PHY489/1489

LECTURE 15:
WEAK INTERACTIONS CONTINUED

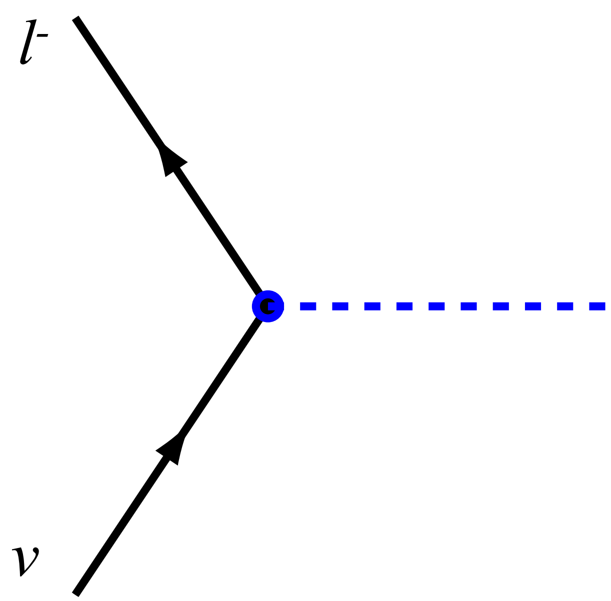
INTRODUCTION

- I hope the midterm went well for everyone
- Solutions are now posted on the website

THE WEAK CHARGED CURRENT

- Feynman rules:

Vertex Factor for Leptons:



$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

W propagator

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2}$$

WEAK VS. QED

$\bar{\psi}\psi$	scalar
$\bar{\psi}\gamma^5\psi$	pseudoscalar
$\bar{\psi}\gamma^\mu\psi$	vector
$\bar{\psi}\gamma^\mu\gamma^5\psi$	pseudovector
$\bar{\psi}\sigma^{\mu\nu}\psi$	antisymmetric tensor

QED vertex

$$-ig_e\gamma^\mu$$

photon propagator

$$\frac{-ig_{\mu\nu}}{q^2}$$

Weak CC vertex

$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

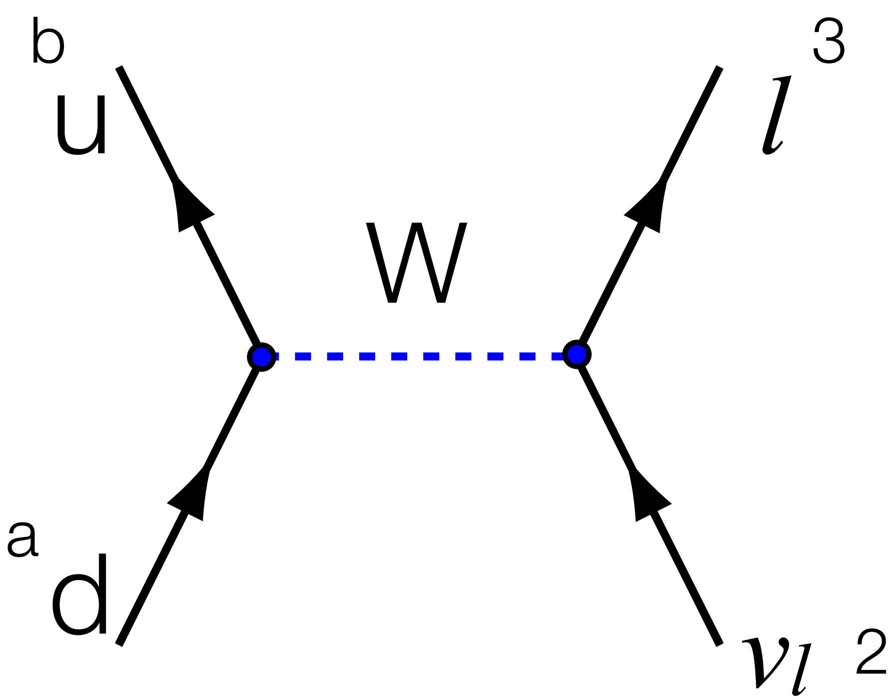
W propagator

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2}$$

- Vertex:
  - coupling constant
  - γ^μ vs. $\gamma^\mu - \gamma^\mu\gamma^5$
  - charge: W carries one unit
- Propagator
  - massive particle (3 polarizations)
  - at low energies: $q << M_W c^2$

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2} \Rightarrow \frac{ig_{\mu\nu}}{M_W^2c^2}$$

