

PHY489/1489

LECTURE 13:

QUANTUM CHROMODYNAMICS

N . B . :

- We will qualitatively cover various important aspects of QCD in this lecture
- We will not do any detailed calculations, though we will show some elements of how things may go if we did
- These key aspects of QCD are an important part of particle physics and establishing gauge theories as the basis for our description of particle physics.

NON-ABELIAN GAUGE THEORY

- Last time we considered the "SU(2) gauge theory"
 - pair of Dirac fields with local gauge invariance under the group SU(2)

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow e^{i\frac{g}{2}\vec{\theta}\cdot\vec{\sigma}} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- elements of the group parameterized by Pauli matrices as "generators":

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- In order to maintain local gauge invariance

- need to add an extra term:

$$(i\not{\partial} - \frac{g}{2}\vec{\sigma}\cdot\vec{A} - m)\psi = 0$$

- where the A^i transform as follows in the gauge transformation.

$$\vec{A}_\mu \rightarrow \vec{A}_\mu - \frac{1}{2}\partial_\mu\theta - \frac{g}{2}\epsilon_{ijk}\theta_i A_\mu^j$$

CONSEQUENCES

- We have three new fields:

$$\vec{A}_\mu \rightarrow \vec{A}_\mu - \frac{1}{2} \partial_\mu \theta - \frac{g}{2} \epsilon_{ijk} \theta_i A_\mu^j \quad \vec{A}_\mu \Rightarrow A_\mu^i$$

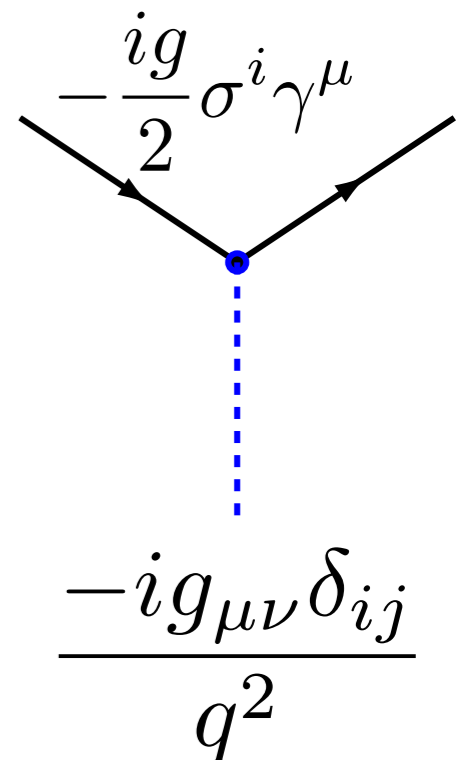
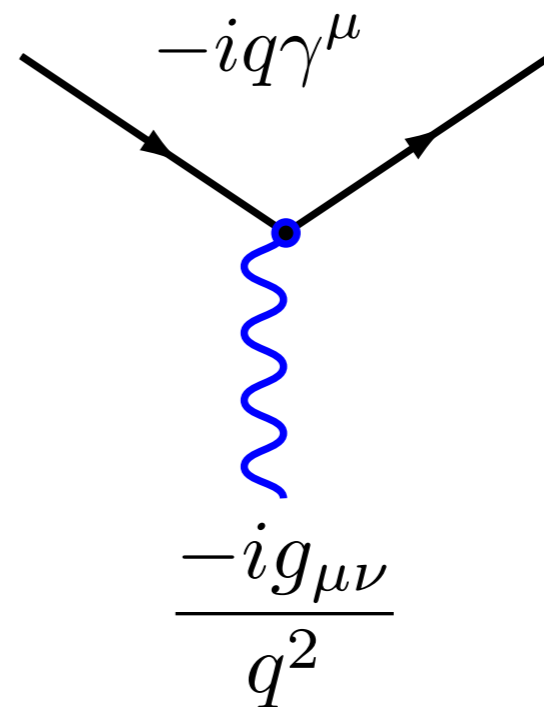
- they are Lorentz vector fields (like the photon)
- last term from non-commutation of the generators (Pauli matrices)
 - the fields couple to themselves
- we can give A life by introducing their own kinetic terms, etc.
 - the same gauge invariance that created them requires them to be massless
- We have a new interaction term:

