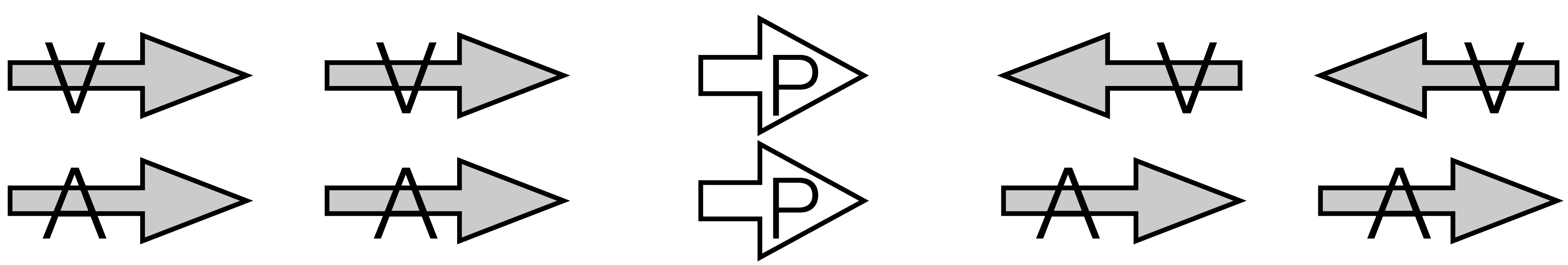
PHYSICS 489/1489

LECTURE 14: PARITY VIOLATION AND THE WEAK INTERACTION

PARITY

- The operation by which we reverse spatial coordinates: **x** → -**x**
- Consider a vector **V**: parity reverses it to -**V**
    - P(**V**) = -**V**
- Another "vector" quantity, the "axial" or "pseudo"vector:
    - consider **c**= **a** x **b**, where **a**, **b** are vectors
    - P(**c**) = P(**a** x **b**) = -**a** x -**b** = **a** x **b** = **c**
- Parity symmetry can be preserved if quantities are constructed entirely as vectors or as axial vectors



- consider:

$$\vec{F} = m\vec{a} \qquad \vec{L} = \vec{r} \times \vec{p}$$

PARITY EIGENSTATES

- If a state is an eigenvector of the parity operator:

$$P|A\rangle = p|A\rangle$$

  - where p is the eigenvalue. If we apply P again, we have:

$$P(P|A\rangle) = P(p|A\rangle) = pP|A\rangle = p^2|A\rangle$$

    - applying P twice brings us back to the initial state, so

$$PP = P^2 = I \qquad p = \pm 1$$

- The parity eigenvalue is conserved if parity is a symmetry of the system (e.g. . [H, P]= 0)

PARITY OF A BOUND STATE

- The bound state (e.g. meson = quark + antiquark) has two components that determine its parity
  - "intrinsic" parity of the particle
    - quarks/leptons have intrinsic parity 1, antiquarks/antileptons -1
  - parity due to the wave function of the particle
    - $(-1)^l$ where l is the orbital momentum

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$$Y_1^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta$$

$$Y_1^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\,\cos\theta\, e^{-i\varphi}$$

$$Y_2^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\,\cos\theta\, e^{i\varphi}$$

$$Y_2^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1)$$

$$Y_2^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\varphi}$$

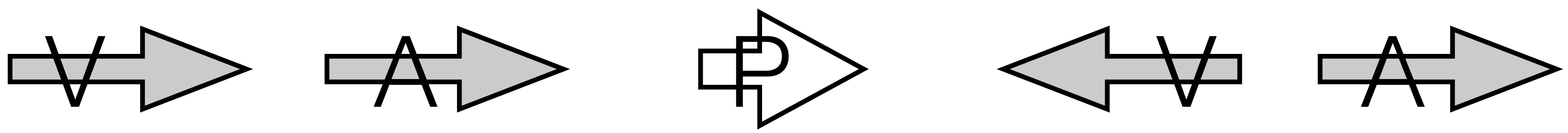
  - Total parity is the product of the two
    - parity eigenvalue is unchanged by an interaction if it conserves parity