EXAMPLE: PION DECAY



- Lepton fermion leg

$$[\bar{u}_3 \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)v_2]$$

- Quark Fermion leg

$$[\bar{v}_b \frac{-ig_w}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)u_a]$$

$$[\bar{v}_b \frac{-ig_w}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)u_a] \Rightarrow F^\nu = f_\pi p^\nu$$

- Propagator

$$\int \frac{1}{(2\pi)^4} d^4q \qquad \frac{ig_{\mu\nu}}{M_W^2 c^2}$$

$$\mathcal{M} = \frac{g_W^2}{8M_W^2 c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\right] f_\pi p_\mu$$

THE MATRIX ELEMENT

so consider only μ=0 here

in rest frame, only μ=0 is non-zero

$$\mathcal{M} = \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\right] f_\pi p_\mu = \frac{g_W^2 f_\pi m_\pi c}{8M_W^2c^2}\left[\bar{u}_3\gamma^0(1-\gamma^5)v_2\right]$$

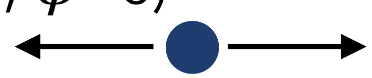
$$= \frac{g_W^2 f_\pi m_\pi c}{8M_W^2c^2}\left[u_3^\dagger\gamma^0\gamma^0(1-\gamma^5)v_2\right] = \frac{g_W^2 f_\pi m_\pi c}{8M_W^2c^2}\left[u_3^\dagger(1-\gamma^5)v_2\right] = \frac{g_W^2 f_\pi m_\pi c}{4M_W^2c^2}\left[u_3^\dagger v_{2\uparrow}\right]$$

$$\frac{1}{2}(1-\gamma^5)v_2 \Rightarrow v_{2\uparrow}$$

s=sin θ/2

c= cos θ/2

p_3 (μ) (θ=0, ϕ=0)



p_2 (ν) (θ=π, ϕ=π)

$$v_\uparrow = \sqrt{E_2+m_2}\begin{pmatrix} \frac{|\mathbf{p}_2|}{E_2+m_2}s \\ -\frac{|\mathbf{p}_2|}{E_2+m_2}ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix} \Rightarrow \sqrt{|\mathbf{p}_3|}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

- Evaluate u_3v_2 with two configurations of u_3

$$u_\uparrow = \sqrt{E_3+m_3}\begin{pmatrix} c \\ se^{i\phi} \\ \frac{|\mathbf{p}_3|}{E_3+m_3}c \\ \frac{|\mathbf{p}_3|}{E_3+m_3}se^{i\phi} \end{pmatrix} \Rightarrow \sqrt{E_3+m_3}\begin{pmatrix} 1 \\ 0 \\ \frac{|\mathbf{p}_3|}{E_3+m_3} \\ 0 \end{pmatrix}$$

$$u_\downarrow = \sqrt{E_3+m_3}\begin{pmatrix} -s \\ ce^{i\phi} \\ \frac{|\mathbf{p}_3|}{E_3+m_3}s \\ -\frac{|\mathbf{p}_3|}{E_3+m_3}ce^{i\phi} \end{pmatrix} \Rightarrow \sqrt{E+m_3}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{|\mathbf{p}_3|}{E_3+m_3} \end{pmatrix}$$

$$u_{3\uparrow}^\dagger v_{2\uparrow} = \sqrt{E_3+m_3}\left(1-\frac{|\mathbf{p}_3|}{E_3+m_3}\right)$$

$$u_{3\downarrow}^\dagger v_{2\uparrow} = 0$$

$$\mathcal{M} = \frac{g_W^2 f_\pi m_\pi}{4M_W^2}\sqrt{E_3+m_3}\sqrt{|\mathbf{p_3}|}\left(1-\frac{|\mathbf{p_3}|}{E_3+m_3}\right)$$

COMPLETING THE CALCULATION

$$\mathcal{M} = \frac{g_W^2 f_\pi m_\pi}{4M_W^2} \sqrt{E_3 + m_3}\sqrt{|\mathbf{p_3}|}\left(1 - \frac{|\mathbf{p_3}|}{E_3 + m_3}\right)$$

$$E_3 = \frac{m_\pi^2 + m_3^2}{2m_\pi} \qquad |\mathbf{p}_3| = \frac{m_\pi^2 - m_3^2}{2m_\pi}$$

