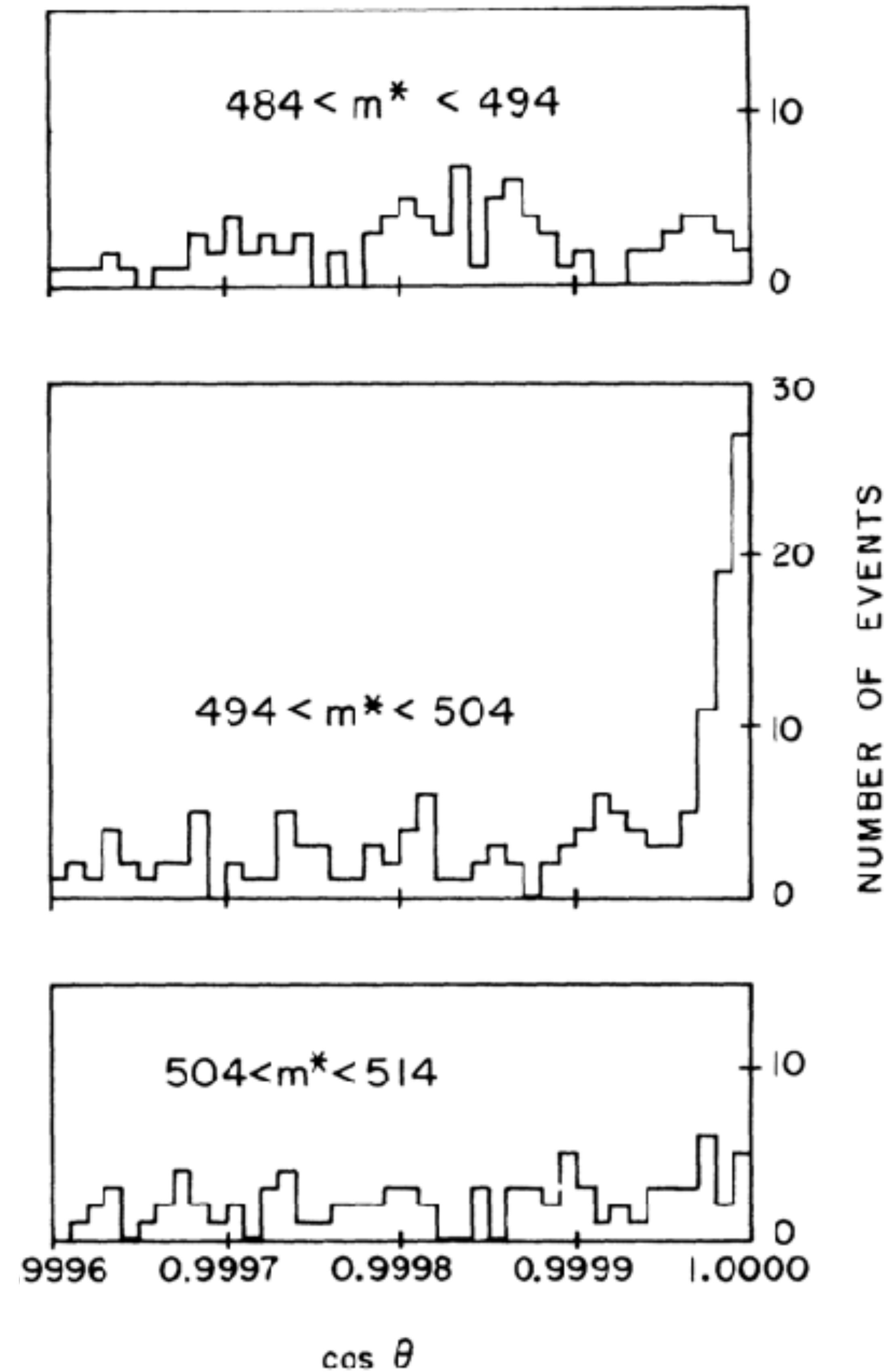
