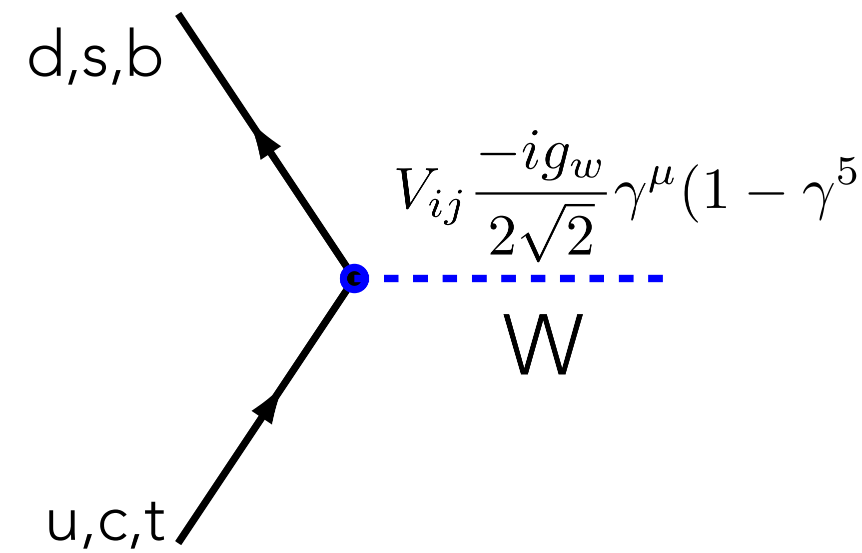
$$\bar{u}_{2\downarrow}\gamma^\mu v_{3\uparrow}\rightarrow 2E(0,-\cos\theta,-i,\sin\theta)$$

MIXING:

u	c	t
d	s	b

- recall the CKM matrix in quark interactions
  - formalizes transitions between generation
  - relation between mass and "flavor" states
    - d' quark is the quark state that couples to u quark

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \times \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- The same situation can arise in neutrinos
  - what we have defined as "ν_e", "ν_μ" and "ν_τ" *are* the flavour states (analogous to d', s' b')
  - ν_1, ν_2, ν_3 are the mass eigenstates analogous to d, s, b

ν_e	ν_μ	ν_τ
e	μ	τ

FLAVOR TRANSITIONS

- Recall that energy eigenstates are "stationary" in QM:

$$|\psi(t)\rangle \to |\psi(0)\rangle e^{-iEt}$$

- a neutrino in a mass eigenstate will stay in the same eigenstate
- However, a flavour state is a linear combination of mass eigenstates:

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle = \sum_i U_{ei}|\nu_i\rangle$$

  - if we consider a neutrino at rest, we would have:

$$|\nu_e\rangle \to \sum_i U_{ei}\; e^{-im_i\tau}|\nu_i\rangle$$

    - proper time τ,m are the elapsed time, energy in the rest frame

$$m_i\tau = p \cdot x = E_i t - \mathbf{p}_i \cdot \mathbf{x}$$

    - so that in other reference frames we can write:

$$|\nu_e\rangle \to \sum_i U_{ei}\; e^{-i(E_i t - \mathbf{p_i}\cdot\mathbf{x}}|\nu_i\rangle$$

KINEMATICS

- If we assume that the flavour state is composed of mass states of common energy E, with E ≫ m_i

$$p_i = \sqrt{E^2 - m_i^2} = E\sqrt{1 - \frac{m_i^2}{E^2}} \sim E\left(1 - \frac{m_i^2}{2E^2}\right)$$

$$Et - p_i x \sim Et\left(1 - \frac{m_i^2}{2E}\right)$$

- so then our flavour state evolves as

$$|\nu_e\rangle \to \sum_i U_{ei}\, e^{-i(E_i(t-L) + \frac{m_i^2}{2E}L)}|\nu_i\rangle$$

- the first term in the exponential is a common overall phase that can be dropped:

$$|\nu_\alpha\rangle \to \sum_i U_{\alpha i}\, e^{-i\frac{m_i^2}{2E}L}|\nu_i\rangle$$

AMPLITUDE → PROBABILITY

- We can find the amplitude for a $\nu_\alpha \rightarrow \nu_\beta$ transition:

$$\langle \nu_\beta | \nu_\alpha(L) \rangle = \sum_i U_{\alpha i} \, e^{-i \frac{m_i^2}{2E} L} \langle \nu_\beta | \nu_i \rangle$$

  - if $\langle \nu_i | \nu_\alpha \rangle$ = $U_{\alpha i}$, then $\langle \nu_\beta | \nu_i \rangle$ = = $U^*_{\beta i}$ so that

$$\langle \nu_\beta | \nu_\alpha(L) \rangle = \sum_i U_{\alpha i} U^*_{\beta i} \, e^{-i \frac{m_i^2}{2E} L}$$

  - to get a probability, we take $|\langle \nu_\beta | \nu_\alpha(L) \rangle|^2$ and we get

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta; L) \quad = \quad & \delta_{\alpha\beta} \\ & -4\mathrm{Re}\left[\textstyle\sum_{i>j} U_{\alpha i} U^*_{\beta i} U_{\alpha j} U^*_{\beta j}\right] \sin^2 \frac{\Delta m^2_{ij} L}{4E} \\ & +2\mathrm{Im}\left[\textstyle\sum_{i>j} U_{\alpha i} U^*_{\beta i} U_{\alpha j} U^*_{\beta j}\right] \sin \frac{\Delta m^2_{ij} L}{2E} \end{aligned}$$

$$\Delta m^2_{ij} \equiv m_i^2 - m_j^2$$

NEUTRINO OSCILLATIONS

$$P(\nu_\alpha \to \nu_\beta; L) = \delta_{\alpha\beta} - 4\mathrm{Re}\left[\sum_{i>j} U_{\alpha i}U^*_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin^2\frac{\Delta m^2_{ij}L}{4E} + 2\mathrm{Im}\left[\sum_{i>j} U_{\alpha i}U^*_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin\frac{\Delta m^2_{ij}L}{2E}$$

$$\Delta m^2_{ij} \equiv m^2_i - m^2_j$$

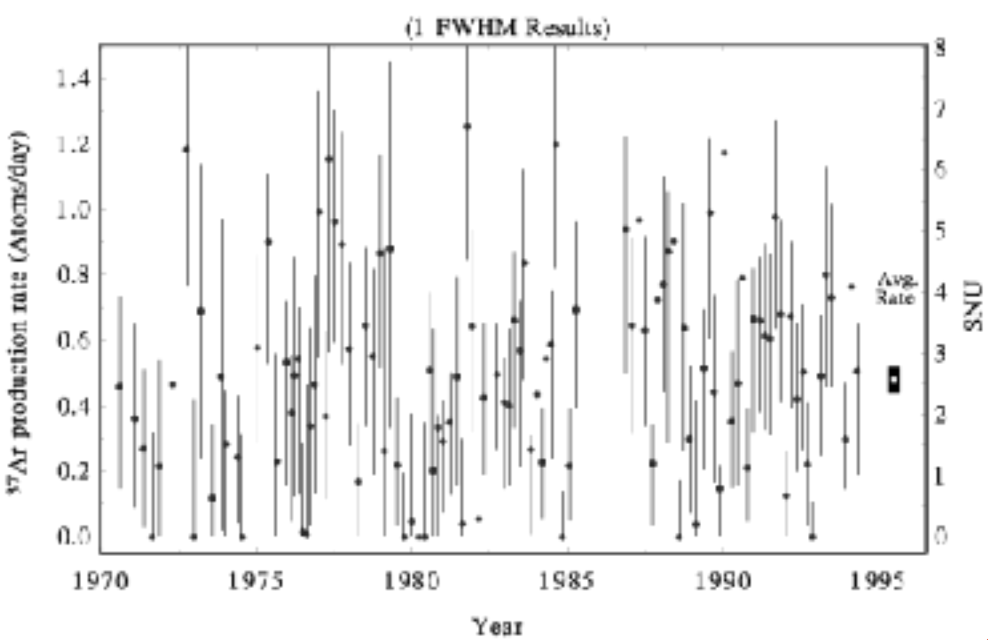
- "Oscillations": probability is sinusoidal in L/E
- "Amplitude" is determined by mixing matrix U
  - if U is diagonal (i.e. mass eigenstates = flavour eigenstates) then amplitude of oscillation is 0.
- "Wavelength" is determined by $\Delta m_{ij}^2 = m_i^2 - m_j^2$
  - non-zero and non-degenerate masses needed for $P(\nu_\alpha \to \nu_\beta) \neq 0$

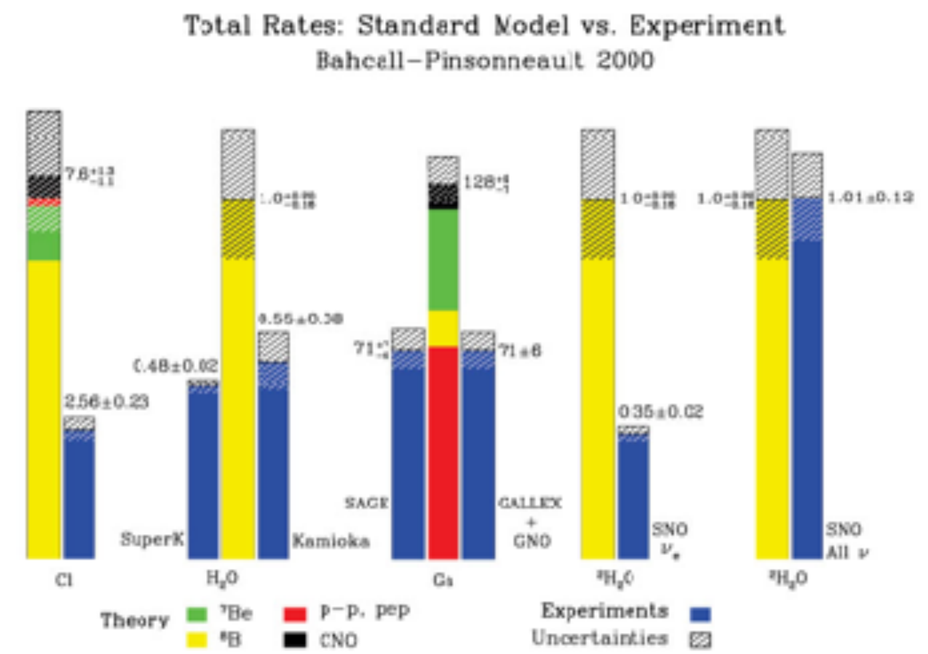
NEUTRINO MASS

- Why would one assume that neutrinos are massless?

ANOMALIES:

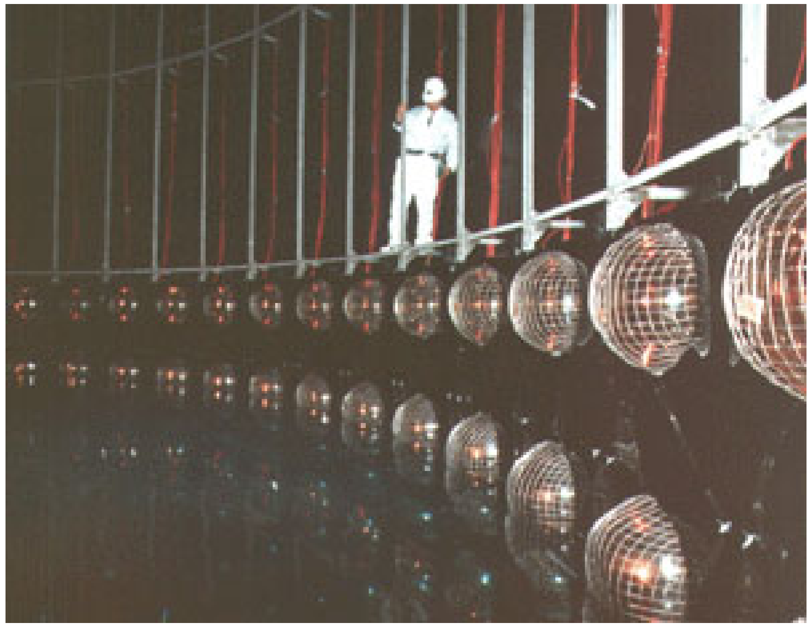
$$\nu_e + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}$$





$$
\begin{aligned}
p + N \to \quad & \pi^\pm + X \\
& \hookrightarrow \mu^\pm + \nu_\mu \\
& \qquad \hookrightarrow e^\pm + \nu_\mu + \nu_e
\end{aligned}
$$

- Persistent deficit in
  - ν_e from the sun
  - ν_μ produced in the atmosphere
- why do we not see as many neutrinos as expected?