$$(i\cancel{\partial} - \frac{g}{2} \vec{\sigma} \cdot \vec{A} - m)\psi = 0$$

$$\frac{g}{2} \sigma_i \cdot A_\mu^i \gamma^\mu \psi$$

- in analogy to the U(1) case:

$$(i\cancel{\partial} - qA - m)\psi = 0$$



SU(3) GAUGE SYMMETRY

- Basically the same same story:

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \rightarrow e^{i \frac{g_s}{2} \vec{\alpha} \cdot \vec{T}} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

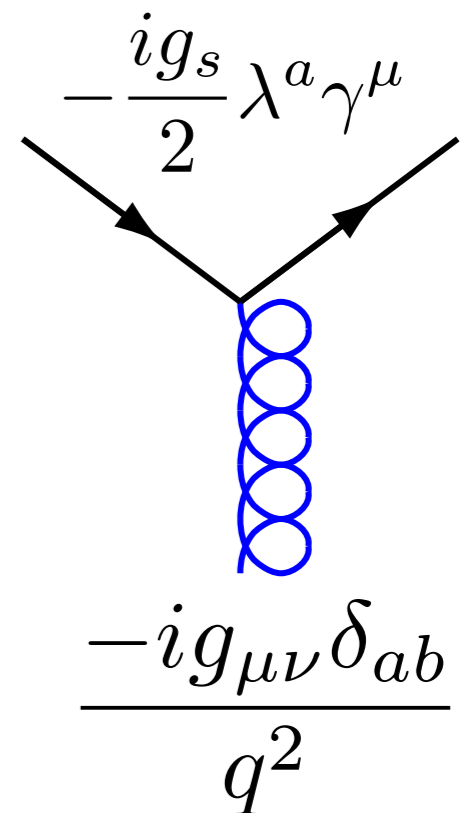
- where T_a are eight 3x3 matrices which parameterize the SU(3) transformation.
 - Conventionally, we can use the "Gell-Mann" matrices which are the analog of the Pauli matrices in SU(3)

$$T_a = \frac{1}{2} \lambda_a$$

- This time, we need to add eight fields

$$(i\cancel{\partial} - g_s T^a G^a - m)\psi = 0$$

$$G^a \rightarrow G_\mu^a - \frac{1}{2} \partial_\mu \alpha^a - g_s f_{ijk} \alpha_i G_\mu^j$$



QUANTUM CHROMODYNAMICS

- This is the theory of QCD as we know it
 - local gauge theory under SU(3)
 - eight massless "gluons": bosons introduced for local gauge invariance
 - acts on triplets of Dirac fields

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \rightarrow e^{i\frac{g_s}{2}\vec{\alpha}\cdot\vec{T}} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \begin{matrix} \text{"red"} \\ \text{"green"} \\ \text{"blue"} \end{matrix}$$