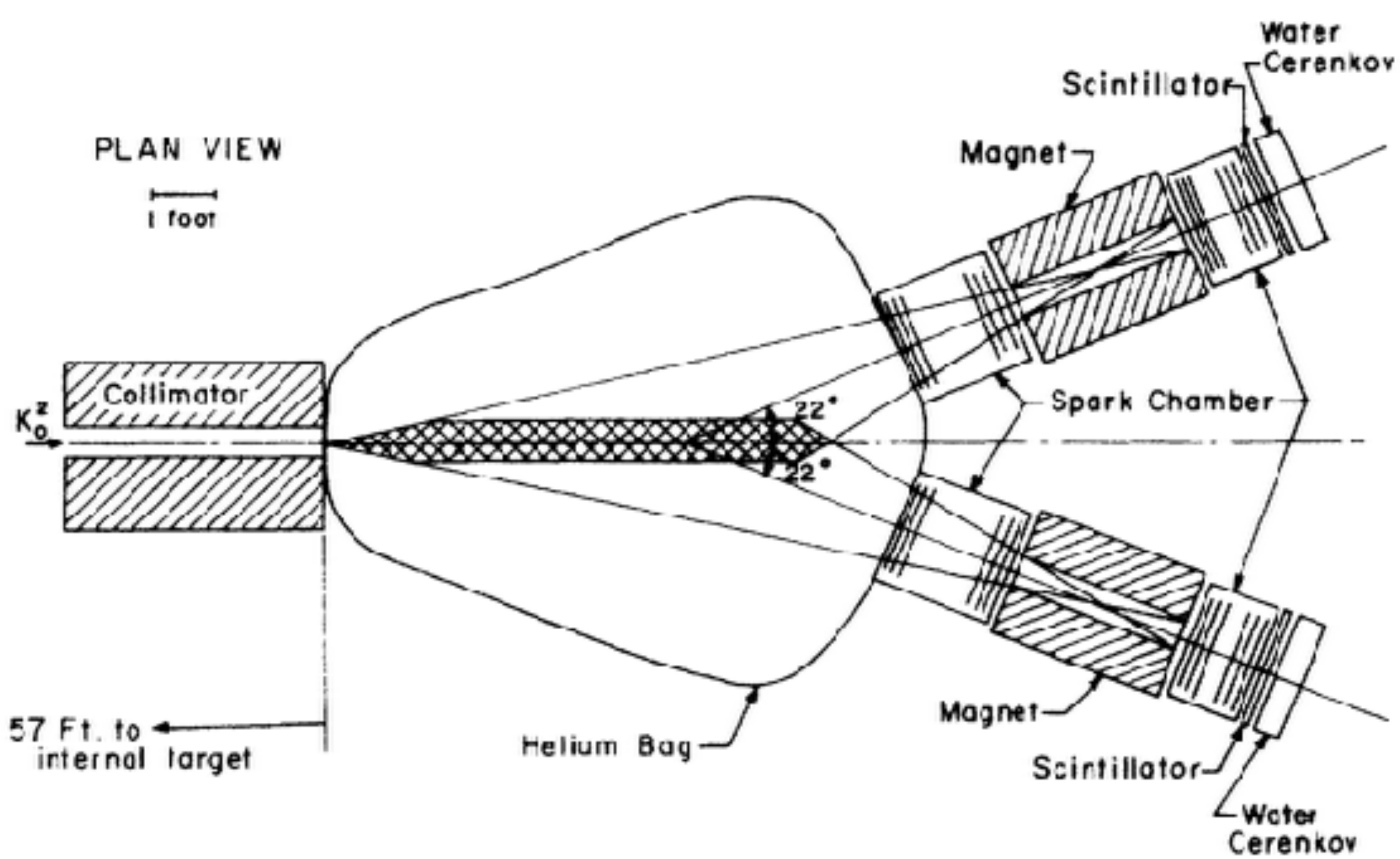
PHY489/1489:

LECTURE 17: ELECTROWEAK MIXING

CP VIOLATION IN KAON DECAY

- Produce a beam of K^0
  - propagate ~20 meters to decay K_1
  - all that is left is K_2
  - Do we see any $K \rightarrow \pi\pi$ decay?

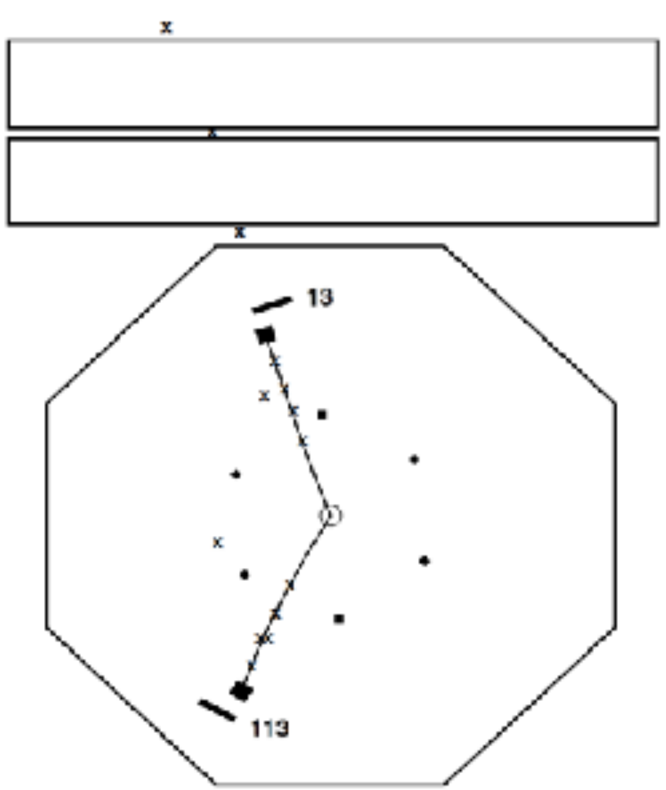




THE THIRD GENERATION

ν_e	ν_μ	ν_τ
e	μ	τ

u	c	t
d	s	b

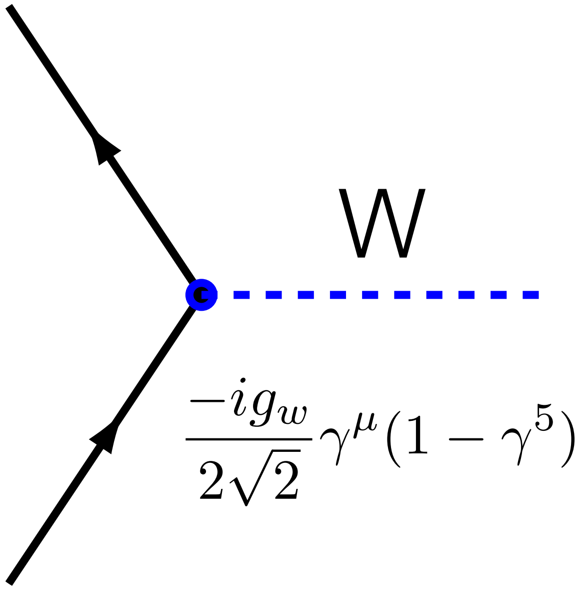


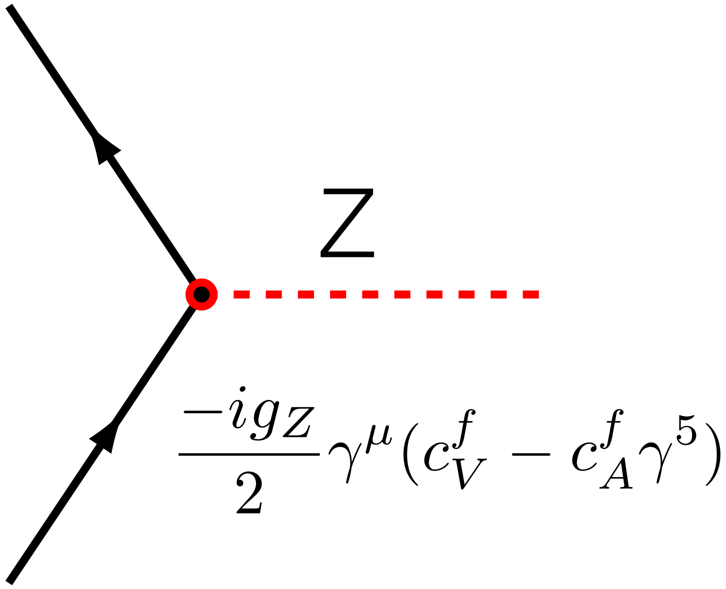
- Kobayashi and Maskawa contemplated that CP violation comes from mixing
  - phase in the mixing will switch sign when considering quark vs antiquark transitions
  - Impossible to generate phase in mixing with only two generations
  - at least three are needed
- First indication came from the discovery of the τ in 1975 at SLAC
  - bottom quark discovered in 1977
  - top quark in 1994
  - ν_τ in 2000
- Experiments (kaon, B-factories, etc.) confirm Kobayashi and Maskawa's explanation for CP violation in quarks