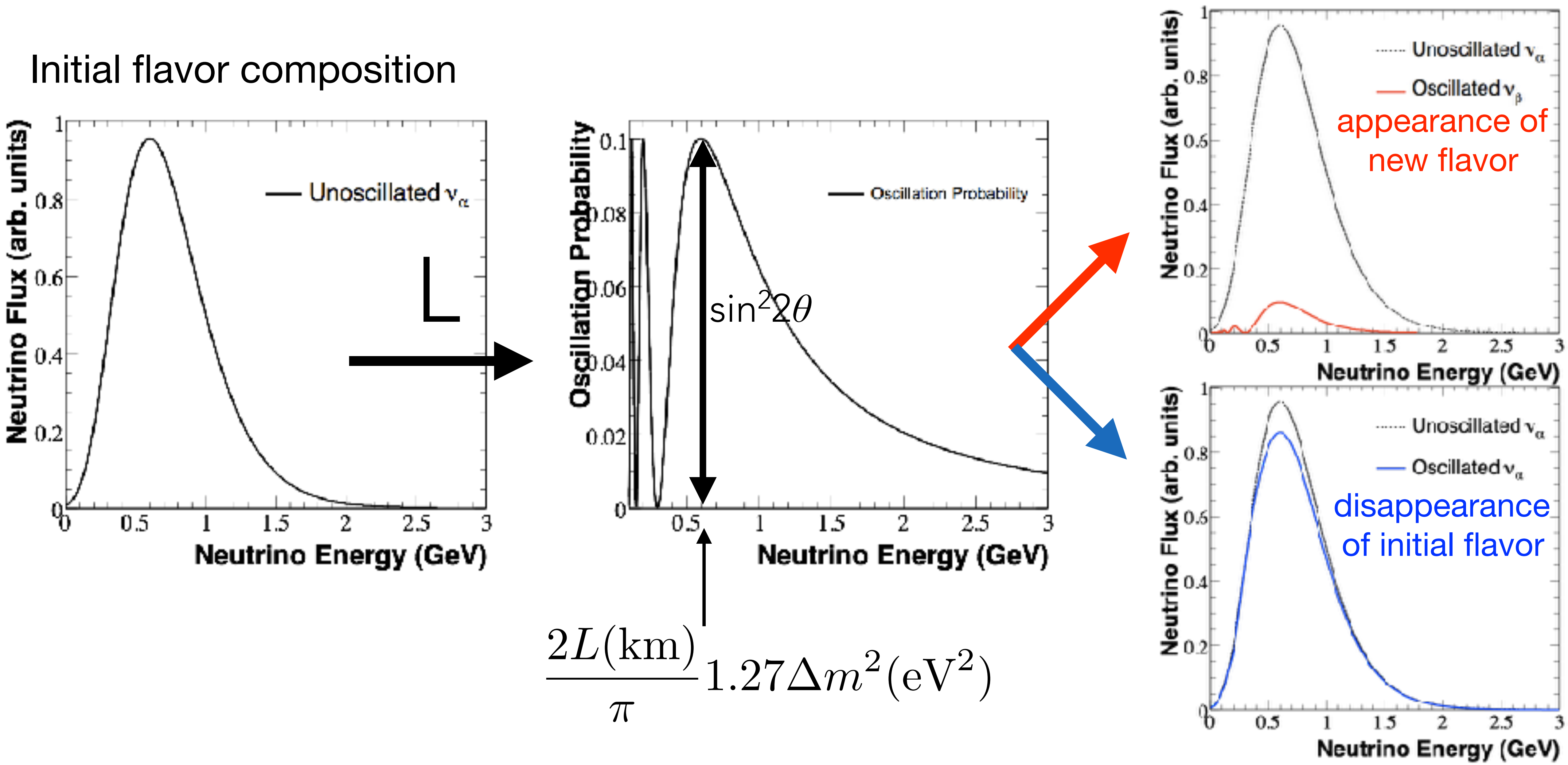
TWO FLAVOR MODEL

- With two flavours, we can write U as

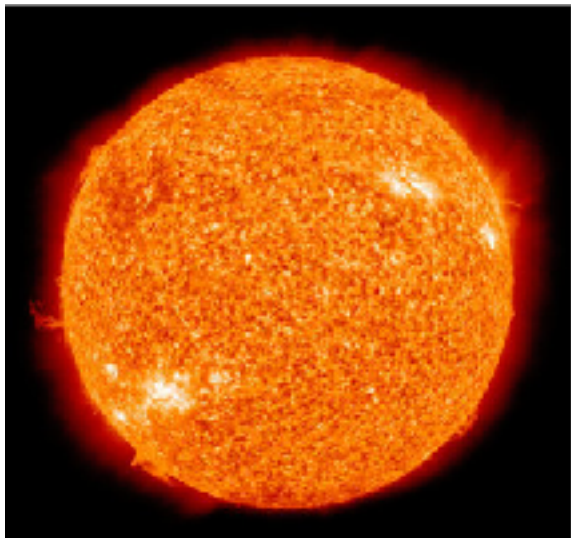
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

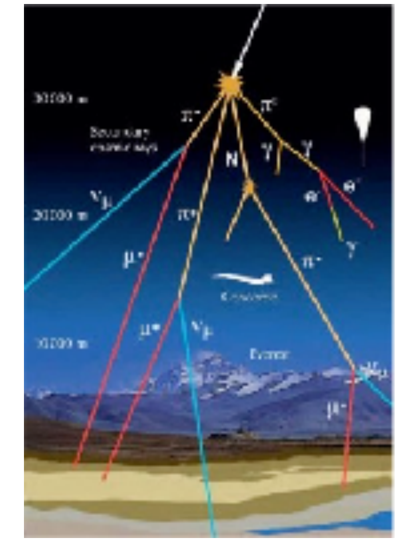
$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta \times \sin^2 \Delta m^2_{21} \frac{L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \times \sin^2 \Delta m^2_{21} \frac{L}{4E}$$



NEUTRINO SOURCES

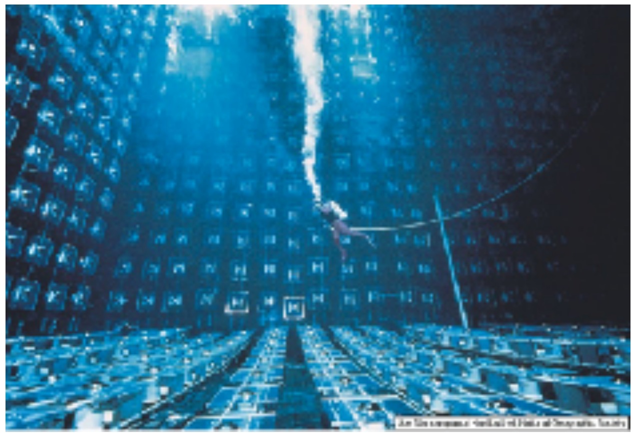


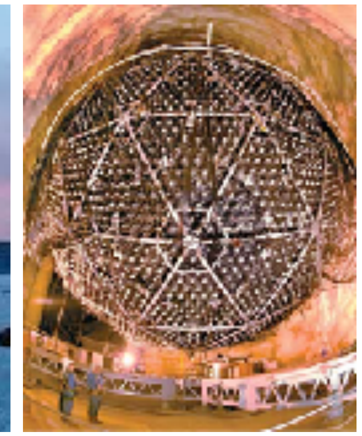


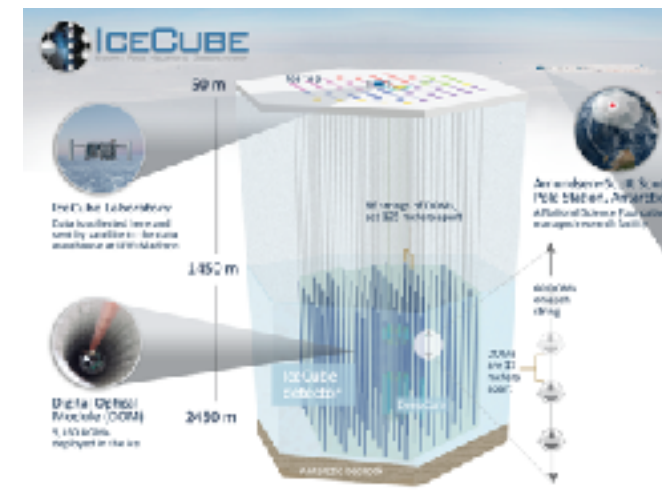
- Nuclear fission/Fusion
  - solar
    - 3% of sun's energy radiated as neutrinos
    - 10^{11} $\bar{\nu}/cm^2$/sec on surface of earth
  - reactor:
    - ~5% of reactor power emitted as $\bar{\nu}$
    - 10^{20} $\bar{\nu}$/sec emitted by typical GW reactor
  - Typical energy ~O(MeV)
    - only ν_e charged-current and neutral current interactions visible

- Meson/muon decays
  - e.g. pion decay ($\pi \rightarrow \nu_\mu + \mu$)
  - atmospheric neutrinos
    - π/K/μ produced in atmosphere by cosmic ray protons
  - accelerator-based neutrinos
    - π/K/μ produced by high energy protons produced by accelerators
  - Typical energy ~O(GeV)
    - can observe charged current interactions of ν_e, ν_μ, sometimes ν_τ

NEUTRINO DETECTORS

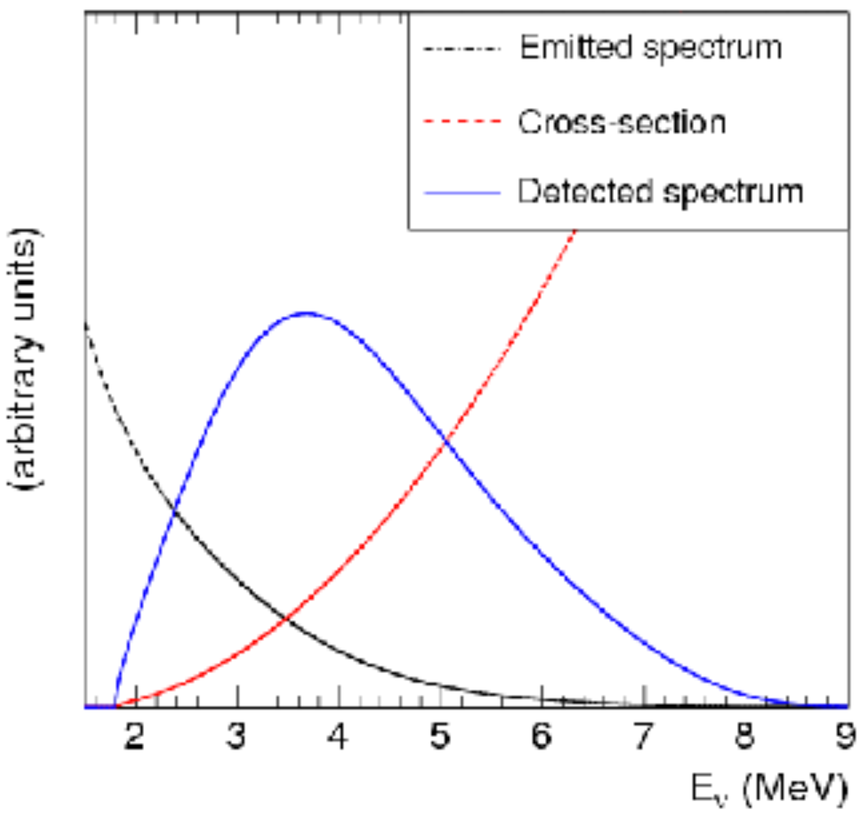


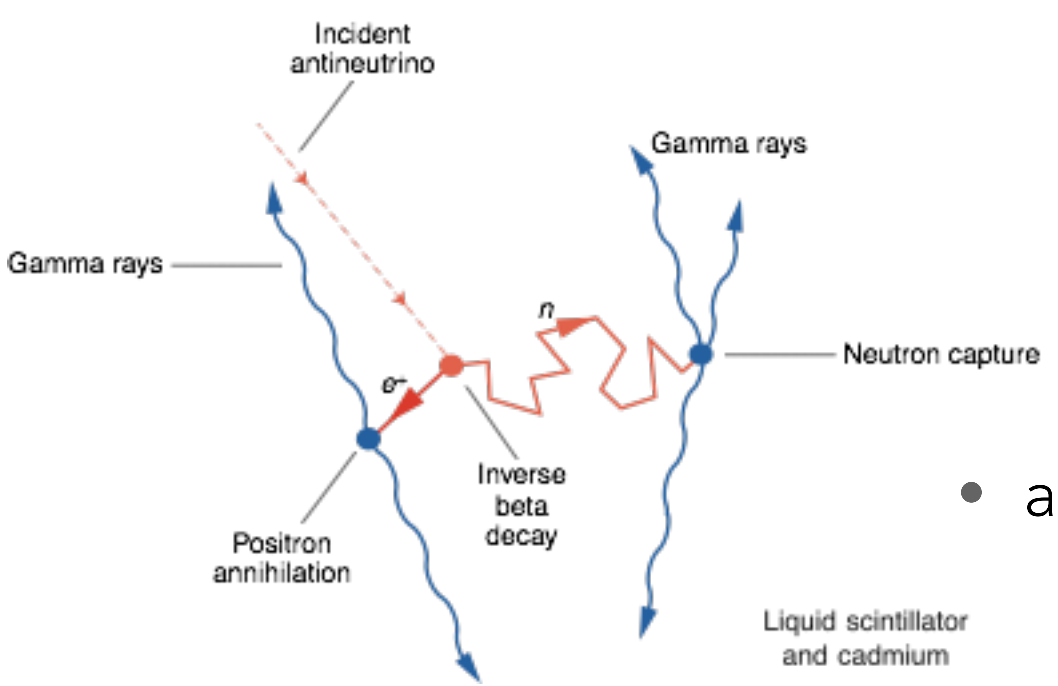




- Large detector/volume needed to gather neutrino interactions
    - neutrino detectors have long been about scalability
        - massive detectors that can still provide the information we need
        - Neutrino detectors have been produced with:
            - steel from decommissioned battleships
            - mineral oil/scintillator
            - large extruded PVC cells