$$\sqrt{|\mathbf{p}_3|}\frac{E_3 + m_3 - |\mathbf{p}_3|}{\sqrt{E_3 + m_3}}$$

$$\sqrt{\frac{m_\pi^2 - m_3^2}{2m_\pi}} \qquad \sqrt{\frac{2m_\pi}{(m_\pi + m_3)^2}} \qquad \frac{m_3(m_3 + m_\pi)}{m_\pi}$$

$$\sqrt{m_\pi^2 - m_3^2}\frac{m_3}{m_\pi}$$

$$\mathcal{M} = \frac{g_W^2 f_\pi}{4M_W^2} m_3 \sqrt{m_\pi^2 - m_3^2}$$

$$|\mathcal{M}|^2 = \frac{g_W^4}{16M_W^4} f_\pi^2 m_3^2 (m_\pi^2 - m_3^2)$$

$$\Gamma = \frac{|\mathbf{p}|}{8\pi m_\pi^2}|\mathcal{M}|^2$$

$$\Gamma = \frac{g_W^4}{256 M_W^4 m_\pi^3} f_\pi^2 m_3^2 (m_\pi^2 - m_3^2)^2$$

RATIOS:

- Consider the two decays:
  - $\pi^- \rightarrow e^- + \bar{\nu}_e$
  - $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

$$\Gamma = \frac{g_W^4}{256 M_W^4 m_\pi^3} f_\pi^2 m_3^2 (m_\pi^2 - m_3^2)^2$$

$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2 (m_\pi^2 - m_e^2)^2}{m_\mu^2 (m_\pi^2 - m_\mu^2)^2}$$

- Using the masses
  - m_π = 139.57 MeV
  - m_μ = 105.65 MeV
  - m_e = 0.511 MeV

$$\frac{\Gamma_e}{\Gamma_\mu} = 1.28 \times 10^{-4}$$

PIENU

$$\frac{\Gamma_e}{\Gamma_\mu} = 1.2344 \pm 0.0023(\text{stat}) \pm 0.0019(\text{syst.}) \times 10^{-4}$$