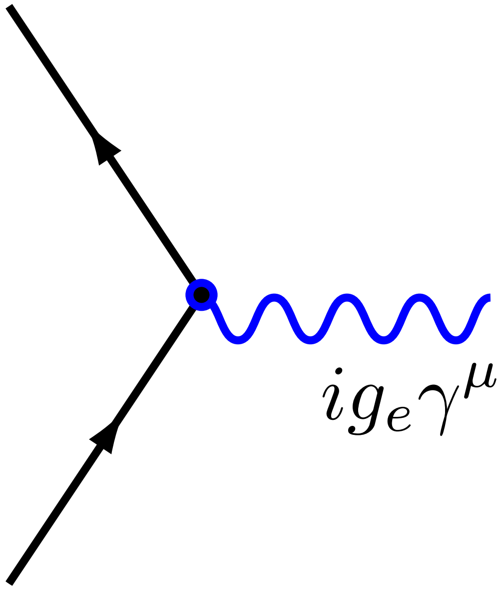
MISSION IMPOSSIBLE



- There are hints that EM and weak interactions have a common origin
  - similar gauge structure, universal coupling constant, etc.
- But there are obvious and dramatic differences:
  - Structure of the vertex is different . . …







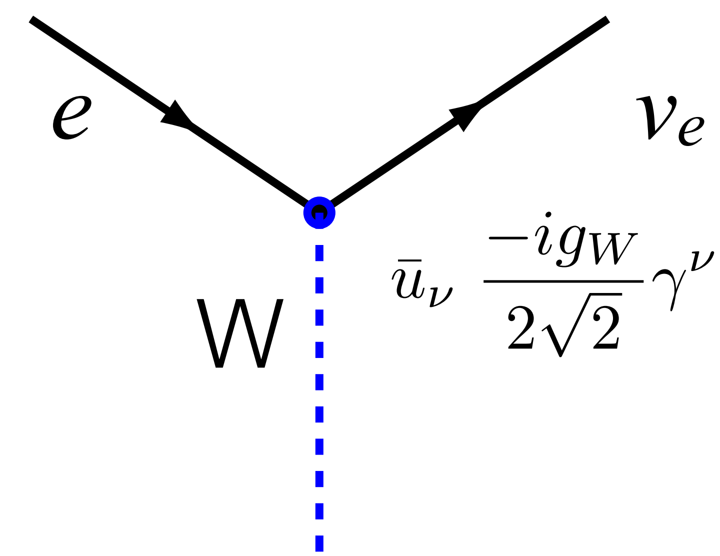
- masses of the intermediaries . . . . .

$$\frac{-i(g_{\mu\nu} - q_\mu q_\nu/M_W^2 c^2)}{q^2 - M_W^2 c^2} \qquad \frac{-i(g_{\mu\nu} - q_\mu q_\nu/M_Z^2 c^2)}{q^2 - M_Z^2 c^2} \qquad \frac{-ig_{\mu\nu}}{q^2}$$

- Let's deal with the first of these issues . . . . ..

CHIRAL STATES

- Recall that the chiral projection operators built into the vertex factor

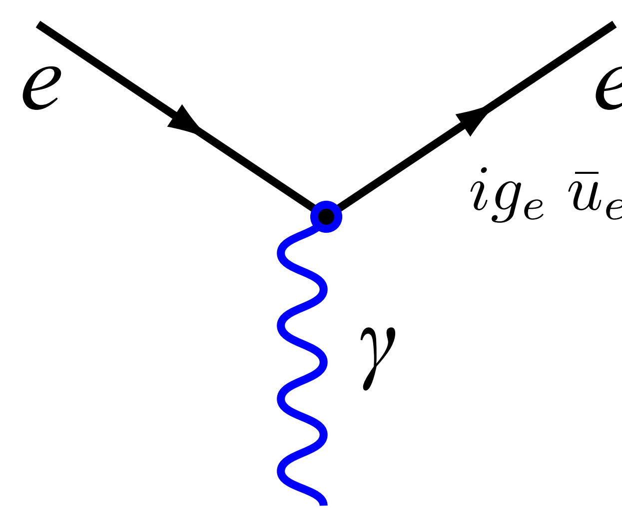


$$\bar{u}_\nu \; \frac{-ig_W}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)\; u_e$$