REACTOR EXPERIMENTS





- detect antineutrinos using "inverse beta decay"

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

- two-step signature pioneered by Reines and Cowan
  - "prompt" signature from positron
  - "delayed" signature from neutron capture
- Due to low energies involved, large liquid scintillator detectors have been the preferred technology
  - large light yield from scintillation for good energy resolution
  - neutron detection from capture process

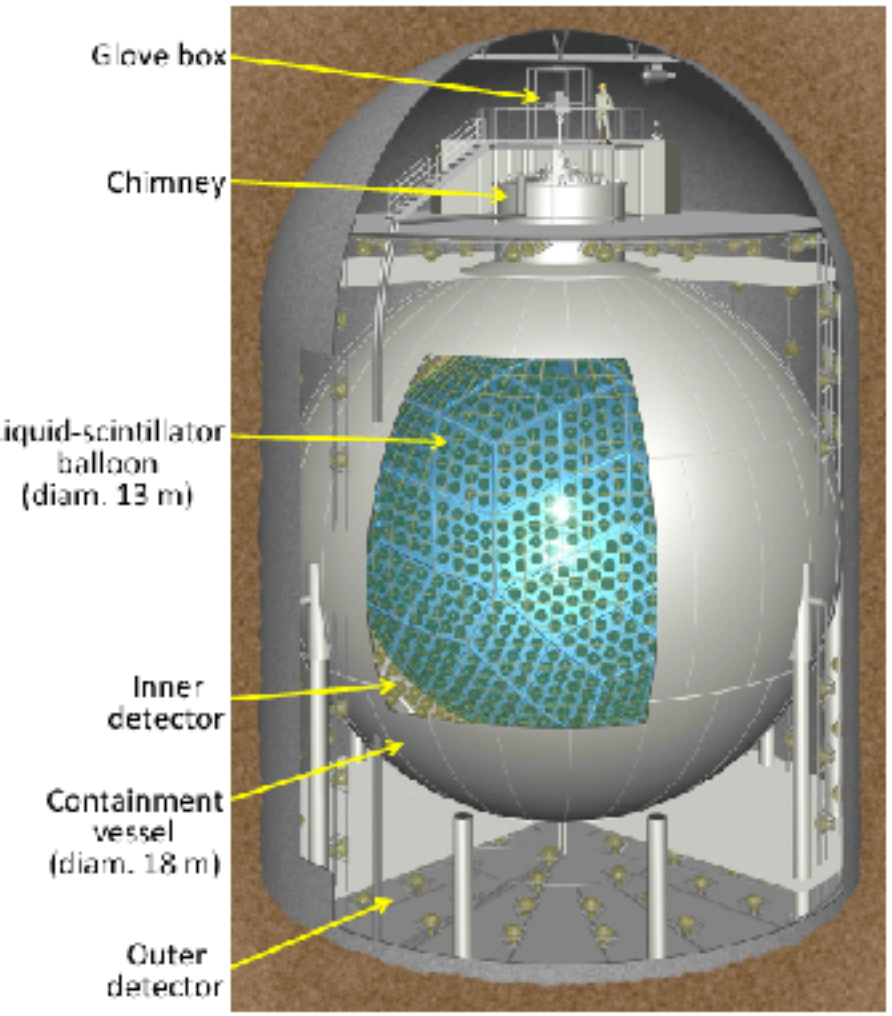
$$n + p \rightarrow d + \gamma(2.2 \text{ MeV})$$

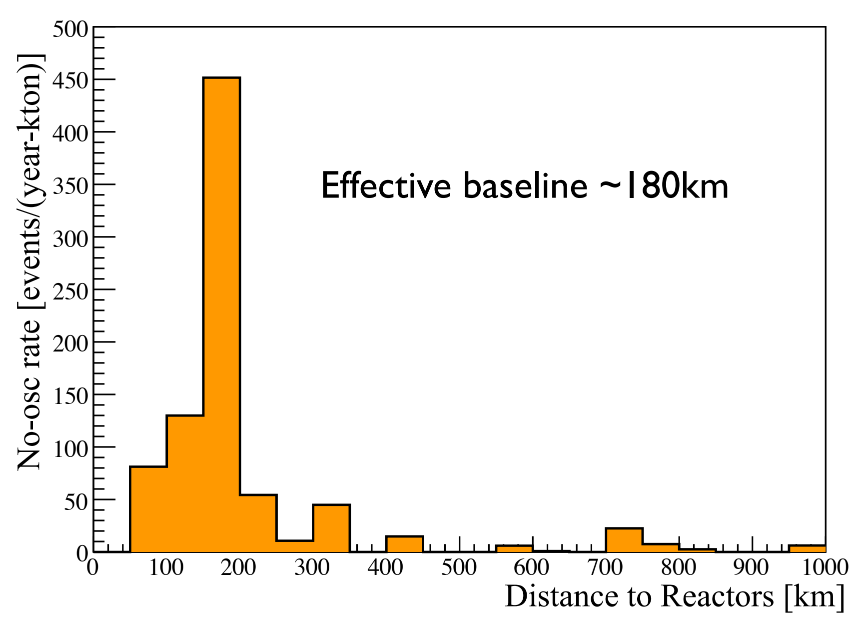
    - photon detection can be enhanced by doping with other nuclei with high neutron capture cross section and photon energy emission
- antineutrino energy can be reconstructed as:

$$E_{\bar{\nu}} \sim E_e + \langle E_n \rangle + 0.8\text{MeV}$$

KAMLAND

- Large liquid scintillator detector in the Kamioka mine (2002-2007)
  - 1 kT of liquid scintillator suspended in pure mineral oil
  - 1879 50 cm photomultiplier tubes to detect scintillation light
- antineutrinos from 55 nuclear reactors in Japan
  - 80% of antineutrinos produced by reactors between 130-220 km