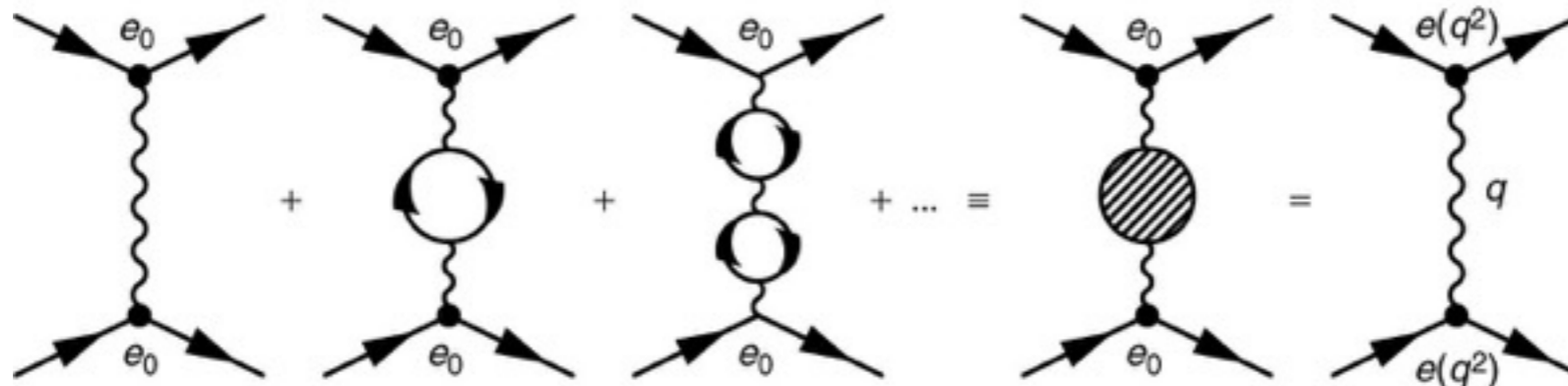
- Fundamental connection between "transformation" and "interaction"
 - fields that do not transform under the gauge transformation do not interact
 - objects that do not transform are called "singlets" and do not participate
 - quarks are "triplets" under SU(3) ("fundamental")
 - leptons and other "uncolored" objects are "singlets"
- similarity of QED, QCD arise because they are both (local) gauge theories

COLOR CONFINEMENT

- “colored” objects are not observed directly in nature
 - i.e. all objects we see take the form of objects with net zero color
 - “color singlet states”
 - in particular, bare gluons and quarks never appear to be seen
- Examples:
 - mesons $q\bar{q}$ objects with opposite color
 - baryons: qqq objects which combine to form a zero color state
 - and antibaryons
- There is no rigorous proof that this must be the case, but conceptually it can be understood in the context of the strength of the interaction.
 - “pulling apart” two coloured objects leads to additional gluons being produced which then form and “neutralize” them.

"RUNNING" COUPLING CONSTANT

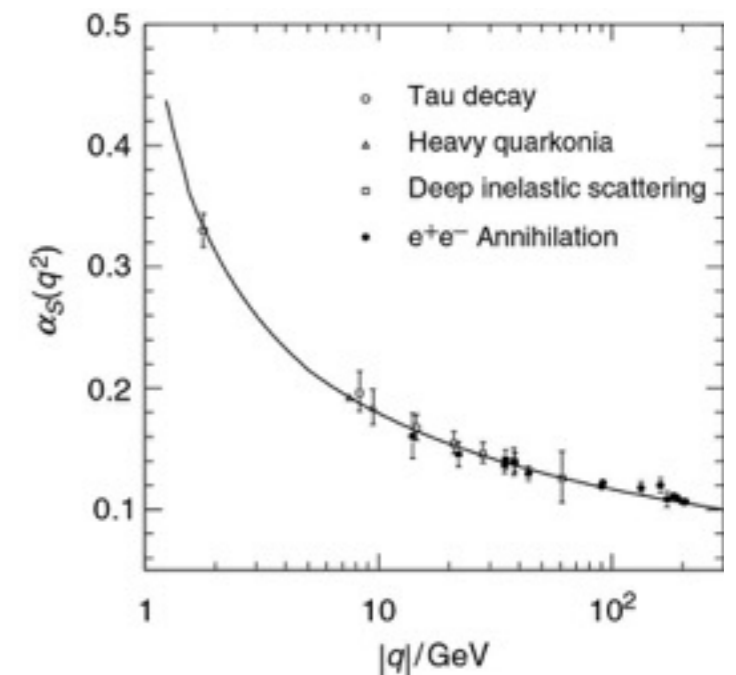
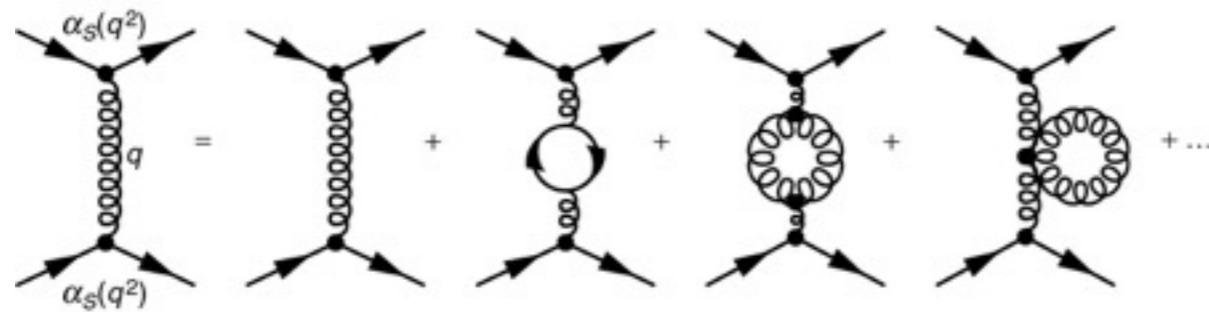
- We have talked about "coupling constants" in QED and QCD
- A major development was the acceptance/incorporation of the idea that these are not constants at all
 - higher order diagrams result in a q^2 dependence for a process