Improved Measurement of the $\pi \to e\nu$ Branching Ratio

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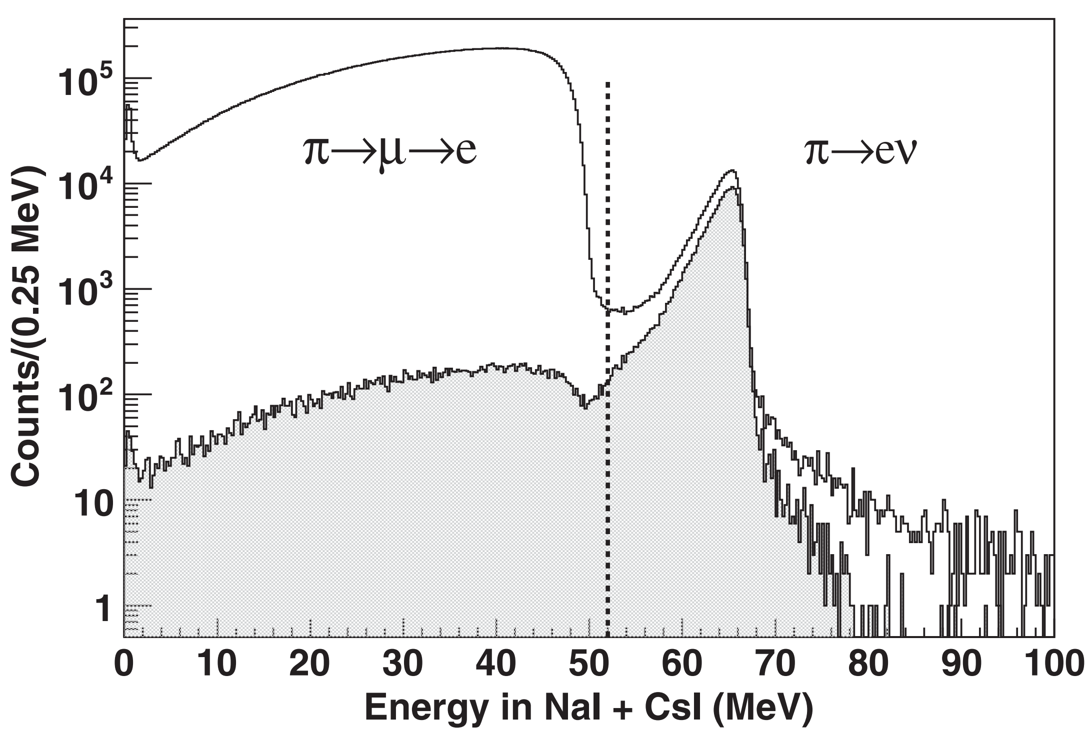
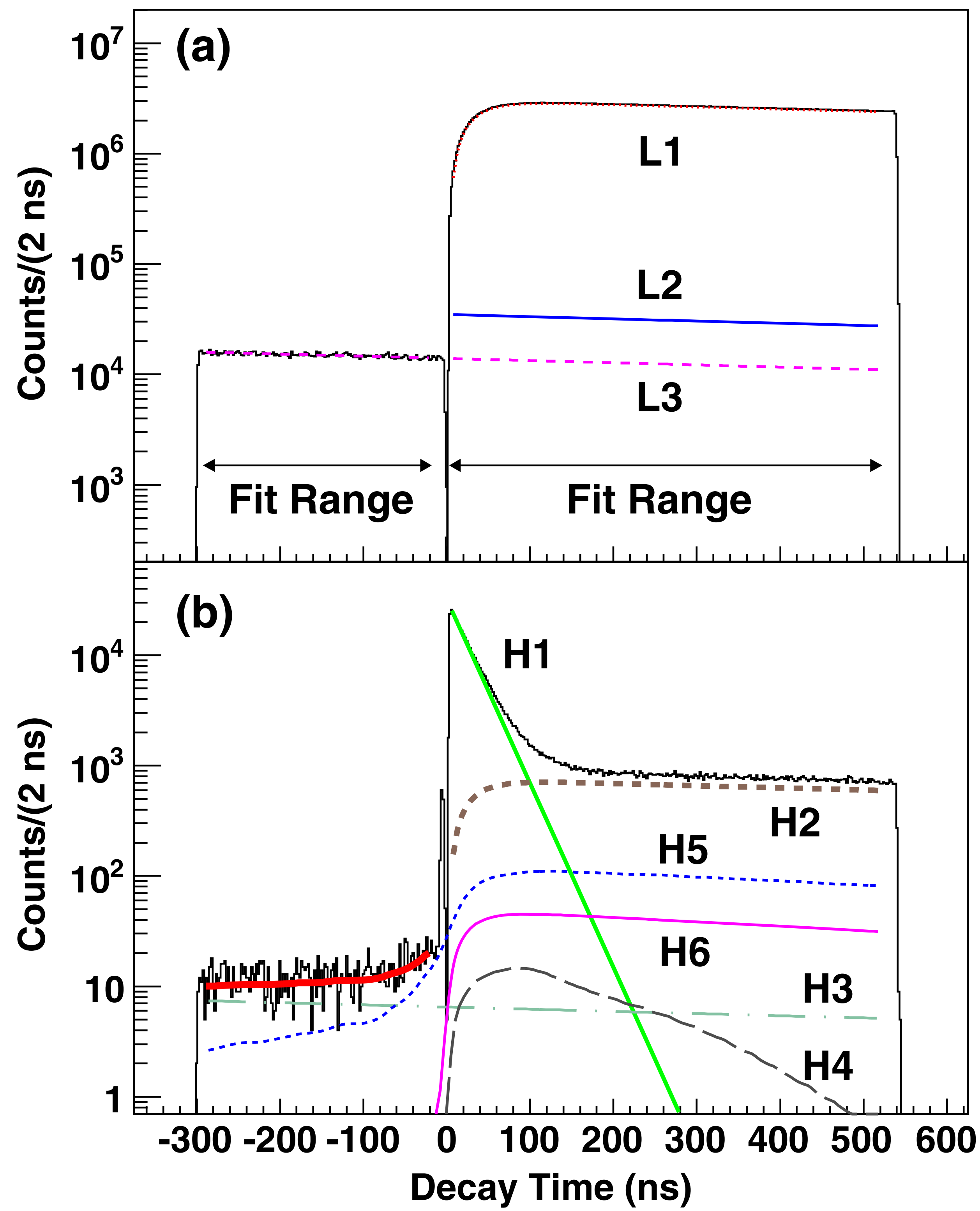
(Received 8 June 2015; published 13 August 2015)

A new measurement of the branching ratio $R_{e/\mu} = \Gamma(\pi^+ \to e^+\nu + \pi^+ \to e^+\nu\gamma)/\Gamma(\pi^+ \to \mu^+\nu + \pi^+ \to \mu^+\nu\gamma)$ resulted in $R_{e/\mu}^{\text{exp}} = [1.2344 \pm 0.0023(\text{stat}) \pm 0.0019(\text{syst})] \times 10^{-4}$. This is in agreement with the standard model prediction and improves the test of electron-muon universality to the level of 0.1%.