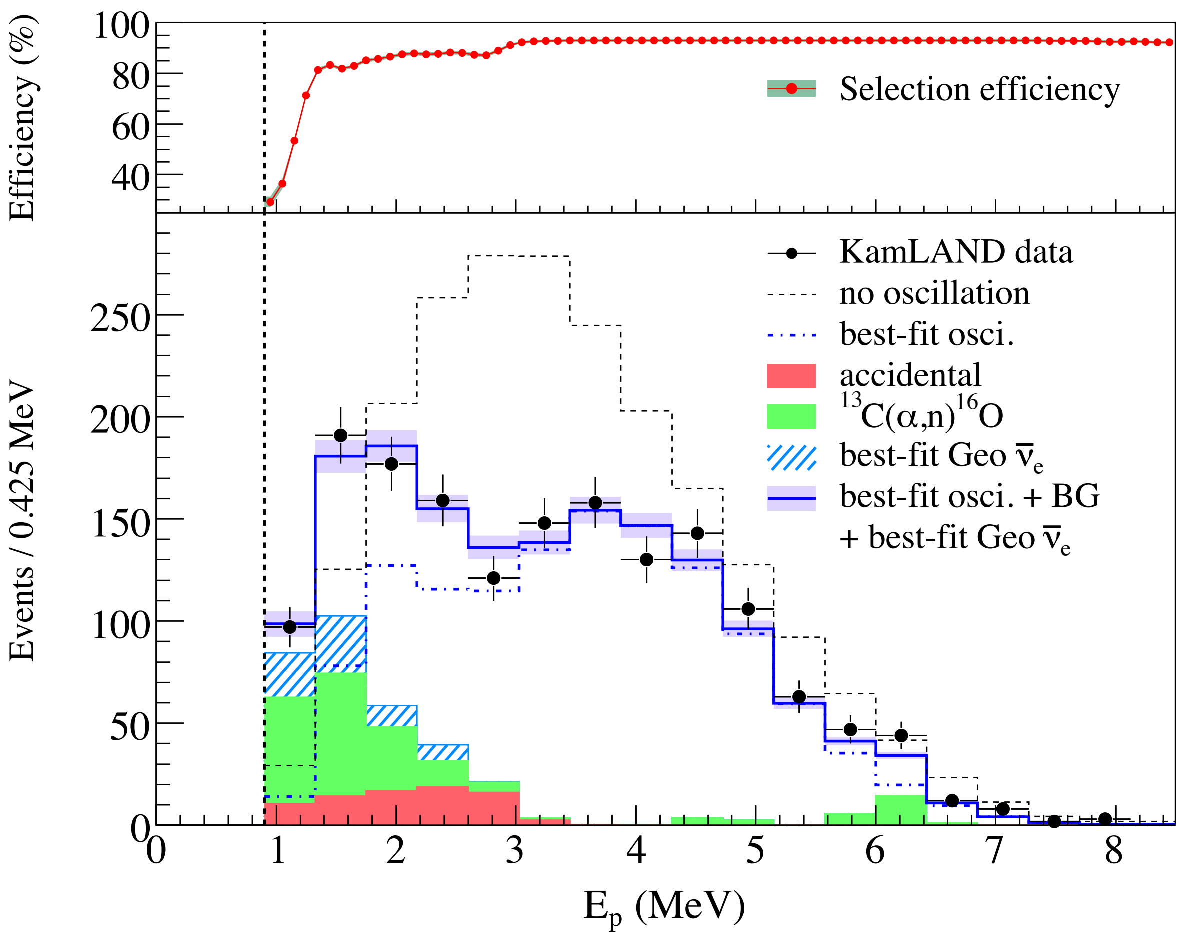


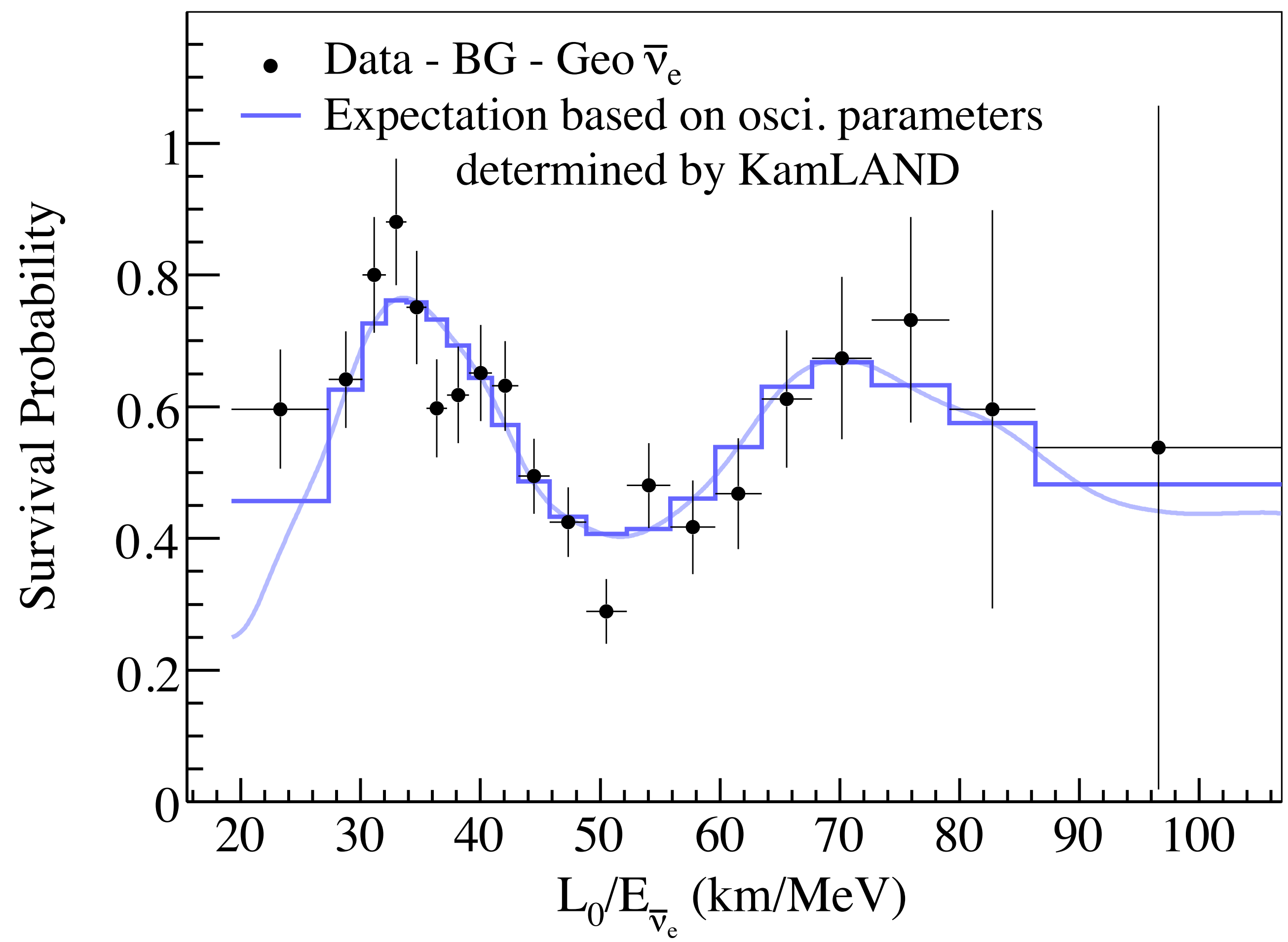


- Known distances to reactors allow $\bar{\nu}_e$ disappearance vs. L/E to be measured

RESULTS

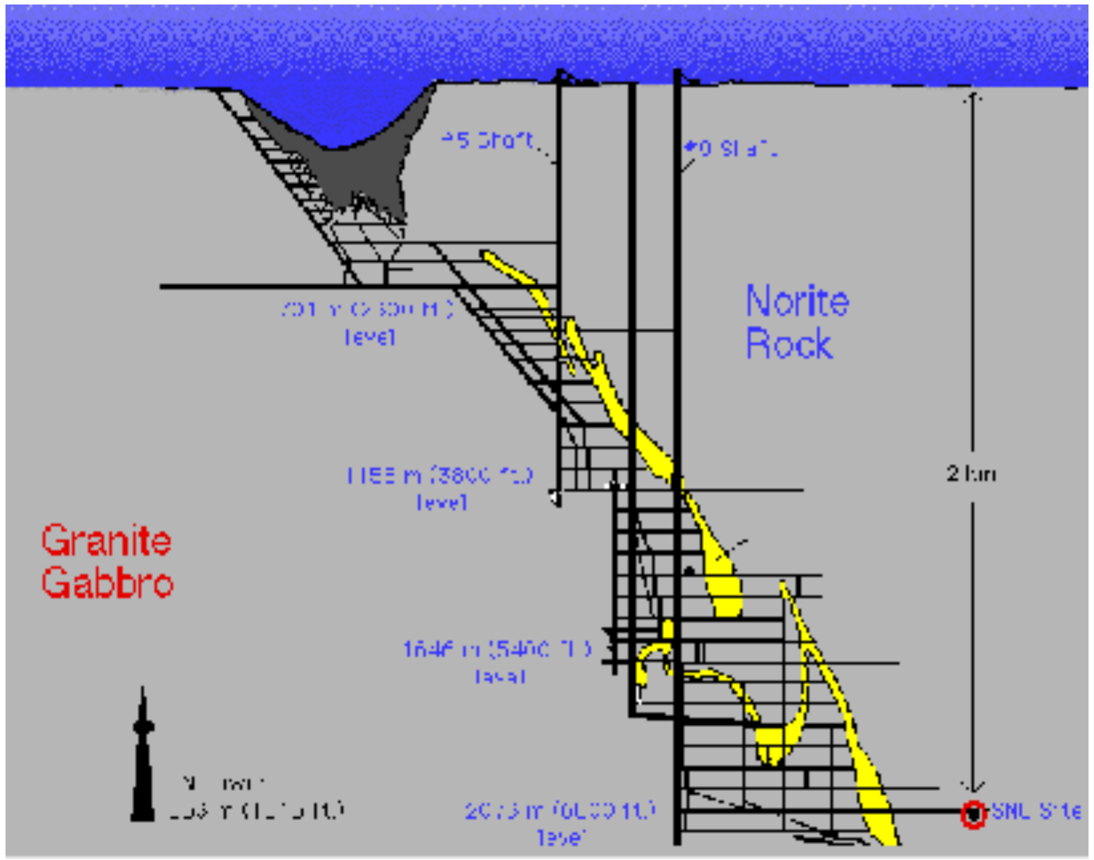
$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \times \sin^2 \Delta m^2_{21} \frac{L}{4E}$$

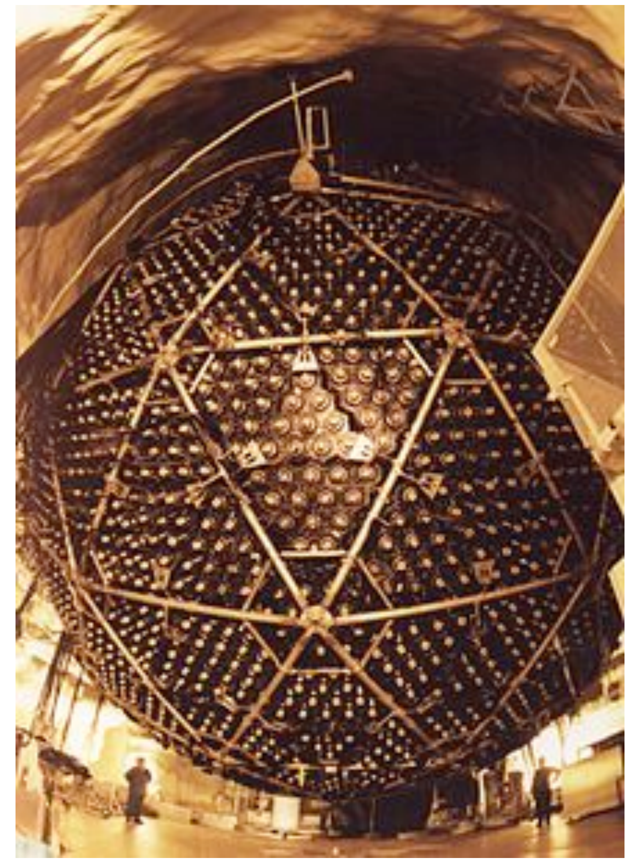


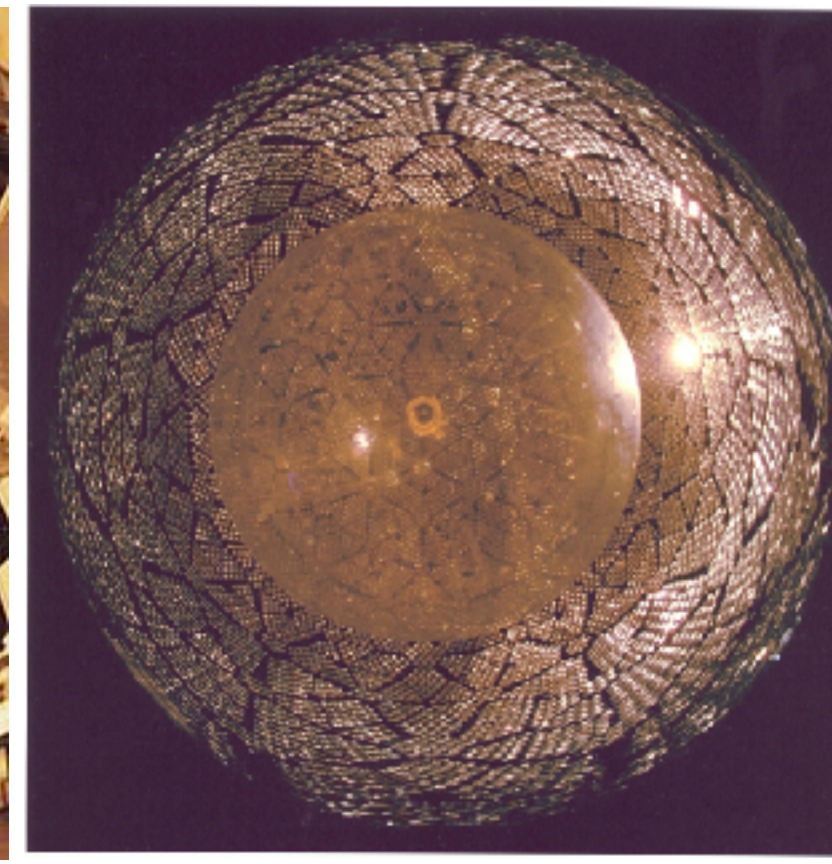


- Energy-dependent deficit of $\bar{\nu}_e$ measured
- Deficit (ratio to expectation without oscillations) versus L/E shows oscillation pattern

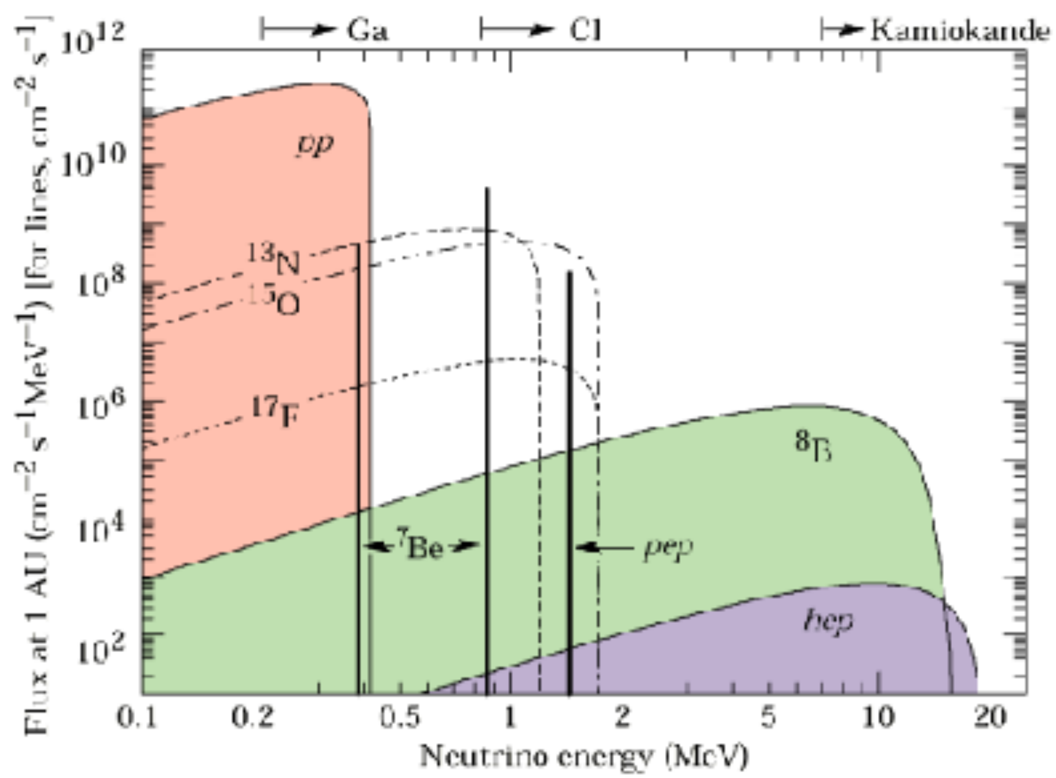
SNO





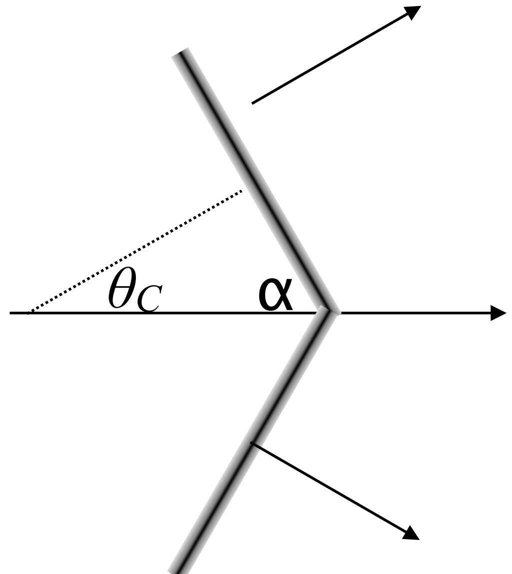


- Large (heavy) water Cherenkov detector 2 km underground in Sudbury, ON
  - "Sudbury Neutrino Observatory"
- 1 kton of heavy water (D_2O) in an acrylic vessel suspended in light water (H_2O)
- viewed by 9456 20 cm photomultiplier tubes
- Observe neutrinos from solar fusion processes