- contribution from all the diagrams can be summarized as a coupling "constant" at a fixed q^2
 - for electromagnetism, this results in large coupling as q^2 increases.
- "renormalization"

QCD

- The first higher order correction to the gluon propagator



- this results in:

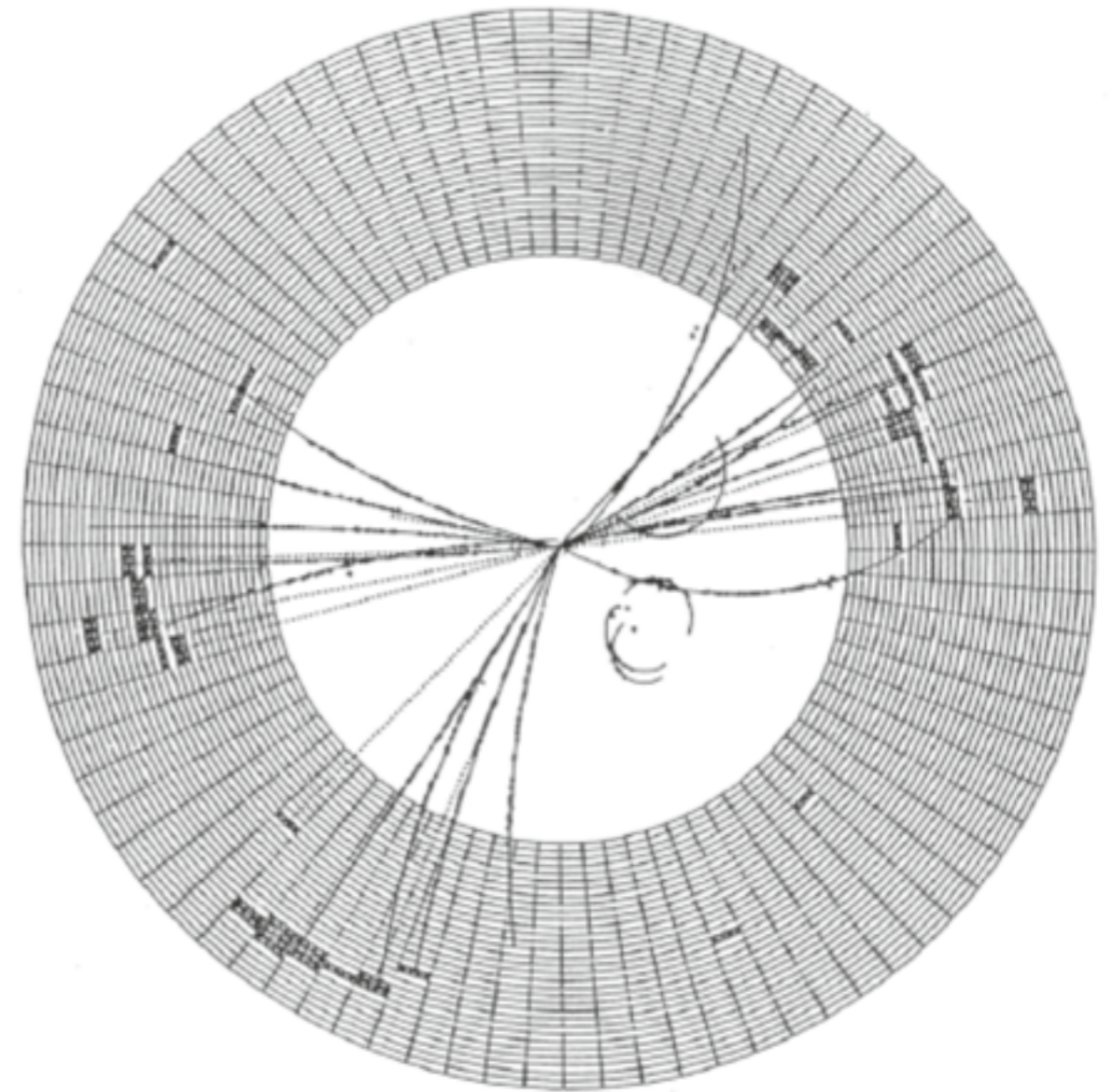
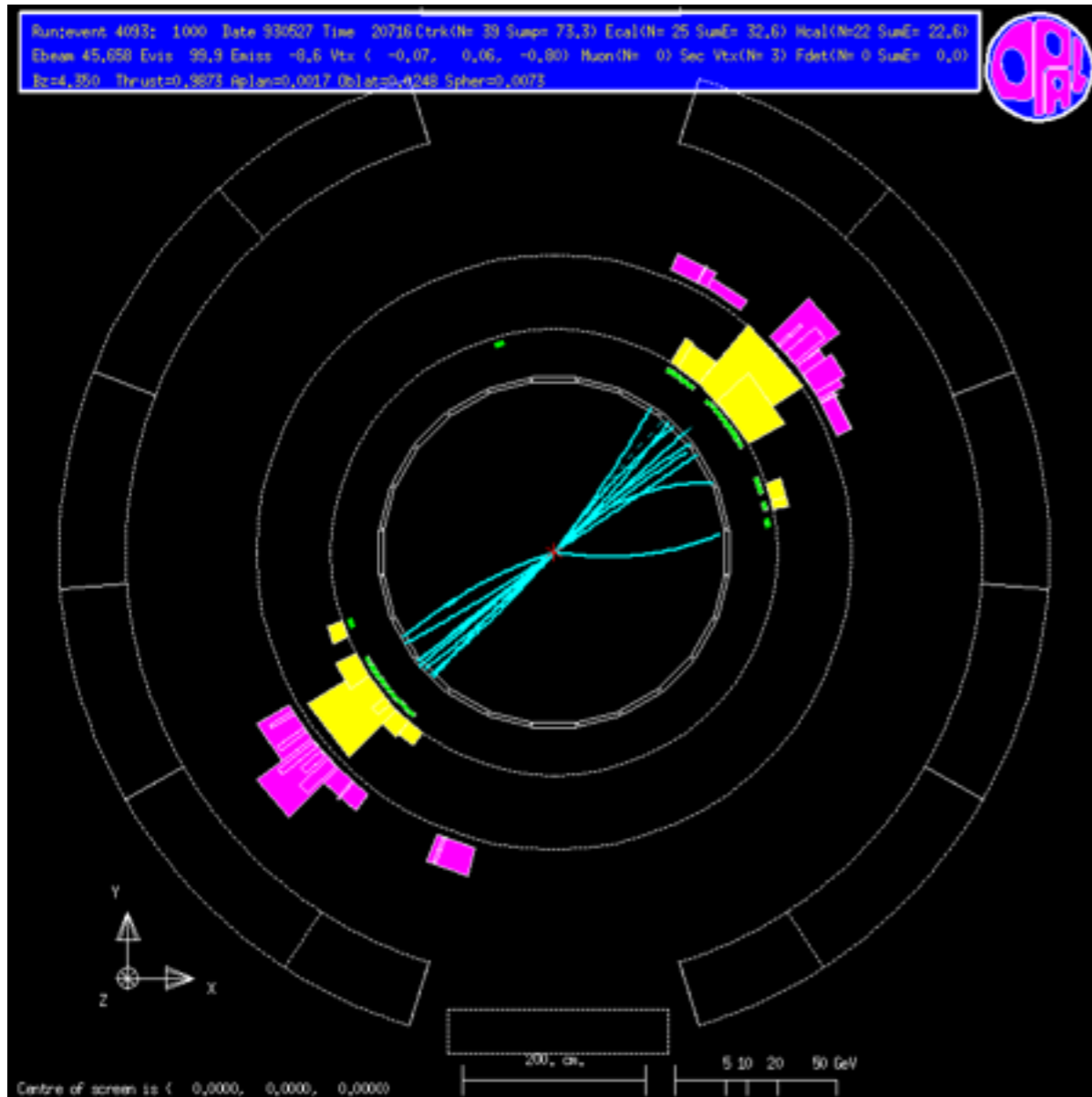
$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + B\alpha_s(\mu^2) \log \frac{q^2}{\mu^2}}$$