FORM OF INTERACTION

- We said this is a “vector - axial vector” interaction

$$\mathcal{M} = \frac{g_W^2}{8M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1 - \gamma^5) v_2\right] f_\pi p_\mu$$

- What would a:
  - vector interaction look like?
  - scalar interaction?
- The form of the interaction affects the decay rate
  - if we assume a scalar interaction we would get

$$\frac{\Gamma_e}{\Gamma_\mu} =\sim 5$$

CONNECTION

- Consider our various propagators
  - photons $\dfrac{-ig_{\mu\nu}}{q^2}$
  - W boson $\dfrac{-i(g_{\mu\nu} - q_\mu q_\nu / M_W^2 c^2)}{q^2 - M_W^2 c^2} \sim \dfrac{ig_{\mu\nu}}{M_W^2 c^2}$
  - These actually arise partly from "completeness" relations:

$$\sum_s \epsilon_s^{*\mu}(q)\epsilon_s^\nu(q) = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m^2} \qquad \sum_s \epsilon_s^{*\mu}(q)\epsilon_s^\nu(q) = -g^{\mu\nu}$$

- (Pseudo)scalar interactions likewise imply a (pseudo) scalar mediator/propagator $\Rightarrow \dfrac{i}{q^2 - M^2}$

HELICITY SUPPRESSION

- For massless particles, chiral states are also helicity states, i.e.
  - we concluded $\frac{1}{2}(1-\gamma^5)v_2 \Rightarrow v_{2\uparrow}$
  - but we could have also concluded

$$\bar{u}_3 \frac{1}{2}\gamma^\mu(1-\gamma^5) \Rightarrow \bar{u}_3 \frac{1}{2}(1+\gamma^5)\gamma^\mu \Rightarrow u_3^\dagger \gamma^0 \frac{1}{2}(1+\gamma^5)\gamma^\mu \Rightarrow \left(\frac{1}{2}(1-\gamma^5)u_3\right)^\dagger \gamma^0\gamma^\mu \Rightarrow \bar{u}_{3L}\gamma^\mu$$

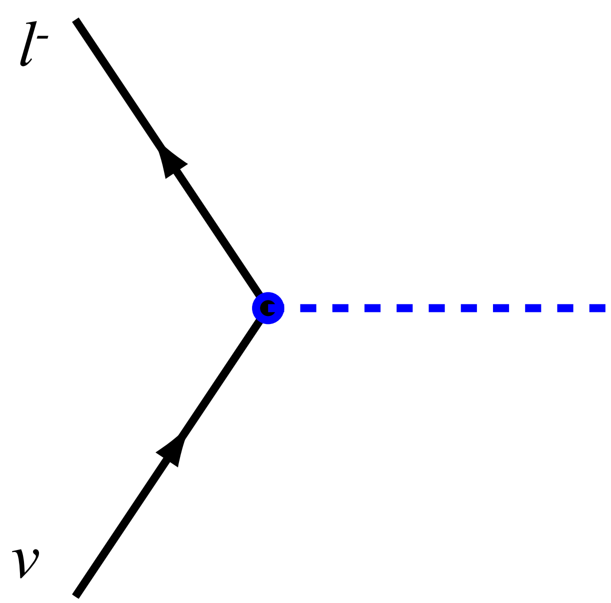
- i.e. neutrino is right helicity, muon is left helicity
  - impossible to conserve angular momentum so the decay will not happen!
  - this is apparent also from the matrix element

$$\mathcal{M} = \frac{g_W^2 f_\pi m_\pi}{4M_W^2}\sqrt{E_3+m_3}\sqrt{|\mathbf{p_3}|}\left(1-\frac{|\mathbf{p_3}|}{E_3+m_3}\right)$$

- The decay only happens to the extent that the chiral projection of one of the particles departs from a helicity state due to its mass.
  - the closer the particles are to the massless limit, the more suppression

$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2(m_K^2-m_e^2)}{m_e^2(m_K^2-m_\mu^2)} = 2.57\times 10^{-5}$$