EXAMPLES

- Pion: quark + antiquark in s = 0, $l = 0$ state:
  - $P = 1 \times -1 \times (-1)^0 = -1$
- ρ: quark, anti-quark in s=1, $l = 0$ state
  - $P = 1 \times -1 \times (-1)^0 = -1$
- Two pions in $l = 0$ state
  - $P = -1 \times -1 \times (-1)^0 = 1$
- Two pions in $l = 1$ state
  - $P = -1 \times -1 \times (-1)^1 = -1$
- Decay of ρ: since s (and $J = l + s$) = 1, we must have $l = 1$ in final state
  - two pions is OK: $P = -1 \times -1 \times (-1)^1 = -1$
  - three pions violates parity: $-1 \times -1 \times -1 \times (-1)^1 = +1$

MXING VECTORS + PSEUDOVECTORS

- geometrically obvious that something has changed



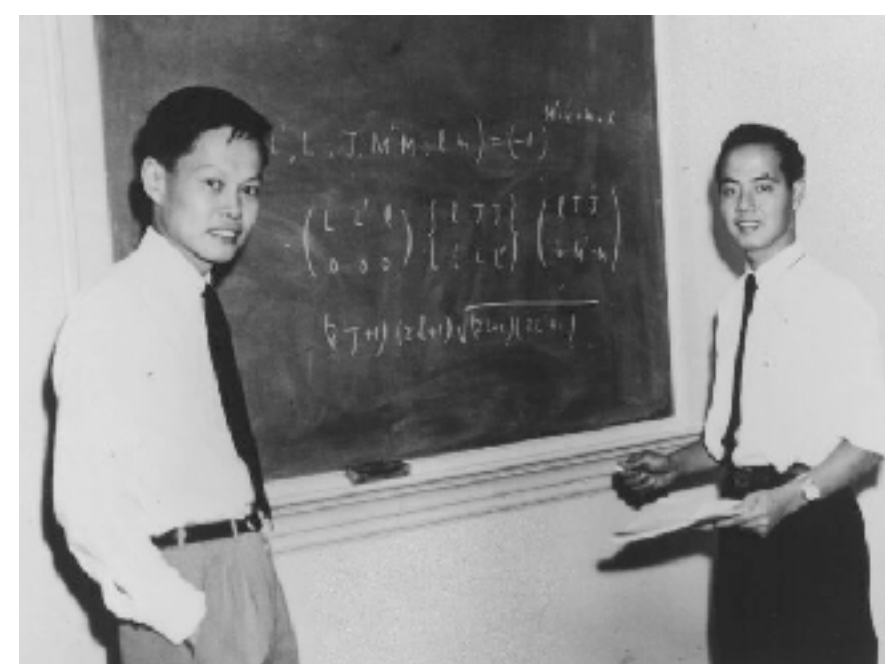
  - two things that were pointing in the same direction are now pointing in opposite directions
- One can formalize this by considering a dot vector:
  - $P(\mathbf{V} \cdot \mathbf{A}) = -\mathbf{V} \cdot \mathbf{A} \neq \mathbf{V} \cdot \mathbf{A}$
- or add a vector and psuedovector
  - $P(|\mathbf{V}+\mathbf{A}|) = |-\mathbf{V}+\mathbf{A}| \neq |\mathbf{V}+\mathbf{A}|$
- Parity is violated by an observable/interaction that:
  - correlates a vector and a pseudovector
  - is the sum of a vector and a pseudovector

"θ/τ" PUZZLE

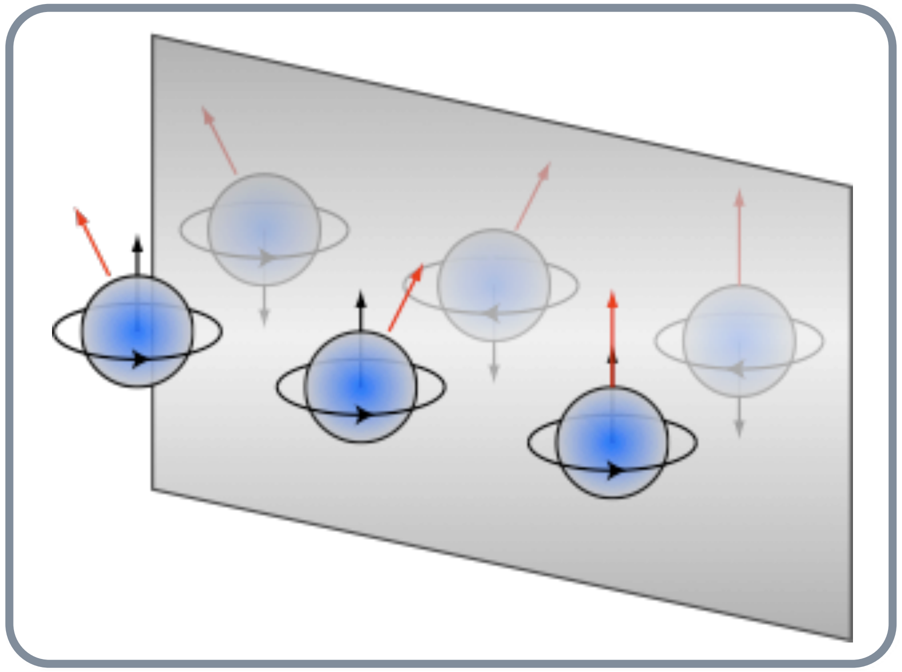
- Imagine two particles identical in every way, but decay to different parity eigenstates:

$$\begin{array}{lcll} \theta^+ & \rightarrow & \pi^+ + \pi^0 & (P = (-1)^2 = 1) \\ \tau^+ & \rightarrow & \pi^+ + \pi^0 + \pi^0 & (P = (-1)^3 = -1) \\ & \rightarrow & \pi^+ + \pi^+ + \pi^- & (P = (-1)^3 = -1) \end{array}$$

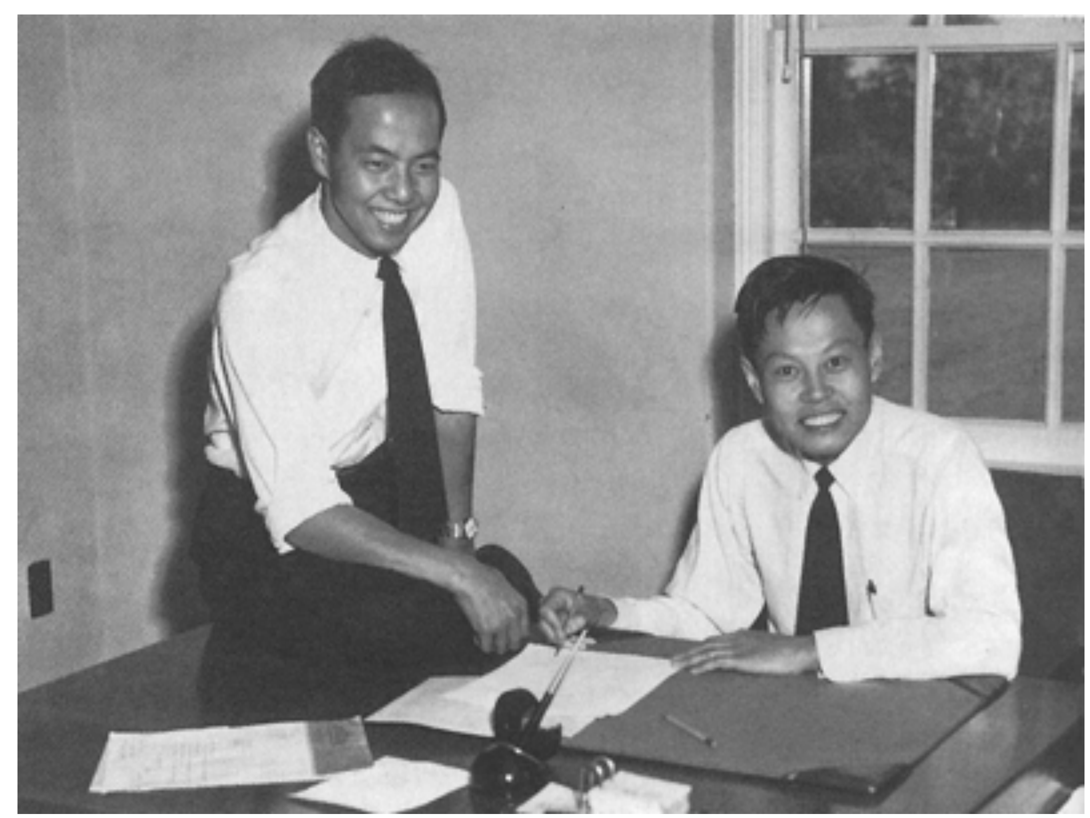
- Coincidence that two such particles exist?
- T. D. Lee, C. N. Yang:
  - θ/τ are the same particle
  - parity not conserved in weak decays