$$\bar{u}_\nu \; \frac{1}{2}\gamma^\nu(1-\gamma^5)\; u_e = u_\nu^\dagger\gamma^0\frac{1+\gamma^5}{2}\gamma^\nu\frac{1-\gamma^5}{2}u_{eL}$$
$$= u_\nu^\dagger\frac{1-\gamma^5}{2}\gamma^0\gamma^\nu u_{eL} = u_{\nu L}^\dagger\gamma^0\gamma^\nu u_{eL}$$
$$= \bar{u}_{\nu L}\gamma^\nu u_{eL}$$

- Recast this by "transferring" the chirality to the particle states
- View the weak CC interaction as the vector interaction left-chiral states
- Effectively view the left and right chiral states of a particle as different particles

- We can also reformulate the EM as a vector interaction with both left and right chiral components
- We note that it treats the left/right chiral states equally



$$ig_e \; \bar{u}_e\gamma^\nu u_e$$

$$ig_e \; (\bar{u}_{eL}\gamma^\nu u_{eL} + \bar{u}_{eR}\gamma^\nu u_{eR})$$

- New notation: label spinors by particle species: $u_{eL} \rightarrow e_L$
  - electromagnetic coupling constant "e"

WEAK INTERACTIONS AS SU(2)

- Recall the SU(2) gauge theory:
  - postulate invariance for a doublet of fields under SU(2) transformations

$$\chi \equiv \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \rightarrow e^{i\frac{g}{2}\vec{\theta}(x)\cdot\vec{\sigma}} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  - θ parameterizes the transformation
  - σ^i as the generators for the group
- Local gauge invariance forces the introduction of three gauge fields:

$$W^i_\mu \rightarrow W^i_\mu - \frac{1}{2}\partial_\mu \theta^i(x) - \frac{g}{2}\epsilon_{ijk}\theta^i W^j_\mu \qquad (i\partial\!\!\!/ - \frac{g}{2}\sigma^i W\!\!\!/^i)\chi$$

- which implies an interaction between the gauge fields and χ

$$\frac{g}{2}\sigma^i W\!\!\!/^i \chi \rightarrow \frac{g}{2}(W^1_\mu \sigma^1 + W^2_\mu \sigma^2 + W^3_\mu \sigma^3)\gamma^\mu \chi$$

- we remarked before that we could rewrite this as:

$$(W^+_\mu \sigma^+ + W^-_\mu \sigma^- + W^3_\mu \sigma^3)$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$$

$$\sigma^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$$

THE WEAK CHARGED INTERACTION

- Let's talk about what χ is:
  - put in the pairs of particles that couple to the weak CC

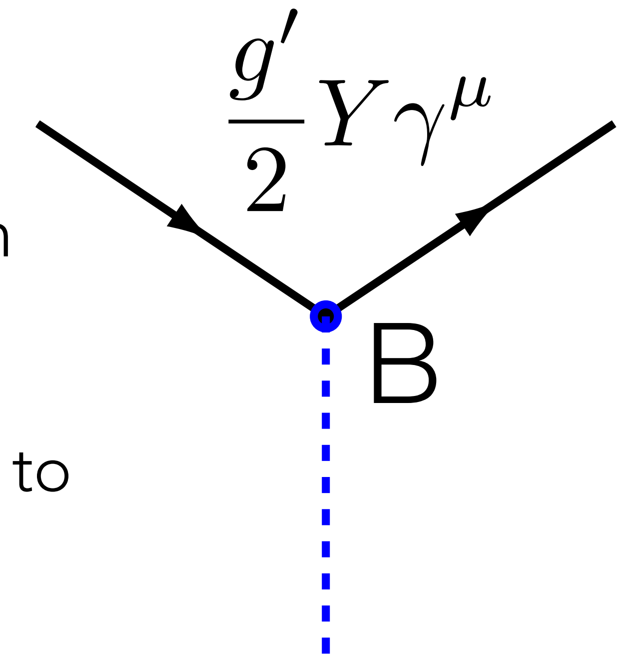
$$\chi = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