$$B = \frac{11N_c - 2N_f}{12\pi}$$

- note that it depends on the number of quark flavours
- with six quarks, we find that $B < 0$
- coupling constant *decreases* with q^2
- Λ_{QCD} : where $\alpha_s \sim 1$: 220 MeV

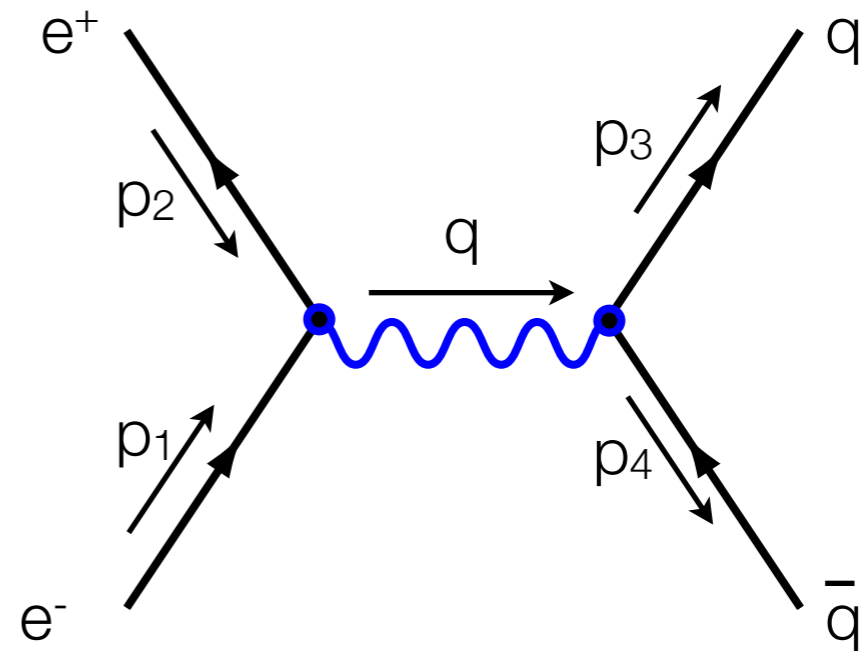
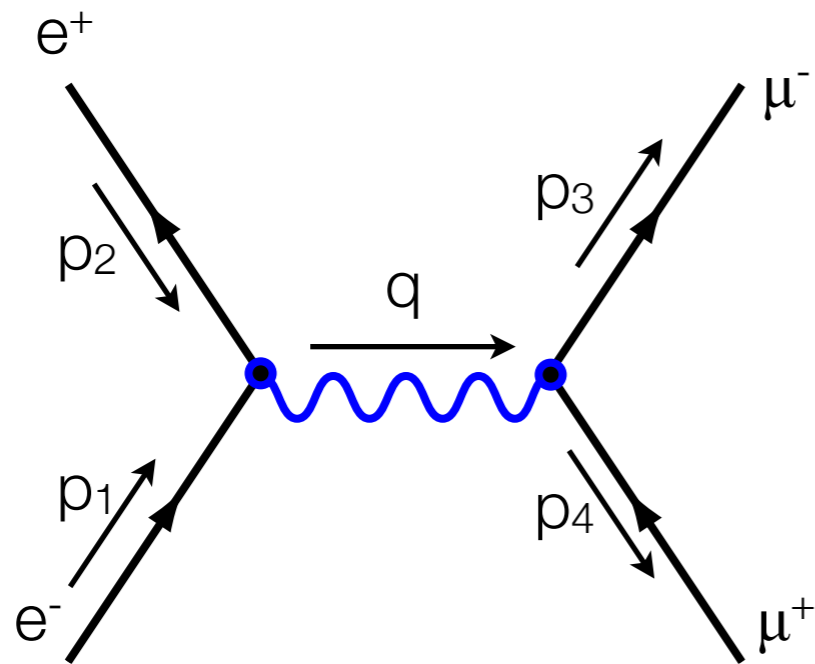