CHERENKOV RADIATION

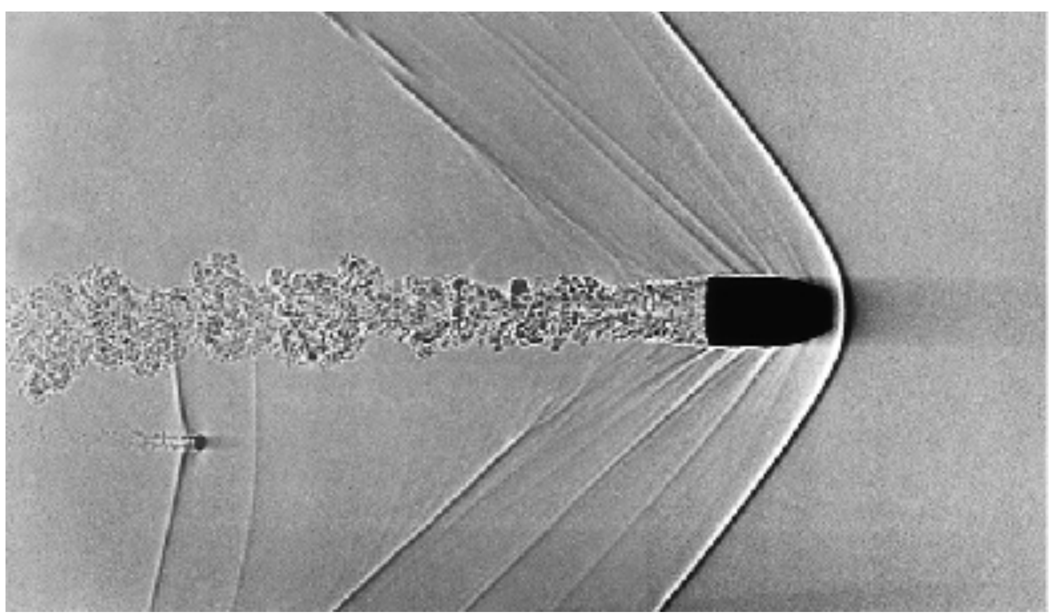
courtesy findagrave.com

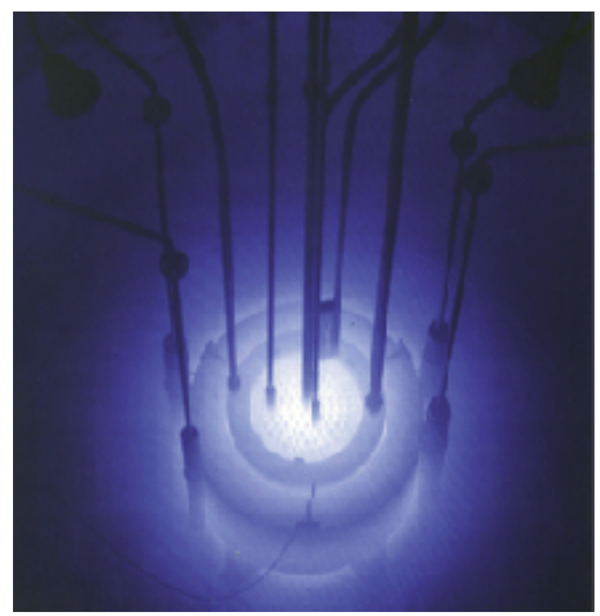


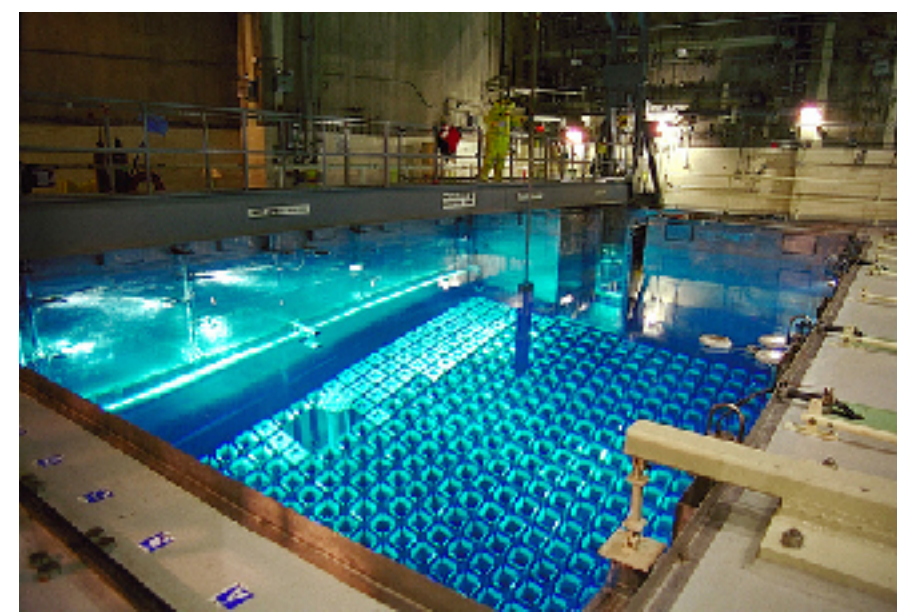
$$\sin\alpha = \frac{v_s}{v} \quad \alpha = \frac{\pi}{2} - \theta_C$$

- Charged particle passing through a dielectric medium (n > 1) induces a EM disturbance
  - If v >c_n, the disturbance piles up
  - EM "shock wave" emitted with angle θ_C