  - $W^{\pm}$, W^3 couplings result in these transitions in "weak isospin space"

$$\frac{1}{2}(\sigma^1 + i\sigma^2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \frac{1}{2}(\sigma^1 - i\sigma^2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  - where ν_e has I_3 = +1/2 and e has I_3=-1/2
- now have formalized how ν_e turns to e (and vice versa)
  - but also have a third interaction that is "neutral current"
    - ν_e stays a ν_e, e stays an e
  - is it the electromagnetic interaction? is it the weak neutral current?
- What about the right chiral fields?

ADDING ANOTHER GAUGE GROUP



- Consider another gauge interaction under U(1)
  - this has exactly the same structure as electromagnetism
  - however, consider that this is a separate interaction
    - a particle has “hypercharge” Y that defines its coupling to the $U(1)_Y$ gauge field which we call B:

$$g' \frac{Y}{2} \gamma^\mu B_\mu \psi$$

  - and allow L/R chiral states to have different hyper charge

$$\frac{1}{2} g' \left(Y_{e_L} \bar{e}_L \gamma^\mu e_L + Y_{e_R} \bar{e}_R \gamma^\mu e_R + Y_{\nu_L} \bar{e}_L \gamma^\mu \nu_L + Y_{\nu_R} \bar{\nu}_R \gamma^\mu \nu_R \right)$$

- Now consider a “combined” gauge group
  - $SU(2)_L xU(1)_Y$
  - can we recover the weak/EM as a combination of W_3 and B interactions?

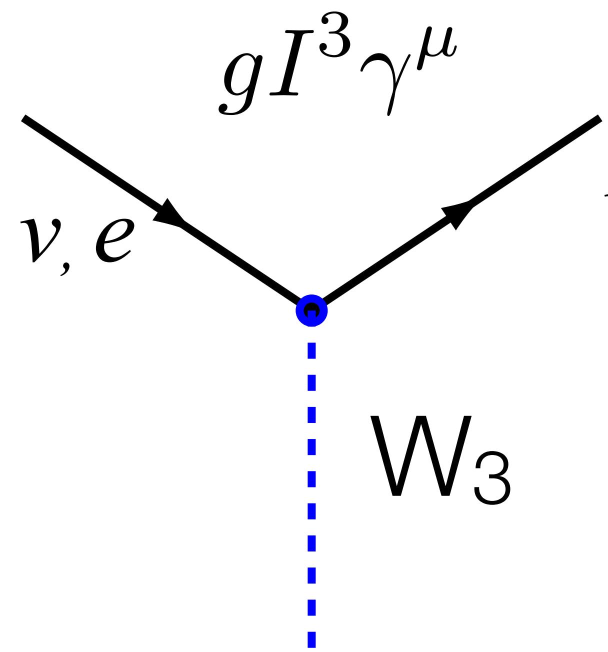
$$A_\mu = +B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$

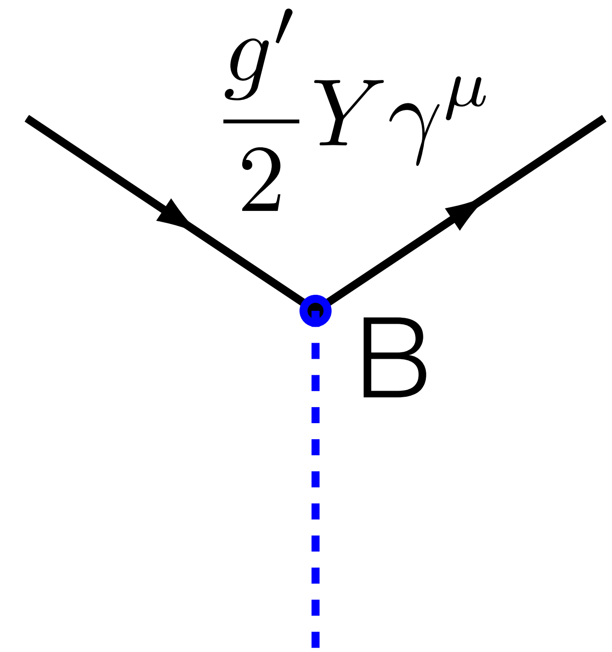
$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$$