$q\bar{q}$ PRODUCTION:



*** SUMS (GEV) *** PTOT 35.768 PTRANS 29.964 PLONG 15.788
 TOTAL CLUSTER ENERGY 15.169 PHOTON ENERGY 4.693 NR OF PHOTONS 1

$$e^+ + e^- \rightarrow q + \bar{q}$$



- All the calculation steps are the same, except . . .
 - quarks do not have unit charge
 - quarks can be emitted in three possible color combinations
 - different quarks have different masses

Cross section

- Recall total cross section for μ pair production

$$\sigma = \frac{\pi}{3} \left(\frac{\hbar c \alpha}{E} \right)^2 \quad \sigma_{\mu^+ \mu^-} = \frac{\pi}{3} \left(\frac{\hbar c \alpha}{E} \right)^2 \Rightarrow \sigma_{q_i \bar{q}_i} = 3Q_i^2 \times \sigma_{\mu^+ \mu^-}$$

- now take the ratio

$$R = \frac{\sum_i \sigma_{q_i \bar{q}_i}}{\sigma_{\mu^+ \mu^-}} = 3 \times \sum Q_i^2$$

- Which quarks can be produced depends on energy

$$\sqrt{s} < 2m_c \sim 3.7\text{GeV}$$

$$R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2) = 2$$

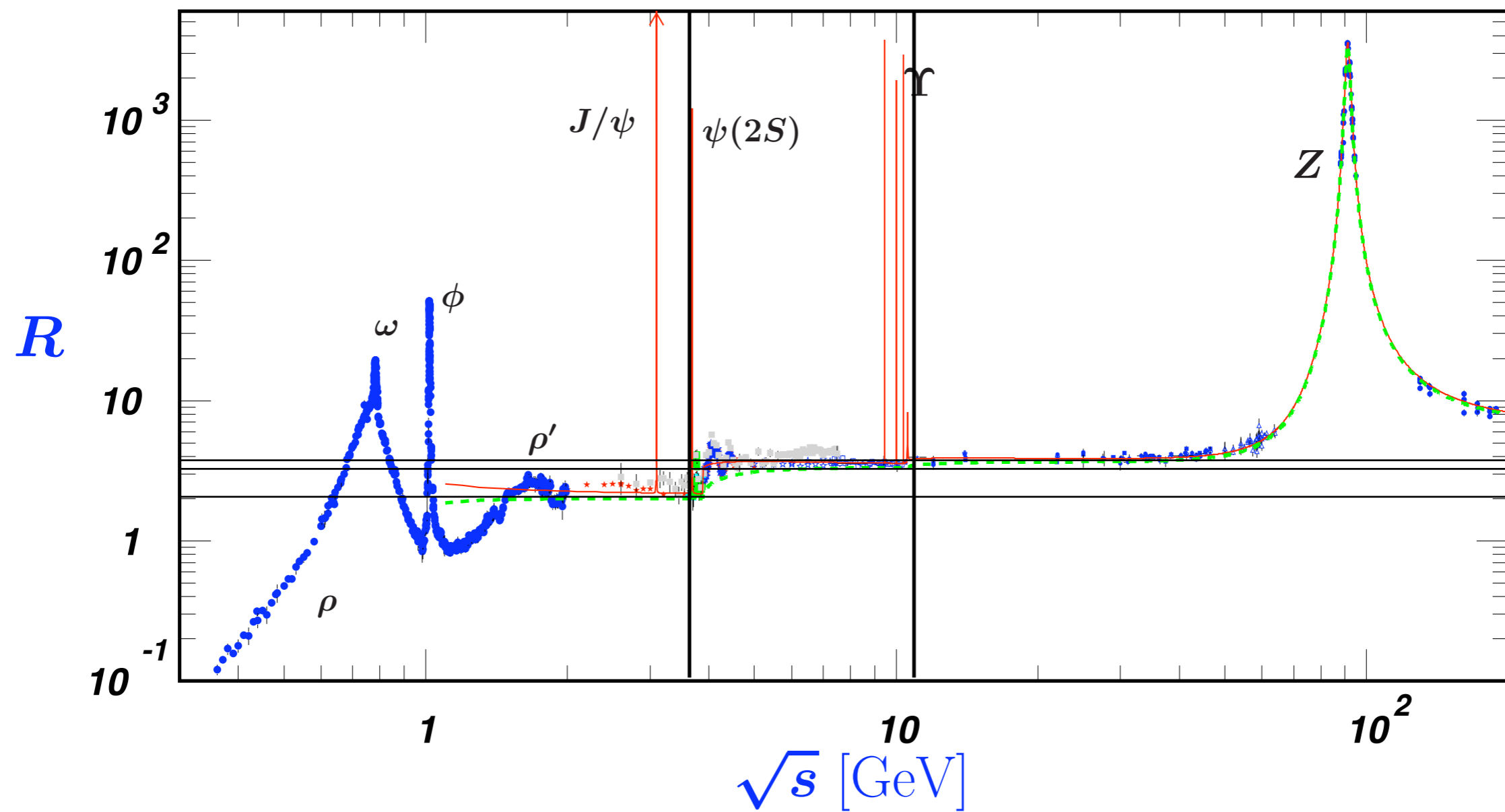
$$\sqrt{s} < 2m_b \Rightarrow 10.6\text{GeV}$$

$$R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2 + (2/3)^2) = 10/3$$