DEMONSTRATION OF PARITY VIOLATION

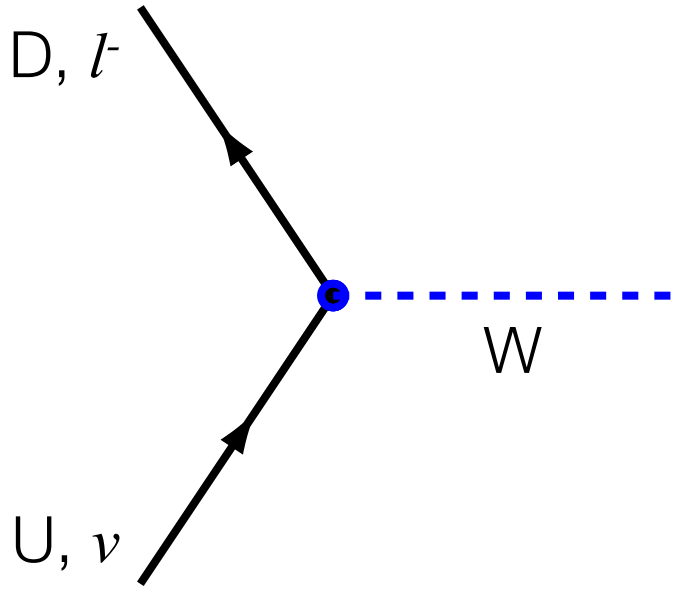


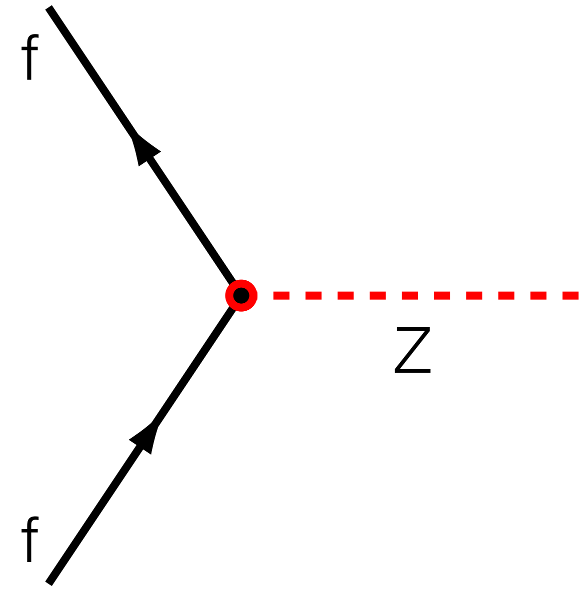
- β decay in polarized ^{60}Co
  - cool ^{60}Co atoms to polarize them
    - magnetic moment/spin become aligned
    - angular momentum is a pseudo vector
    - momentum of positron is a vector
  - Correlation would demonstrate parity violation
  - N.B. left shows "mirror" transformation
    - parity transformation + rotation by 180 degrees



WEAK INTERACTIONS

- Fundamental vertices



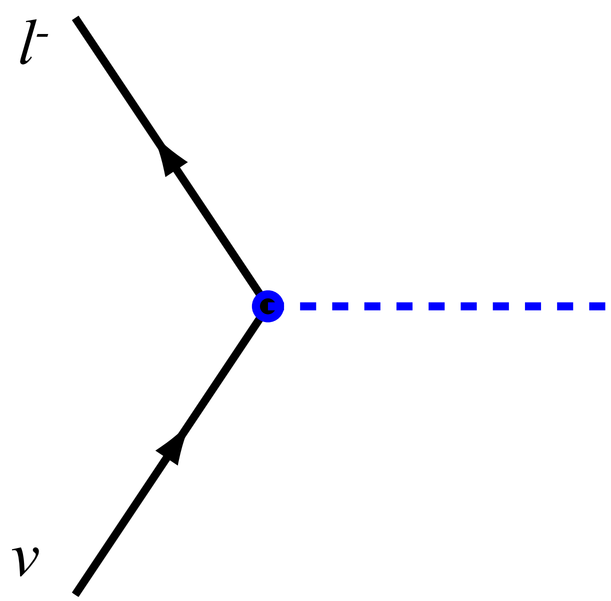


- Weak charged current:
  - couples charged lepton (e, μ, τ) into its corresponding neutrino (ν_e, ν_μ, ν_τ)
  - change a "down"-type quake (d, s, b) into an up-type quark (u,c,t)
- Weak neutral current:
  - particle identity does not change
  - same particle in and out