WEAK CC FOR LEPTONS

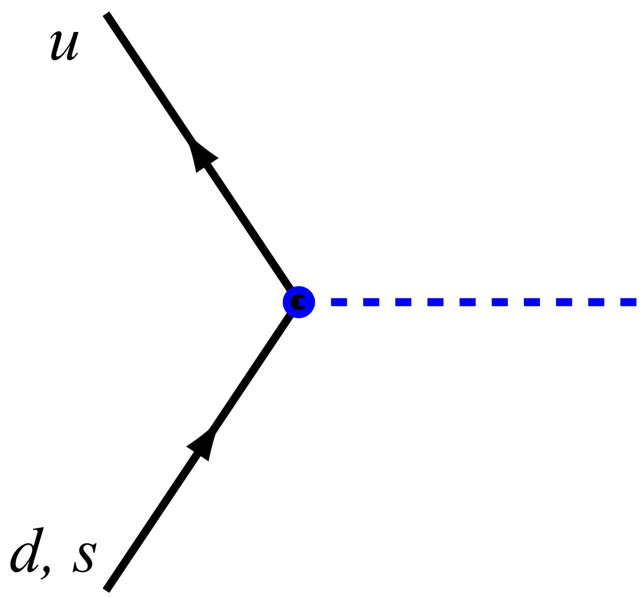


$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

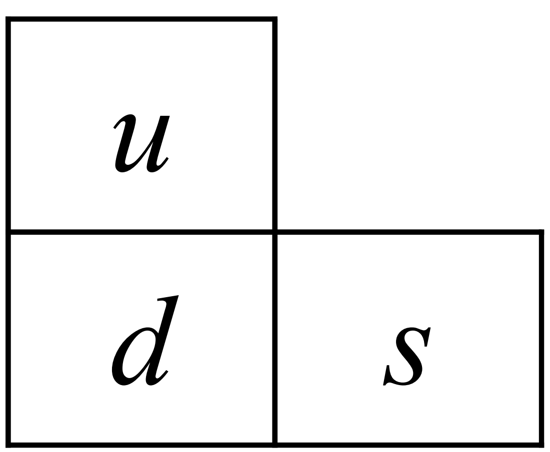
v_e	v_μ	v_τ
e	μ	τ

- Reasonably simple
  - charged lepton connects to corresponding neutrino

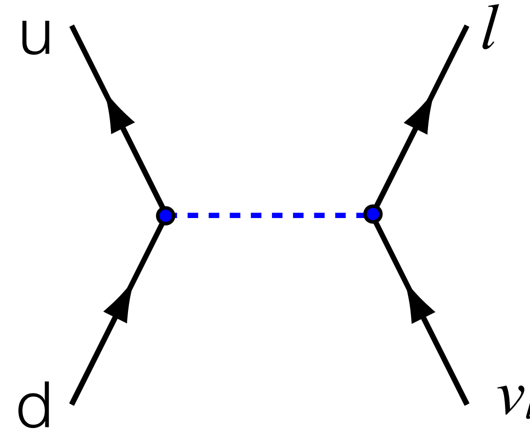
WEAK INTERACTION OF QUARKS

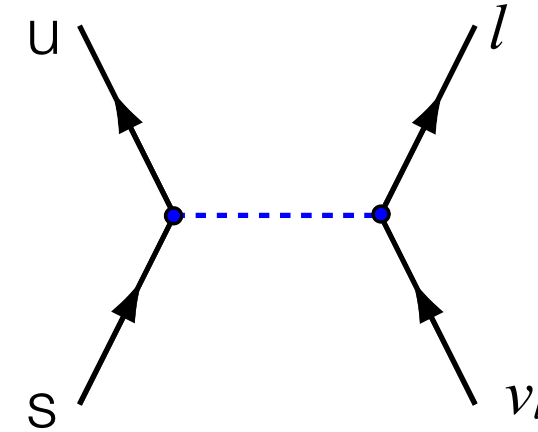


$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



- Step back to 1960s when we "knew" of three quarks
- Noticed that decays of "strange' particles was much slower than expected
- We can compare pion/kaon decays





m_π = 139.57 MeV

m_K = 493.68 MeV

$$\Gamma = \frac{f_\pi^2}{\pi\hbar m_\pi^3}\left(\frac{g_w}{4M_W}\right)^4 m_l^2(m_\pi^2 - m_l^2)^2$$

$$\frac{\Gamma(K^- \to \mu^- + \nu_\mu)}{\Gamma(\pi^- \to \mu^- + \nu_\mu)} = \left(\frac{m_\pi}{m_K}\right)^3\left(\frac{m_K^2 - m_\mu^2}{m_\pi^2 - m_\mu^2}\right)^2 \sim 18$$

CONCLUSIONS

- Please read 12.1, 14.1-3, 14.7
- Are you interested in neutrino oscillations?
- You might consider looking at Appendix D