NEUTRINO AND ELECTRON

$$A_\mu = +B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$$

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$





- implied electromagnetic coupling of each field:

$$\bar{e}_L\gamma^\mu e_L \rightarrow \frac{g'}{2}Y_{e_L}\cos\theta_W - \frac{1}{2}g\sin\theta_W \qquad \rightarrow Q_e e$$

$$\bar{e}_R\gamma^\mu e_R \rightarrow \frac{g'}{2}Y_{e_R}\cos\theta_W \qquad \rightarrow Q_e e$$

$$\bar{\nu}_R\gamma^\mu \nu_R \rightarrow \frac{g'}{2}Y_{\nu_R}\cos\theta_W \qquad \rightarrow 0$$

$$\bar{\nu}_L\gamma^\mu \nu_L \rightarrow \frac{g'}{2}Y_{\nu_L}\cos\theta_W + \frac{1}{2}g\sin\theta_W \qquad \rightarrow 0$$

$$Y = 2(Q - I^3_W)$$

$$e = g\sin\theta_W$$
$$= g'\cos\theta_W$$

- we can repeat this with the other "weak isospin doublets"

WHAT ABOUT Z?

$$g_Z \equiv \frac{g}{\cos\theta_W} = \frac{e}{\sin\theta_W \cos\theta_W}$$

$$g' = g_Z \sin\theta_W$$

$$g = g_Z \cos\theta_W$$

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

$$\bar{\nu}_L \gamma^\mu \nu_L \to -\frac{g'}{2} Y_{\nu_L} \sin\theta_W + \frac{1}{2} g \cos\theta_W \qquad \to \frac{g_Z}{2}$$

$$\bar{\nu}_R \gamma^\mu \nu_R \to -\frac{g'}{2} Y_{\nu_R} \sin\theta_W \qquad \to 0$$

$$\bar{e}_L \gamma^\mu e_L \to -\frac{g'}{2} Y_{e_L} \sin\theta_W - \frac{1}{2} g \cos\theta_W \qquad \to \frac{g_Z}{2}(\sin^2\theta_W - \cos^2\theta_W)$$

$$\bar{e}_R \gamma^\mu e_R \to -\frac{g'}{2} Y_{e_R} \sin\theta_W \qquad \to \frac{g_Z}{2} 2\sin^2\theta_W$$

- Interaction properties of the Z are completely determined completely once we set e (EM coupling), g (weak CC coupling)!

$$Y = 2(Q - I^3_W)$$

$$e = g\sin\theta_W = g'\cos\theta_W$$

OTHER DOUBLETS:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$Y = 2(Q - I_W^3)$$

WHAT HAPPENED?

- First, we considered the chiral states of each particle separately

$$e \rightarrow e_L, e_R \qquad \nu \rightarrow \nu_L, \nu_R$$

  - We considered a SU(2) gauge theory that couples only to left chiral particles
    - This gave us $W^{\pm}$ (weak CC interaction) plus another "neutral current" interaction W_3
    - we know this can't be the photon (nor the Z)
  - Introduced a new gauge field B with $U(1)_Y$ gauge symmetry (like EM) but where we can assign different "hypercharge" Y to each chiral state
  - postulate a $SU(2)_L x U(1)_Y$ gauge symmetry
- Postulate that the A, Z are linear combinations of W_3 and B
- See what we have to do to:
  - get equal left/right chiral coupling to A (consistent with EM)
  - get appropriate electrons charges (-1 for electron, 0 for neutrino)
- Sets relations between the coupling constants and hyper charge of each state.
  - completely determines the properties of Z interactions with quarks, leptons

$$e = g \sin\theta_W = g' \cos\theta_W = g_Z \cos\theta_W \sin\theta_W$$

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

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Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