THE WEAK CHARGED CURRENT

- Feynman rules:

Vertex Factor for Leptons:



$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

W propagator

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2}$$

WEAK VS. QED

$\bar{\psi}\psi$	scalar
$\bar{\psi}\gamma^5\psi$	pseudoscalar
$\bar{\psi}\gamma^\mu\psi$	vector
$\bar{\psi}\gamma^\mu\gamma^5\psi$	pseudovector
$\bar{\psi}\sigma^{\mu\nu}\psi$	antisymmetric tensor

QED vertex

$$-ig_e\gamma^\mu$$

photon propagator

$$\frac{-ig_{\mu\nu}}{q^2}$$

Weak CC vertex

$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

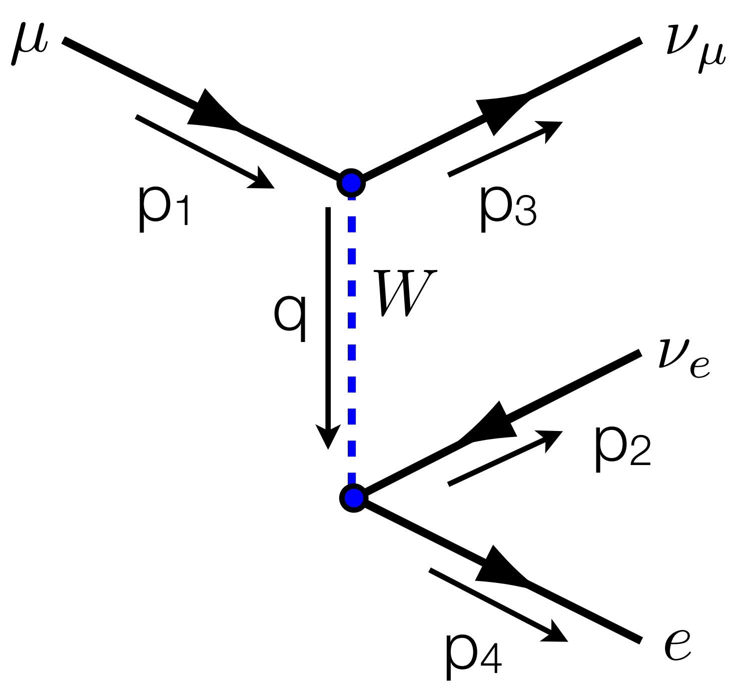
W propagator

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2}$$

- Vertex:
  - coupling constant
  - γ^μ vs. γ^μ-$\gamma^\mu\gamma^5$
  - charge: W carries one unit
- Propagator
  - massive particle (3 polarizations)
  - at low energies: $q << M_W c^2$

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2} \Rightarrow \frac{ig_{\mu\nu}}{M_W^2c^2}$$

EXAMPLE: MUON DECAY:



- upper fermion leg

$$[\bar{u}_3 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) u_1] \quad (2\pi)^4 \delta(p_1 - q - p_3)$$

- lower fermion leg

$$[\bar{u}_4 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) v_2] \quad (2\pi)^4 \delta(q - p_2 - p_4)$$

- Propagator

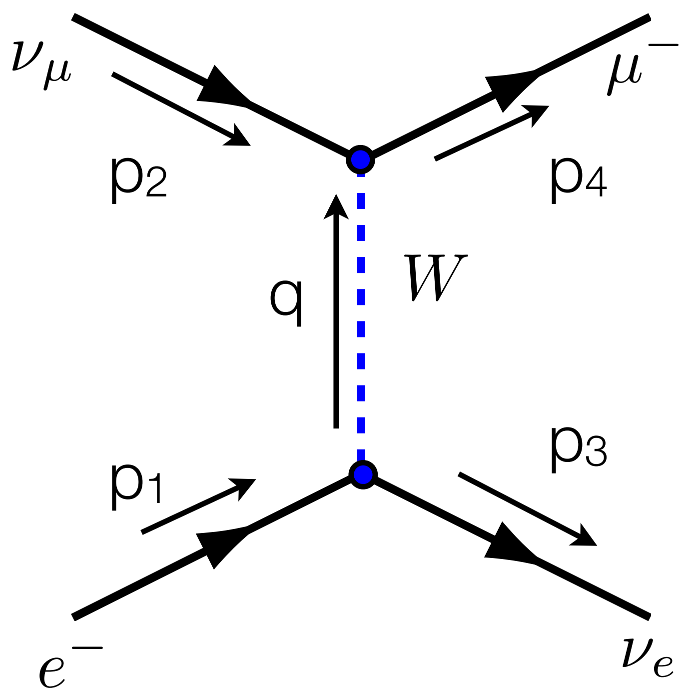
$$\int \frac{1}{(2\pi)^4} d^4 q \quad \frac{ig_{\mu\nu}}{M_W^2 c^2}$$