$$\cos\theta_C = \frac{c}{nv} = \frac{1}{n\beta}$$

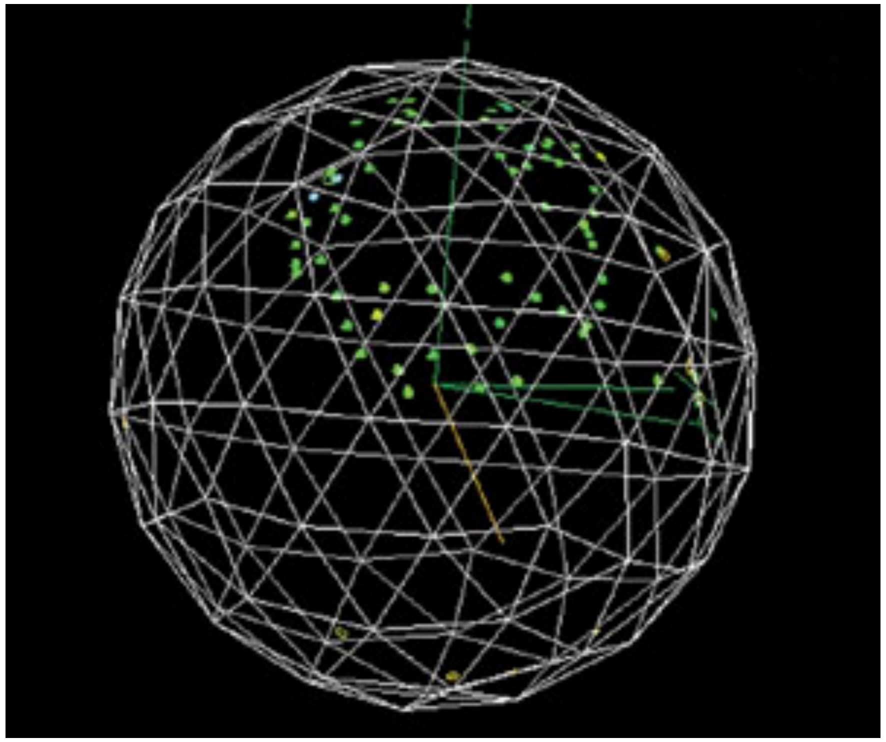




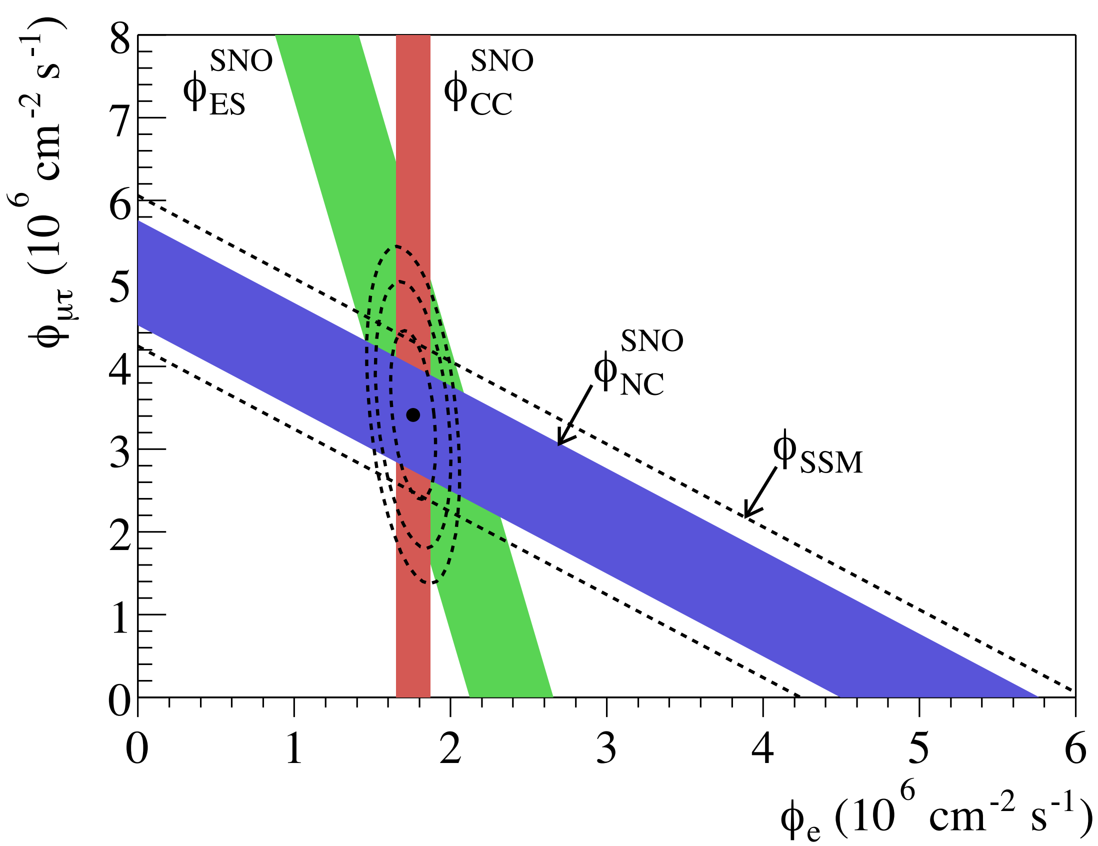


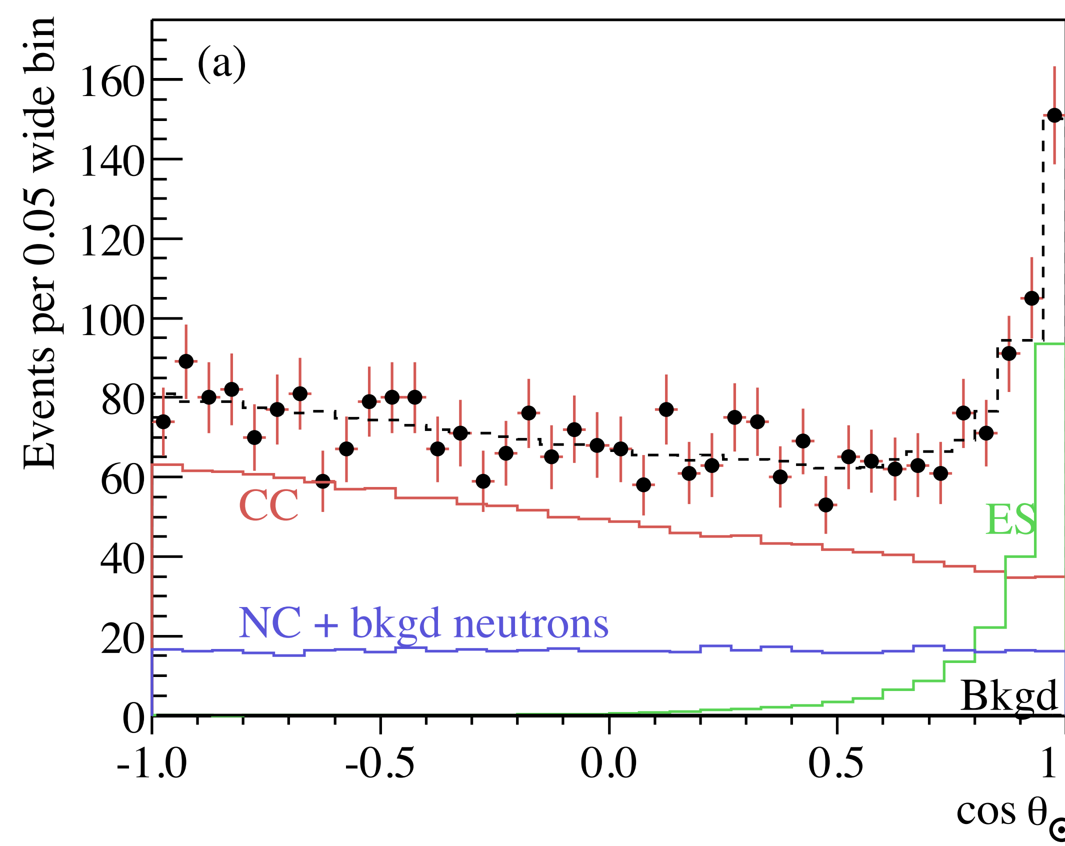
- Analogous to other (mechanical) systems where a disturbance exceeds the propagation velocity
  - e.g. "sonic boom" from supersonic object

NEUTRINO INTERACTIONS AT SNO



- Three channels observed:
- "CC": $\nu_e + d \rightarrow e^- + p + p$
  - sensitive only to ν_e from the sun
- "NC": $\nu_x + d \rightarrow \nu_x + n + p$ [$n + d \rightarrow t + \gamma$(6.25 MeV)]
  - equally sensitive to all neutrino flavours (ν_e, ν_μ, ν_τ)
- "ES": $\nu_x + e^- \rightarrow \nu_x + e^-$
  - interactions in all flavors, but ν_e: $\sigma(\nu_e) \sim 6.5 \times \sigma(\nu_\mu)$ or $\sigma(\nu_\tau)$)





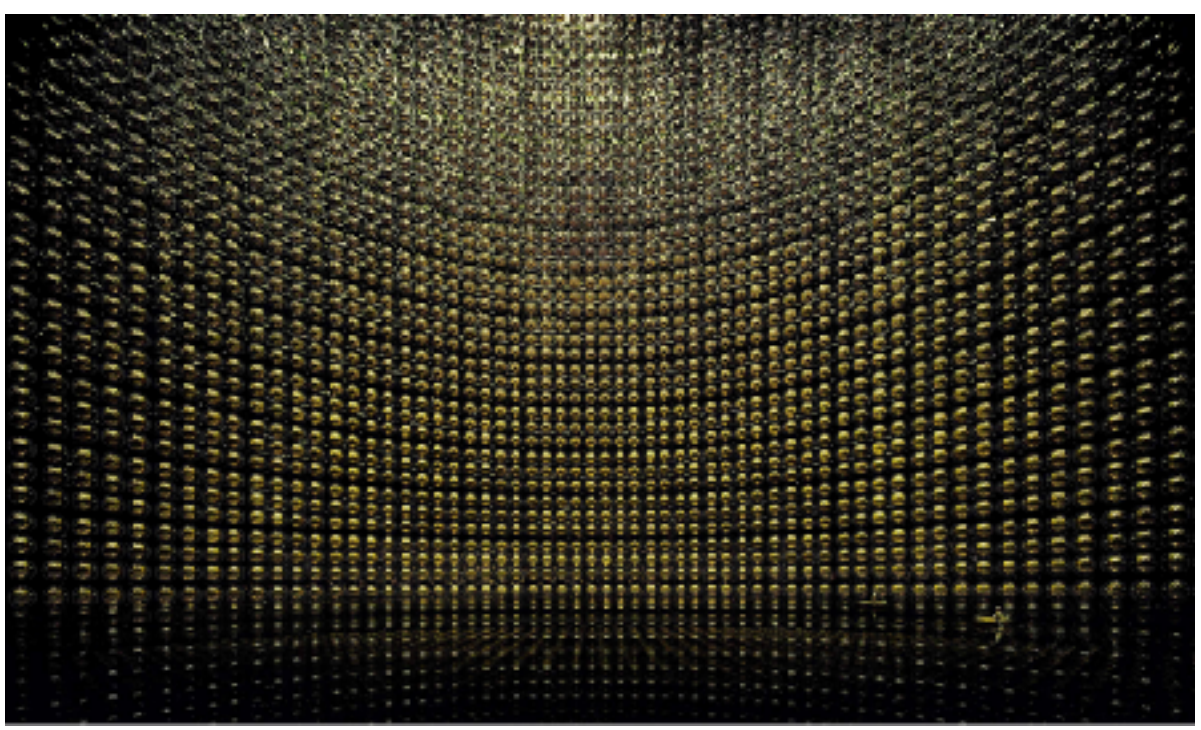
Conclusively resolved the "solar neutrino deficit"

ATMOSPHERIC NEUTRINOS

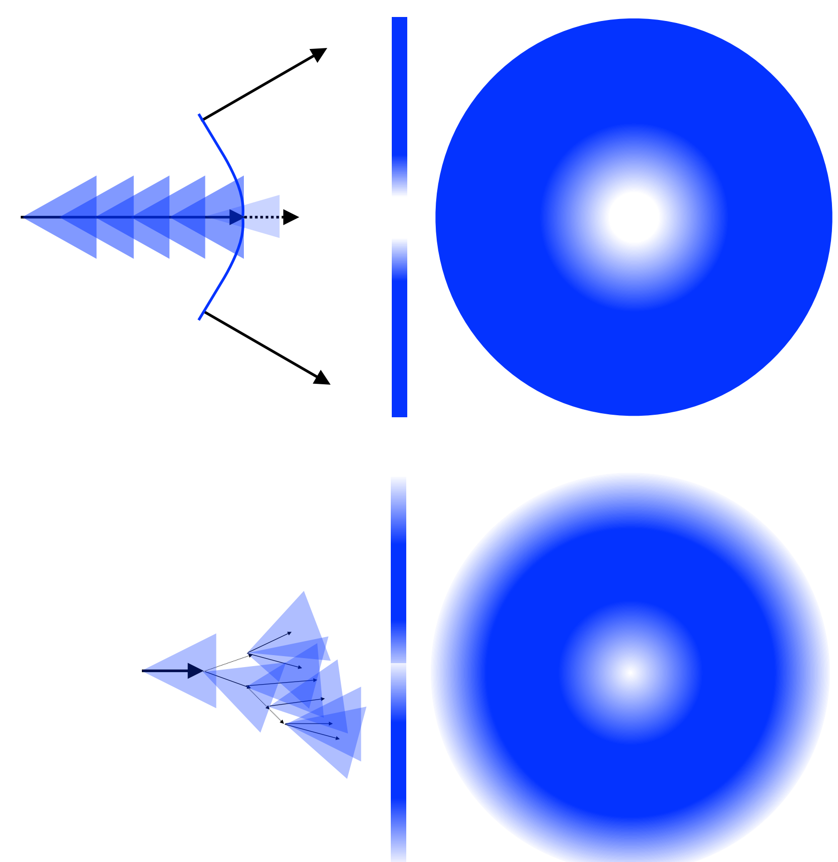
- Atmospheric neutrinos are produced by the interaction of cosmic ray protons:

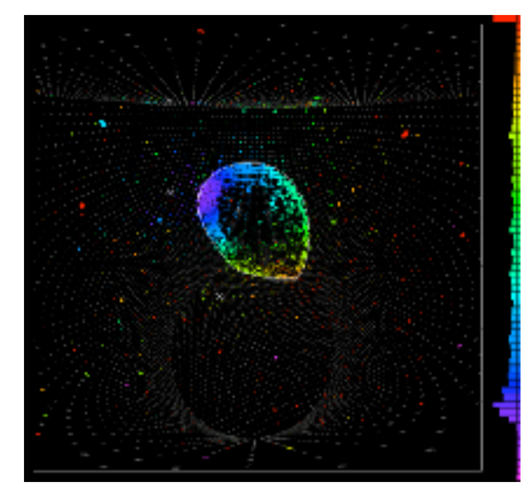
$$p + N \rightarrow \pi^{\pm} + X \quad \Longrightarrow \quad \begin{aligned} \pi^{+} &\rightarrow \mu^{+} + \boxed{\nu_{\mu}} \\ &\qquad \hookrightarrow e^{+} + \nu_{e} + \boxed{\bar{\nu}_{\mu}} \\ \pi^{-} &\rightarrow \mu^{-} + \boxed{\bar{\nu}_{\mu}} \\ &\qquad \hookrightarrow e^{-} + \bar{\nu}_{e} + \boxed{\nu_{\mu}} \end{aligned}$$

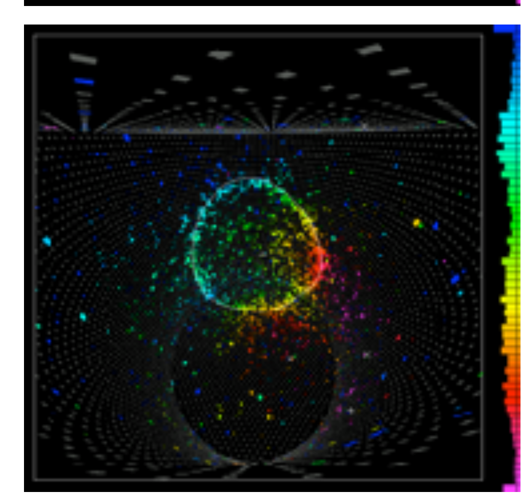
- Naively, expect a 2:1 ratio of muon (anti)neutrino to electron (anti)neutrino ratio
    - can we test this by identifying muon neutrinos and electron neutrinos?
    - look for muon production (from ν_{μ}) and electron production (from ν_{e}).



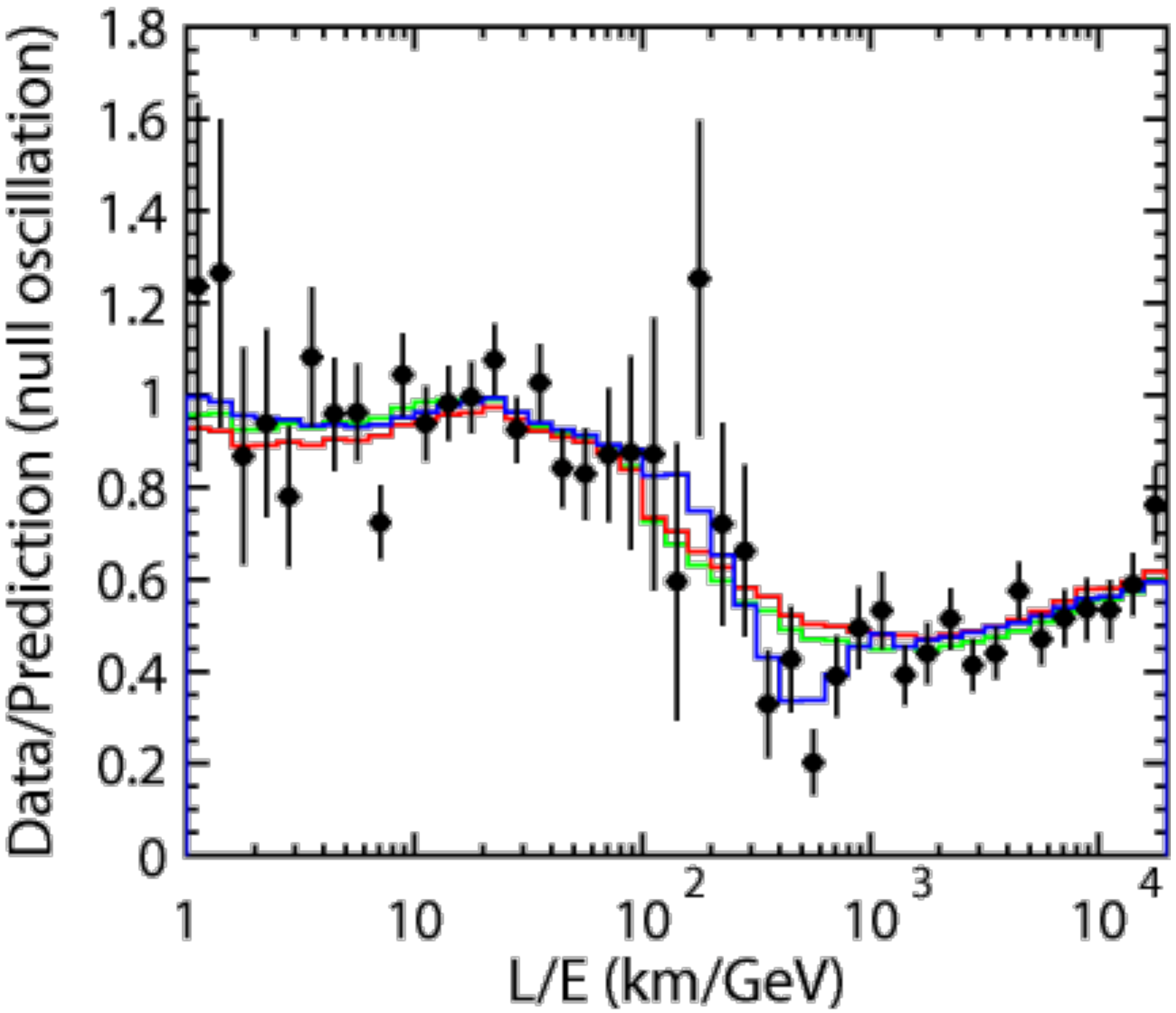
Super-Kamiokande detector
50 kt WC rector with 11k 20" photosensors