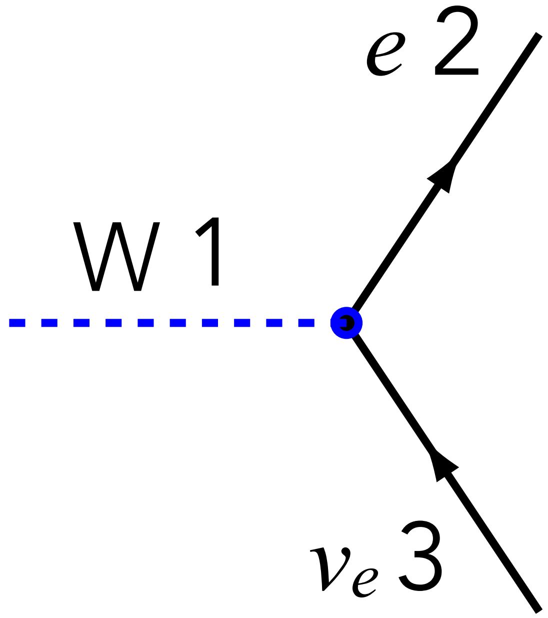
1. Introduction

At first sight there may be little or no similarity between electromagnetic effects and the phenomena associated with weak interactions. Yet certain remarkable parallels emerge with the supposition that the weak interactions are mediated by unstable bosons. Both interactions are universal, for only a single coupling constant suffices to describe a wide class of phenomena: both interactions are generated by vectorial Yukawa couplings of spin-one fields ††. Schwinger first suggested the existence of an "isotopic" triplet of vector fields whose universal couplings would generate both the weak interactions and electromagnetism — the two oppositely charged fields mediate weak interactions and the neutral field is light [2]). A certain ambiguity beclouds the self-interactions among the three vector bosons; these can equivalently be interpreted as weak or electromagnetic couplings. The more recent accumulation of experimental evidence supporting the $\Delta I = \frac{1}{2}$ rule characterizing the non-leptonic decay modes of strange particles indicates a need for at least one additional neutral intermediary [3]).

The mass of the charged intermediaries must be greater than the K-meson mass, but the photon mass is zero — surely this is the principal stumbling block in any pursuit of the analogy between hypothetical vector mesons and photons. It is a stumbling block we must overlook. To say that the decay intermediaries

- Some complain that this not really "unification":
  - usually means embedding two interactions into a larger gauge group with one coupling constant
  - "electroweak mixing" may be a better term
- But the consequences are profound
  - electromagnetic and weak interactions are inextricably linked by the fact that the photon and Z are chimeras contains bits of:
    - $SU(2)_L$ gauge group that governs the weak charge current
    - $U(1)_Y$ gauge group that also has right chiral couplings.
- The Z boson contains obvious hints this mix
  - right chiral couplings
  - modified coupling constant
  - dependence of properties on electric charge
  - (different mass from W)

GAUGE BOSON FEYNMAN RULES



- The Feynman rule for an incoming(outgoing) boson is its polarization vector:
  - ε_μ, $\varepsilon_\mu^{\,*}$

$$\frac{-ig}{2\sqrt{2}} \left[\bar{u}_2\gamma^\mu(1-\gamma^5)v_3\right] \times (2\pi)^4(p_1-p_2-p_3)\ \epsilon_\mu(p_3)$$

$$\mathcal{M} = \frac{g}{2\sqrt{2}} \left[\bar{u}_2\gamma^\mu(1-\gamma^5)v_3\right]\ \epsilon_\mu(p_1)$$

- Relative to the z-axis, we can define

$$\epsilon_{+\mu} = \frac{1}{\sqrt{2}}(0,1,i,0)$$

$$\epsilon_{-\mu} = \frac{1}{\sqrt{2}}(0,-1,+i,0)$$

$$\epsilon_{L\mu} = (0,0,0,-1)$$

- for the rest of the amplitude, we know (in the massless limit):
  - e, ν_e come out with energy $M_W/2$
  - e with left helicity, ν_e with right helicity

$$\bar{u}_3\gamma^\mu(1-\gamma^5)v_2 \to \bar{u}_{2\downarrow}\gamma^\mu v_{3\uparrow}$$

- We evaluated this combination back in QED

$$\bar{u}_{2\downarrow}\gamma^\mu v_{3\uparrow} \to 2E(0,-\cos\theta,-i,\sin\theta)$$

SUMMARY

- Reminder:
  - no class on Tuesday
  - no office hours on Monday, Tuesday
- Please read chapter 13