$$\frac{-ig_W^2}{8M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) u_1\right] \left[\bar{u}_4 \gamma_\mu (1-\gamma^5) v_2\right] \times (2\pi)^4 \delta(p_1 - p_2 - p_3 - p_4)$$

$$\frac{g_W^2}{8M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) u_1\right] \left[\bar{u}_4 \gamma_\mu (1-\gamma^5) v_2\right]$$

INVERSE MUON DECAY:

$\nu_\mu + e^- \to \mu^- + \nu_e$



- upper fermion leg

$$\left[\bar{u}_4 \frac{-ig_W}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) u_2\right] (2\pi)^4 \delta^4(p_2 + q - p_4)$$

- lower fermion leg

$$\left[\bar{u}_3 \frac{-ig_W}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) u_1\right] (2\pi)^4 \delta^4(p_1 - q - p_3)$$

- Propagator

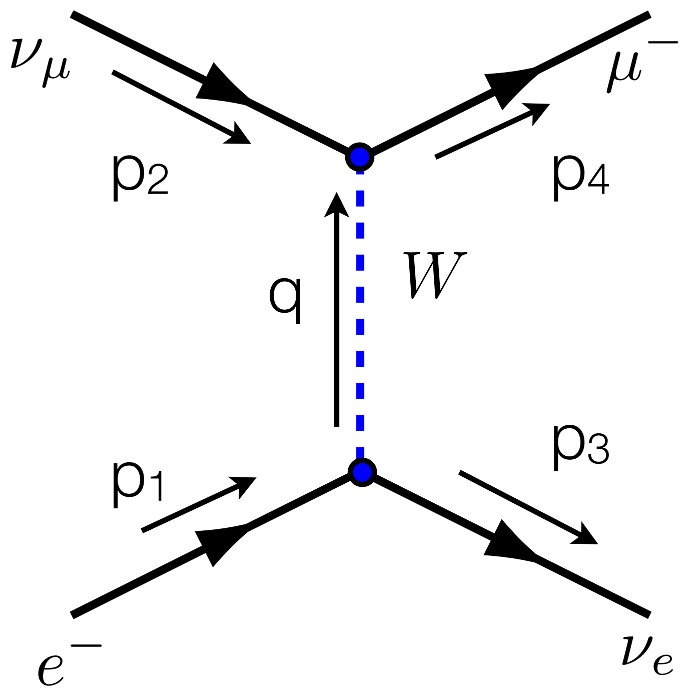
$$\int \frac{1}{(2\pi)^4} d^4 q \quad \frac{ig_{\mu\nu}}{M_W^2 c^2}$$

$$\frac{-ig_W^2}{8M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) u_1\right] \left[\bar{u}_4 \gamma_\mu (1-\gamma^5) u_2\right] \times (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)$$