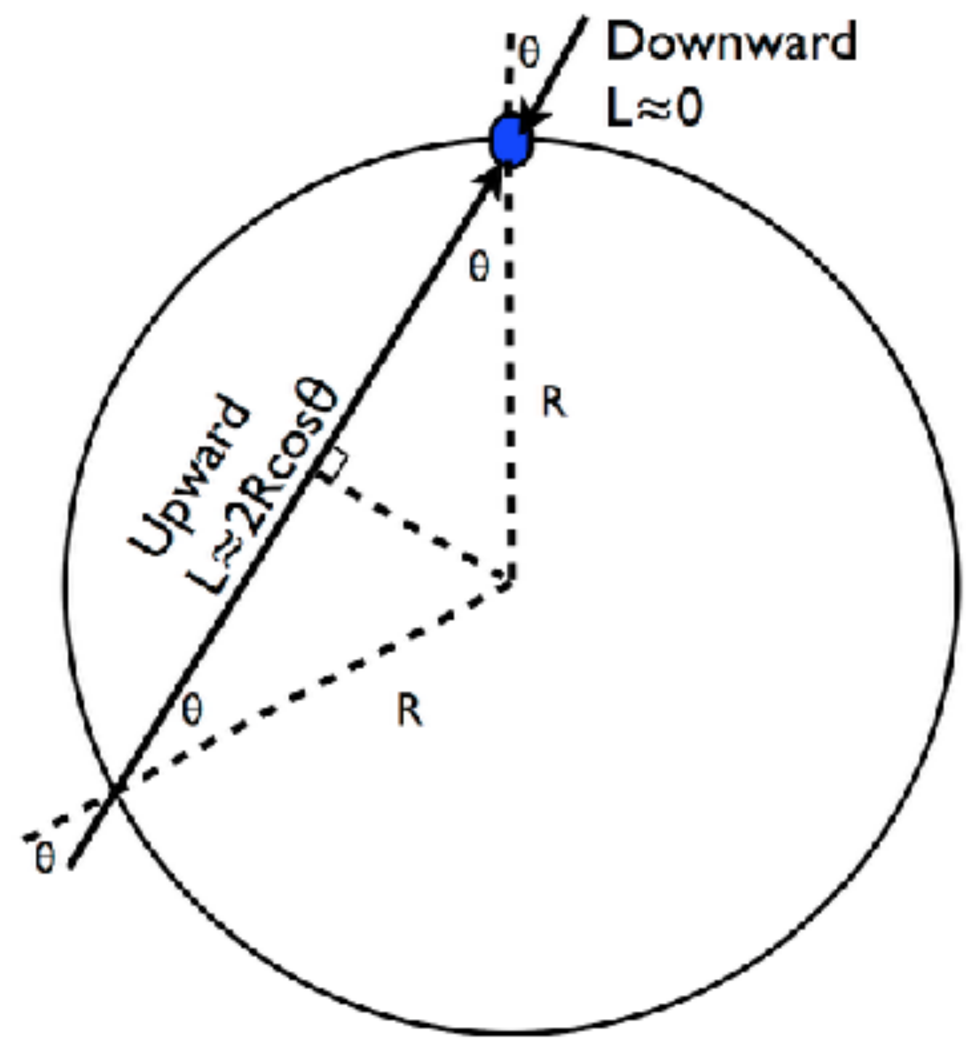


EVIDENCE FOR OSCILLATION



$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \times \sin^2 \Delta m^2 \frac{L}{4E}$$

- Neutrino oscillations should have a dependence on the path length from production to detection.
- For atmospheric neutrinos, is related to the "zenith angle" of the neutrino




The Nobel Prize in Physics 2015

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2015 to

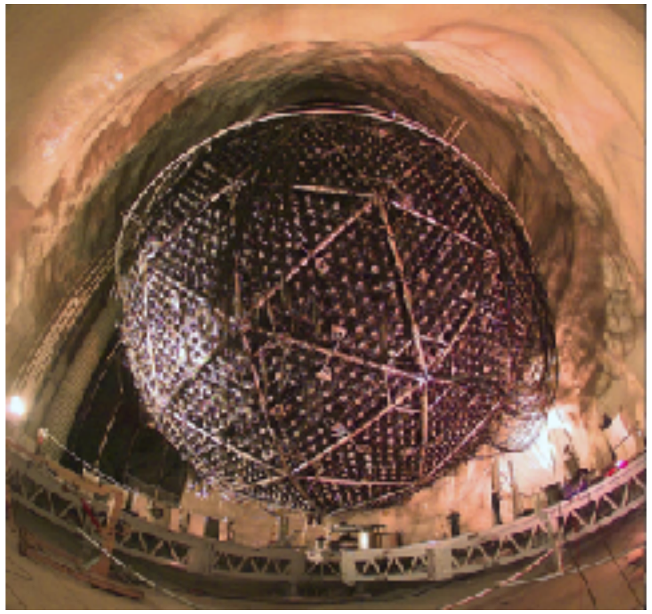
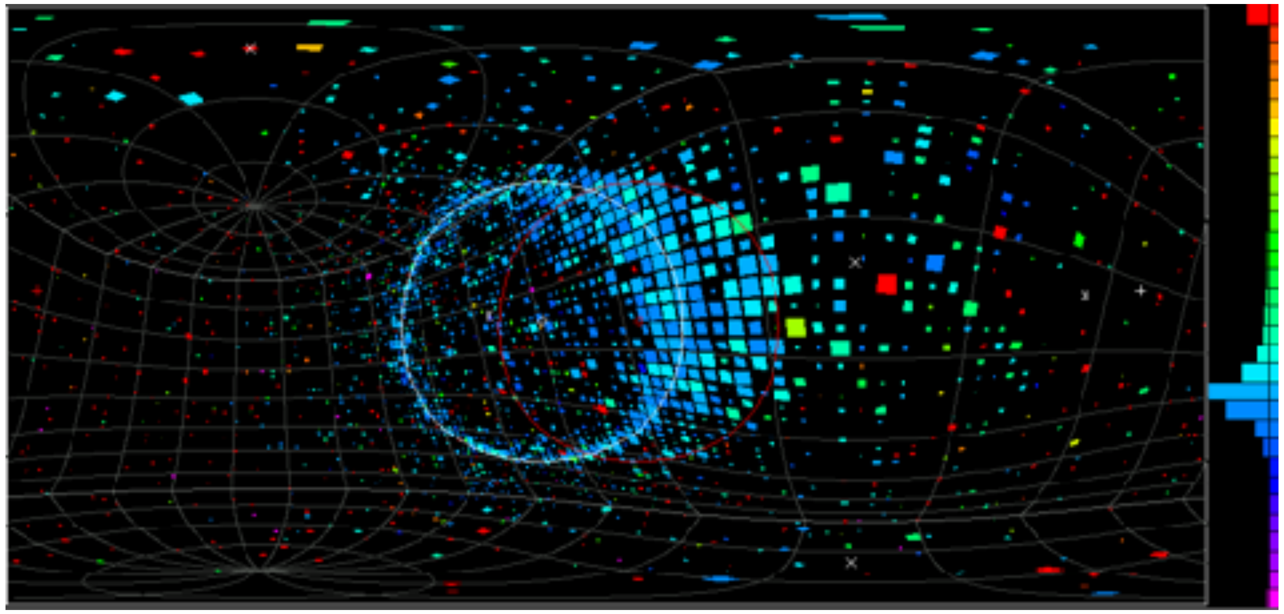
Takaaki Kajita

Super-Kamiokande Collaboration
University of Tokyo, Kashiwa, Japan

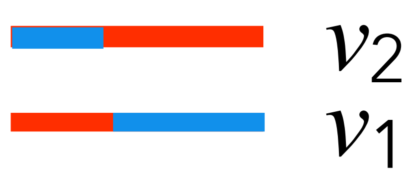
Arthur B. McDonald

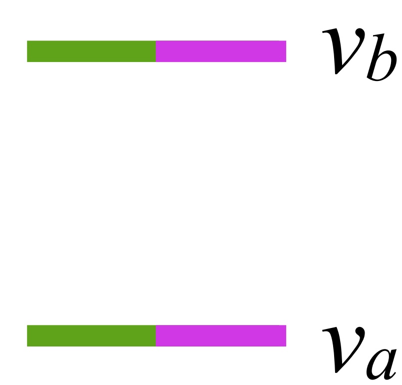
Sudbury Neutrino Observatory Collaboration
Queen's University, Kingston, Canada

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

WHAT DO WE KNOW?





$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

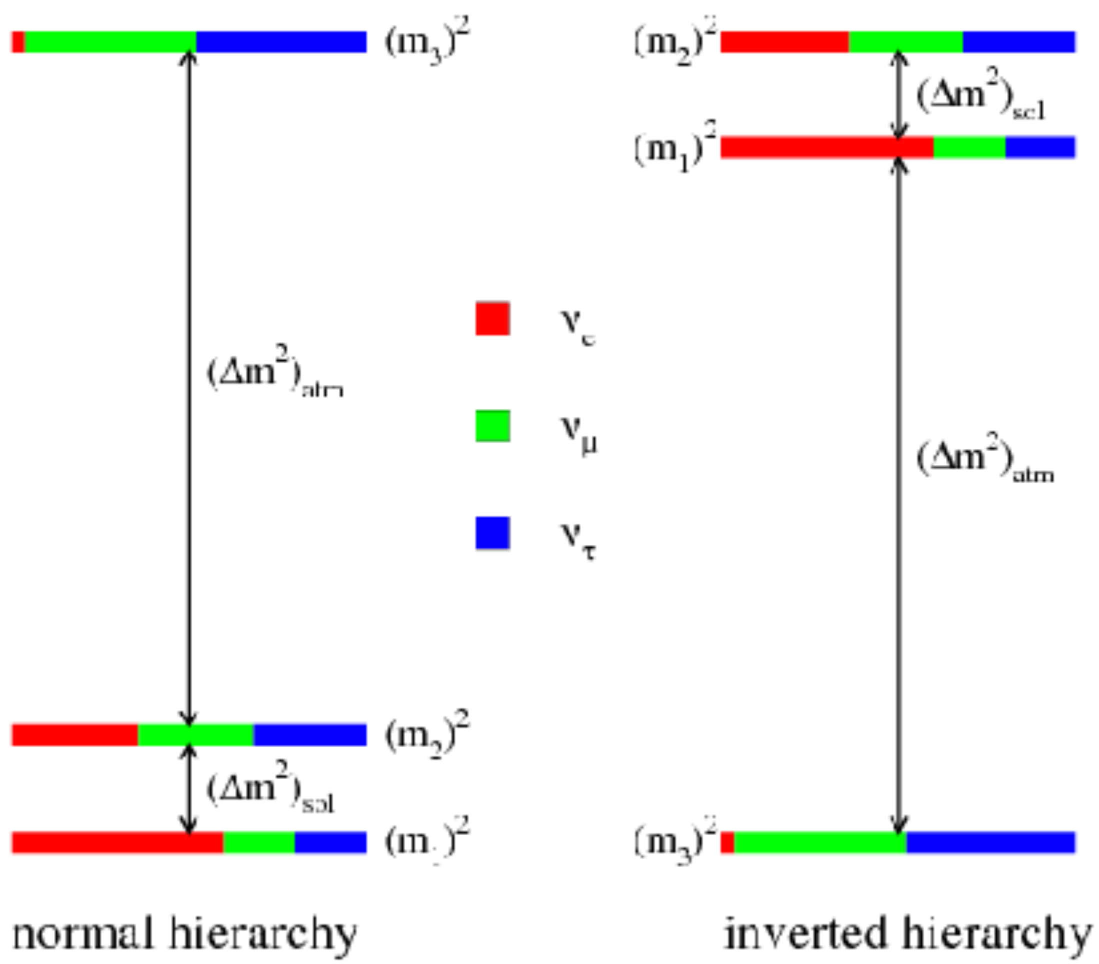
$$\begin{pmatrix} \nu_\mu \\ \nu_y \end{pmatrix} = \begin{pmatrix} \cos\theta_{23} & \sin\theta_{23} \\ -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix}$$