$$\sqrt{s} < 2m_t \sim 330\text{GeV}$$

$$R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2 + (2/3)^2 + (1/3)^2) = 11/3$$

MEASUREMENTS



SUMMARY:

- QED and QCD have a common origin in local gauge theories
 - QED is a U(1) gauge theory, 1 gauge boson
 - QCD is a SU(3) gauge theory, 8 gauge bosons
- The triplet of states associated with the SU(3) transformations is the "color" space
 - objects must transform under SU(3) to feel the QCD interaction
- Confinement:
 - only colorless/"singlet" states are observed
- Asymptotic Freedom:
 - strength of QCD (α_s) decreases as q^2 rises
 - " Λ_{QCD} ": q^2 region where QCD is "strong" ($\alpha_s \sim 1$)

PROBLEM SET 2

- 6.7(c)

$$\bar{u}_{\uparrow}(p_4)\gamma^{\mu}u_{\downarrow}(p_2) = 2(Ms, 0, 0, 0)$$

$$\bar{u}_{\downarrow}(p_4)\gamma^{\mu}u_{\uparrow}(p_2) = -2(Ms, 0, 0, 0)$$

NEXT TIME:

- We'll move onto weak interactions
 - Please read chapter 11
 - Problem set 3 will be posted soon.
- Colloquia Today (1600)
 - **M. Spiropulu (CalTech)**
 - **"The Higgs and beyond: connections and puzzles in particle physics"**