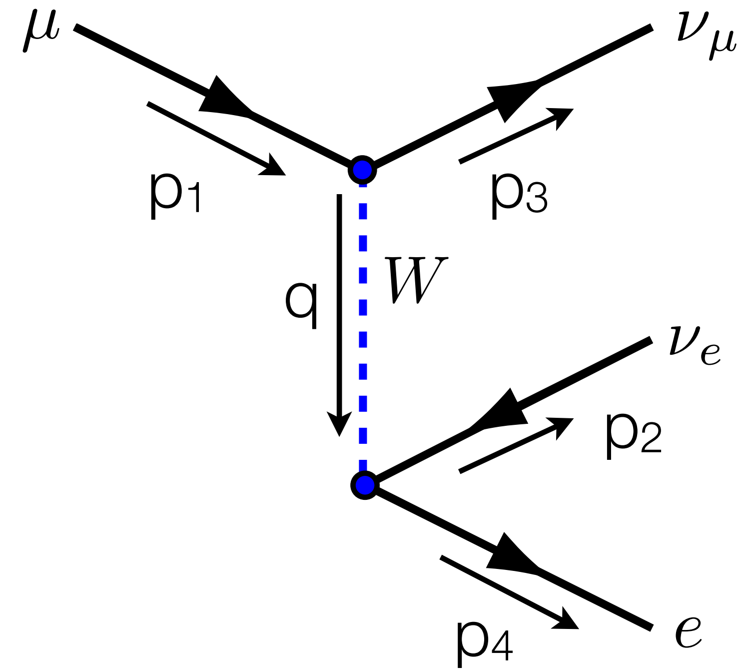
$$\mathcal{M} = \frac{g_W^2}{8M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) u_1\right] \left[\bar{u}_4 \gamma_\mu (1-\gamma^5) u_2\right]$$

COMPARISON

$$\nu_\mu + e^- \to \mu^- + \nu_e$$



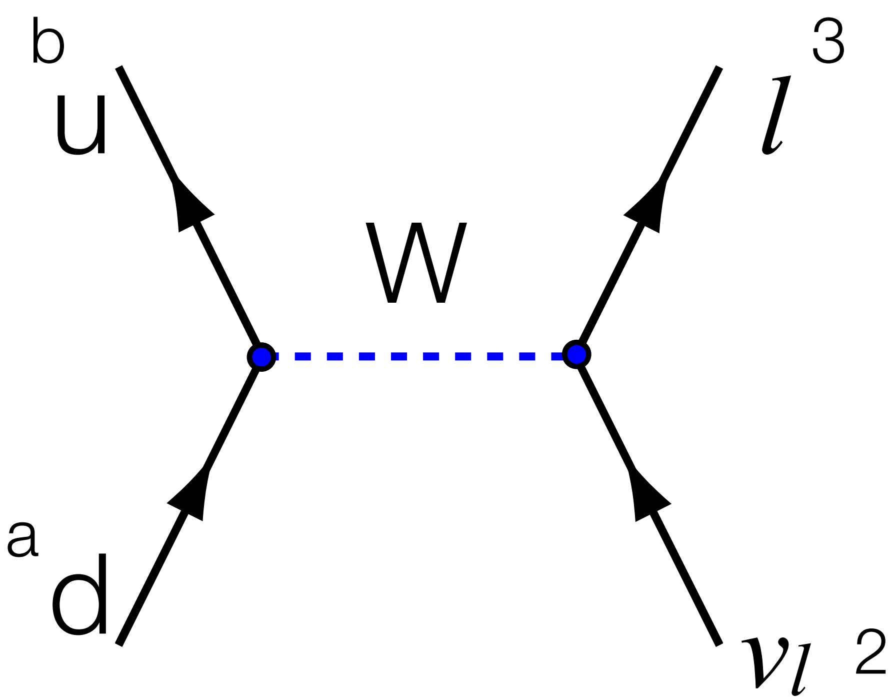
$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$$



$$\mathcal{M} = \frac{g_W^2}{8M_W^2 c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_4\gamma_\mu(1-\gamma^5)u_2\right] \qquad \frac{g_W^2}{8M_W^2 c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_4\gamma_\mu(1-\gamma^5)v_2\right]$$

- The similarity is not a coincidence

EXAMPLE: PION DECAY



- Lepton fermion leg

$$[\bar{u}_3 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) v_2]$$

- Quark Fermion leg

$$[\bar{v}_b \frac{-ig_w}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) u_a]$$

$$[\bar{v}_b \frac{-ig_w}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) u_a] \Rightarrow F^\nu = f_\pi p^\nu$$

- Propagator

$$\int \frac{1}{(2\pi)^4} d^4 q \qquad \frac{ig_{\mu\nu}}{M_W^2 c^2}$$

$$\mathcal{M} = \frac{g_W^2}{8M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) v_2\right] f_\pi p_\mu$$

MIDTERM:

- Two parts:
  - Short answer: a few questions about
    - fundamental particles, interactions, “strengths”
    - Feynman diagrams for basic process
    - basic considerations of phase space, form factors, etc.
    - no detailed calculations, all conceptual
  - Feynman rules, γ matrices
    - some calculations using general properties of γ matrices
    - all explicit forms of spinors, matrices that are needed will be provided
  - Feel free to bring:
    - 1 formula sheet
    - calculator

$$\mathcal{M}\mathcal{M}^* = \left(\frac{g^2}{8M_W^2c^2}\right)^2 \left[\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\right]\left[\bar{u}_3\gamma^\nu(1-\gamma^5)v_2\right]^* f_\pi^2 p_\mu p_\nu$$

$$\sum_{\text{a, b spins}} \left[\bar{u}(a)\Gamma_1 u(b)\right]\left[\bar{u}(a)\bar{\Gamma}_2 u(b)\right]^* = \text{Tr}\left[\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)\right]$$

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^4}{64M_W^4c^4} f_\pi^2 p_\mu p_\nu$$
$$\text{Tr}\left[\gamma^\mu(1-\gamma^5)(\not{p}_2)\gamma^\nu(1-\gamma^5)(\not{p}_3 + m_l c)\right]$$

- We've done this trace already:

$$\text{Tr} \Rightarrow 8 \times \left[p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - g^{\mu\nu} p_2 \cdot p_3 - i\epsilon^{\mu\nu\lambda\sigma} p_{2\lambda} p_{3\sigma}\right]$$

- So:

$$\langle|\mathcal{M}|^2\rangle = \frac{f_\pi^2 g_W^4}{8M_W^4c^4}\left[2 \times (p \cdot p_2)(p \cdot p_3) - p^2(p_2 \cdot p_3)\right]$$

EVALUATING THE MATRIX

- If we work in the centre of mass frame, the pion is at rest:
  - $p_\pi^\mu = (m_\pi, \mathbf{0})$
  - so in the expression, we only need to consider $\mu = 0$

$$\mathcal{M} = \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\right]f_\pi p_\mu \quad \Rightarrow \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^0(1-\gamma^5)v_2\right]f_\pi m_\pi$$

$$\bar{u}_3 = u_3^\dagger\gamma^0$$

$$\Rightarrow \frac{g_W^2}{8M_W^2c^2}\left[u_3^\dagger\gamma^0(1-\gamma^5)v_2\right]f_\pi m_\pi$$

Improved Measurement of the $\pi \to e\nu$ Branching Ratio

A. Aguilar-Arevalo,[1] M. Aoki,[2] M. Blecher,[3] D. I. Britton,[4] D. A. Bryman,[5] D. vom Bruch,[5] S. Chen,[6] J. Comfort,[7] M. Ding,[6] L. Doria,[8] S. Cuen-Rochin,[5] P. Gumplinger,[8] A. Hussein,[9] Y. Igarashi,[10] S. Ito,[2] S. H. Kettell,[11] L. Kurchaninov,[8] L. S. Littenberg,[11] C. Malbrunot,[5,*] R. E. Mischke,[8] T. Numao,[8] D. Protopopescu,[4] A. Sher,[8] T. Sullivan,[5] D. Vavilov,[8] and K. Yamada[2]

(PIENU Collaboration)

[1]*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de Mexico, Distrito Federal 04510 México*
[2]*Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[3]*Physics Department, Virginia Tech, Blacksburg, Virginia 24061, USA*
[4]*Physics Department, University of Glasgow, Glasgow G12 8QQ, United Kingdom*
[5]*Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada*
[6]*Department of Engineering Physics, Tsinghua University, Beijing 100084, People's Republic of China*
[7]*Physics Department, Arizona State University, Tempe, Arizona 85287, USA*
[8]*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada*
[9]*University of Northern British Columbia, Prince George, British Columbia V2N 4Z9, Canada*
[10]*KEK, 1-1 Oho, Tsukuba-shi, Ibaraki 305-0801, Japan*
[11]*Brookhaven National Laboratory, Upton, New York 11973-5000, USA*
(Received 8 June 2015; published 13 August 2015)

A new measurement of the branching ratio $R_{e/\mu} = \Gamma(\pi^+ \to e^+\nu + \pi^+ \to e^+\nu\gamma)/\Gamma(\pi^+ \to \mu^+\nu + \pi^+ \to \mu^+\nu\gamma)$ resulted in $R^{\text{exp}}_{e/\mu} = [1.2344 \pm 0.0023(\text{stat}) \pm 0.0019(\text{syst})] \times 10^{-4}$. This is in agreement with the standard model prediction and improves the test of electron-muon universality to the level of 0.1%.