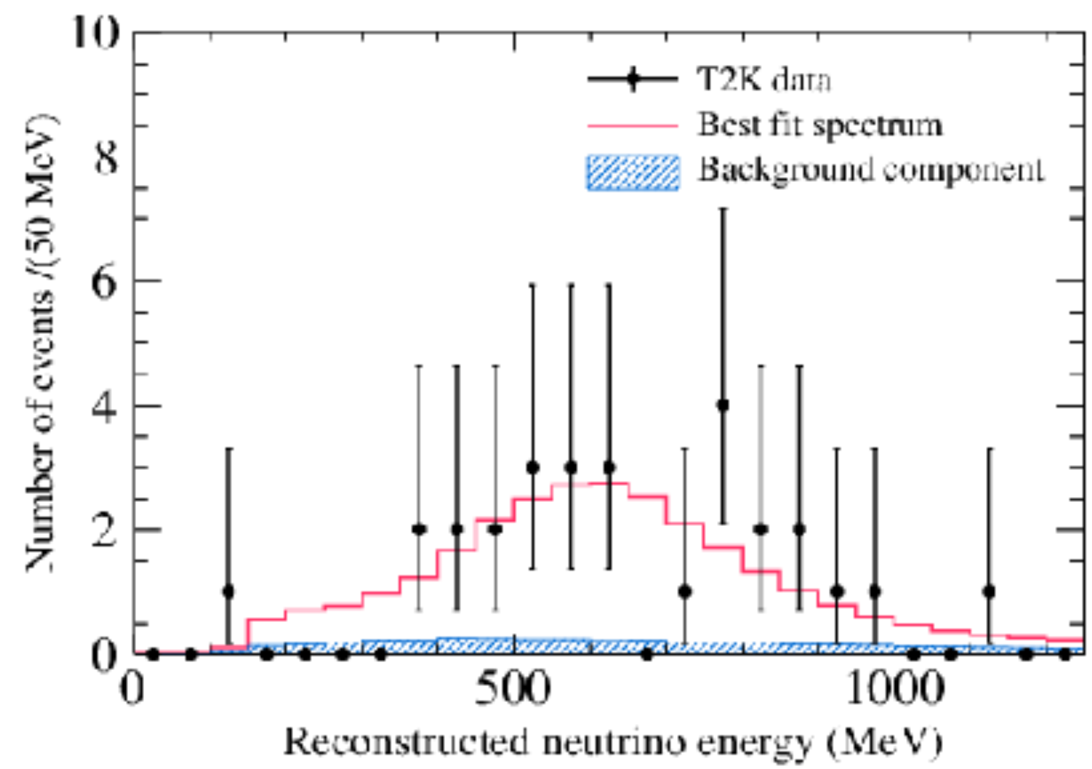
- From solar measurement:
  - ν_e component of ν_2 is ~1/3 → $\sin^2\theta_{12} = 1/3$
  - θ_{12} ~ 35 degrees
- From KamLAND
  - $\sin^2 2\theta_{12} = 0.85$ → θ_{12} ~34 degrees
  - $\Delta m^2_{21} \sim 7.5\times 10^{-5}\ eV^2$
- From atmospheric measurement
  - ν_μ disappearance is ~maximal
  - θ_{23} ~ 45 degrees
  - $\Delta m^2_{ba} \sim 2.5\times10^{-5}\ eV^2$
  - excess of ν_e not observed:
    - ν_y is primarily ν_τ

CONTEMPORARY TOPICS

- CP Violation?

$$P(\nu_\alpha \to \nu_\beta; L) = \delta_{\alpha\beta} - 4\mathrm{Re}\left[\sum_{i>j} U_{\alpha i}U^*_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin^2\frac{\Delta m^2_{ij}L}{4E} + 2\mathrm{Im}\left[\sum_{i>j} U_{\alpha i}U^*_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin\frac{\Delta m^2_{ij}L}{2E}$$

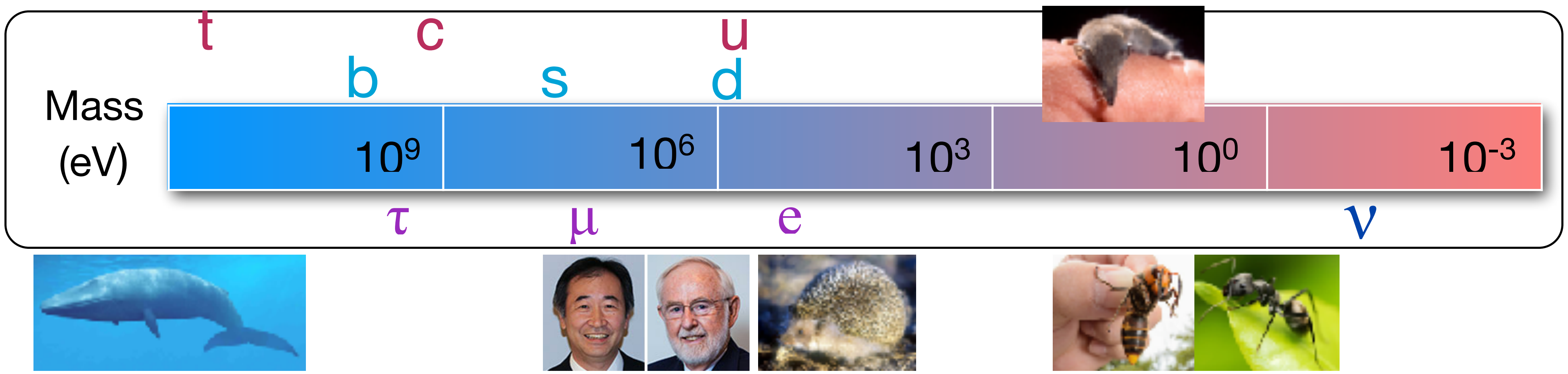


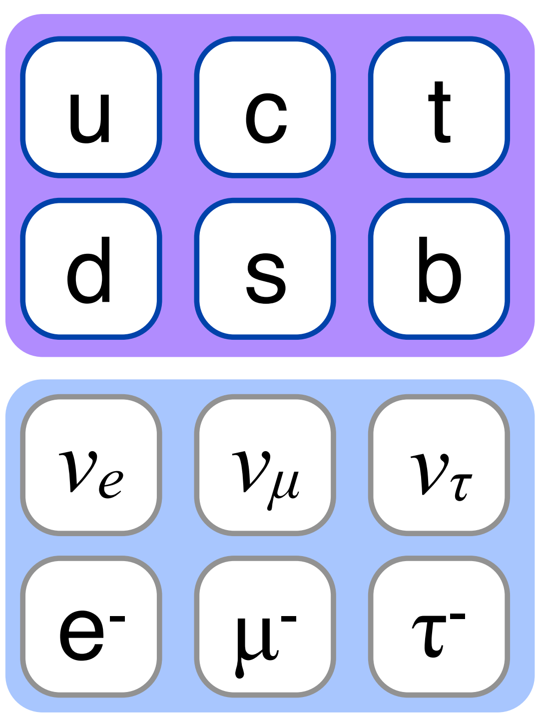


- Mass ordering?

ANSWERS OR MORE QUESTIONS

$$|U_{QUARK}| \sim \begin{pmatrix} 0.97428 & 0.2253 & 0.0034 \\ 0.2252 & 0.93745 & 0.0410 \\ 0.00862 & 0.0403 & 0.99915 \end{pmatrix} \qquad |U_{LEPTON}| \sim \begin{pmatrix} 0.8 & 0.5 & 0.15 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$





- Why are quark and lepton mixings so different?
  - is neutrino mixing "maximal"?
- Why are neutrino masses so tiny?
  - quarks/charged leptons masses from Higgs mechanism
  - do neutrinos get mass some other way?

THE MATTER DOMINATED UNIVERSE

SAKHAROV CONDITIONS:

- BARYON NUMBER (B) VIOLATION
- VIOLATION OF C, CP SYMMETRY (CPV)
- DEPARTURE FROM THERMAL EQUILIBRIUM

- Extremely small?
- Extremely large?
    - Known sources of CPV (quark CKM) cannot produce this asymmetry

$$\frac{\Delta B}{N_\gamma} \sim \mathcal{O}(10^{-10})$$

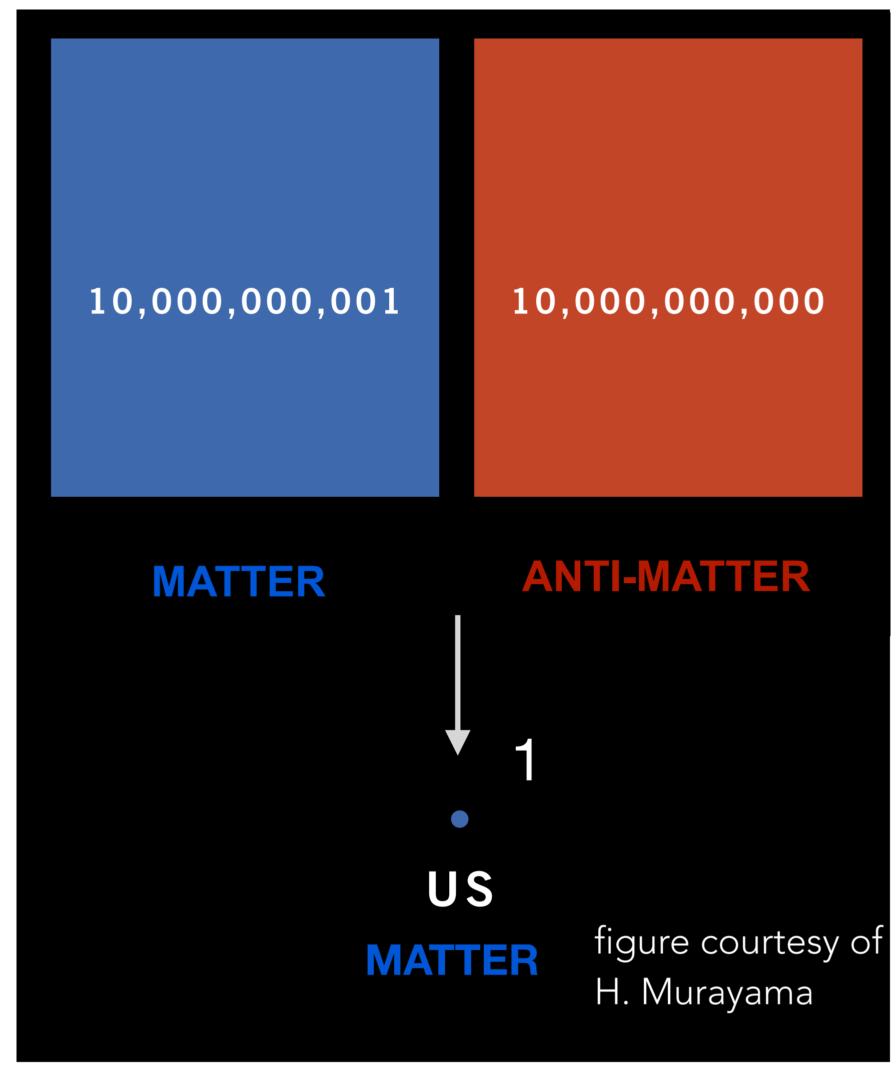


figure courtesy of H. Murayama

NEXT TIME

- Chapter 16.1-16.3
- Chapter 17.1-17.3