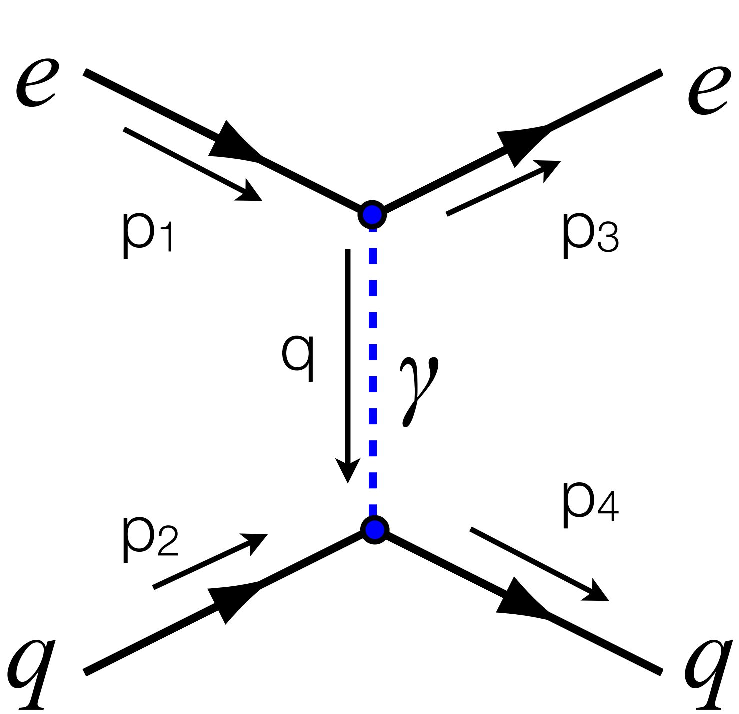
PHYSICS 489/1489

LECTURE 11: ELECTRON-PROTON SCATTERING

ELECTRON-QUARK SCATTERING



$$\mathcal{M} = \frac{Qe^2}{(p_1 - p_3)^2}\left[\bar{u}_3\gamma^\mu u_1\right]\left[\bar{u}_4\gamma_\mu u_2\right]$$

$$|\mathcal{M}|^2 = \frac{Q^2e^4}{(p_1 - p_3)^4}\left[\bar{u}_3\gamma^\mu u_1\right]\left[\bar{u}_3\gamma^\nu u_1\right]^*\left[\bar{u}_4\gamma_\mu u_2\right]\left[\bar{u}_4\gamma_\nu u_2\right]^*$$

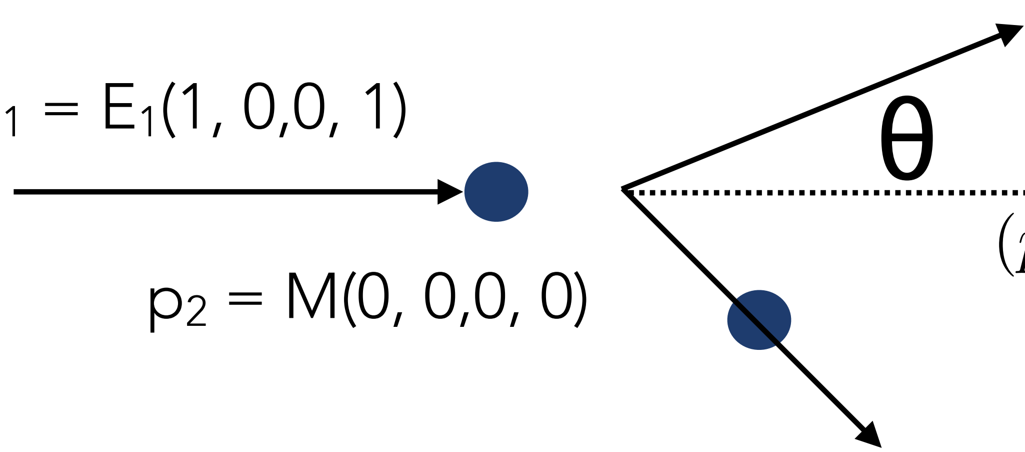
$$\sum_{s_a, s_b}\left[\bar{u}_a\Gamma_1 u_b\right]\left[\bar{u}_a\bar{\Gamma}_2 u_b\right]^*$$
$$\Rightarrow \mathrm{Tr}\left[\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)\right]$$

$$\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{e^4}{(p_1 - p_3)^4}\mathrm{Tr}\left[\gamma^\mu(\not{p}_1 + mc)\gamma^\nu(\not{p}_3 + mc)\right]\mathrm{Tr}\left[\gamma_\mu(\not{p}_2 + Mc)\gamma_\nu(\not{p}_4 + Mc)\right]$$

$$\langle|\mathcal{M}|^2\rangle = \frac{8Q^2e^4}{(p_1 - p_3)^4}\left[(p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) - (M^2c^2)(p_1\cdot p_3)\right]$$

with m~0

$p_1 = E_1(1, 0,0, 1)$



$p_2 = M(0, 0,0, 0)$

$p_3 = E_3(1, 0, \sin\theta, \cos\theta)$

$$(p_1 - p_3)^2 \sim -2p_1\cdot p_3 = -4E_1E_3\sin^2\frac{\theta}{2} = q^2 \equiv -Q^2$$

$$\langle|\mathcal{M}|^2\rangle = \frac{M^2e^4}{E_1E_3\sin^4\frac{\theta}{2}}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2M}\sin^2\frac{\theta}{2}\right]$$

A FEW NOTES

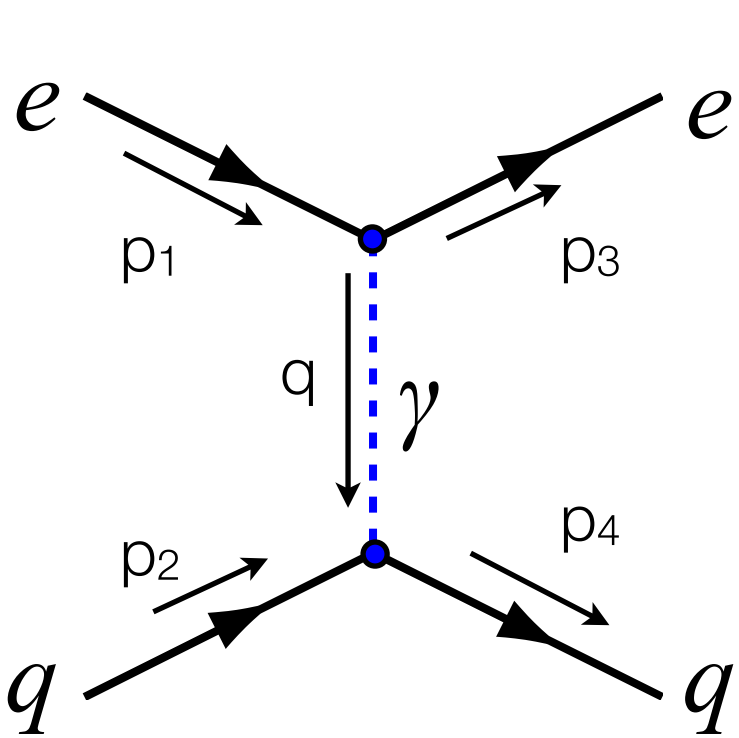
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{E_3}{ME_1}\right)^2 |\mathcal{M}|^2 \qquad\qquad e^2 = 4\pi\alpha$$

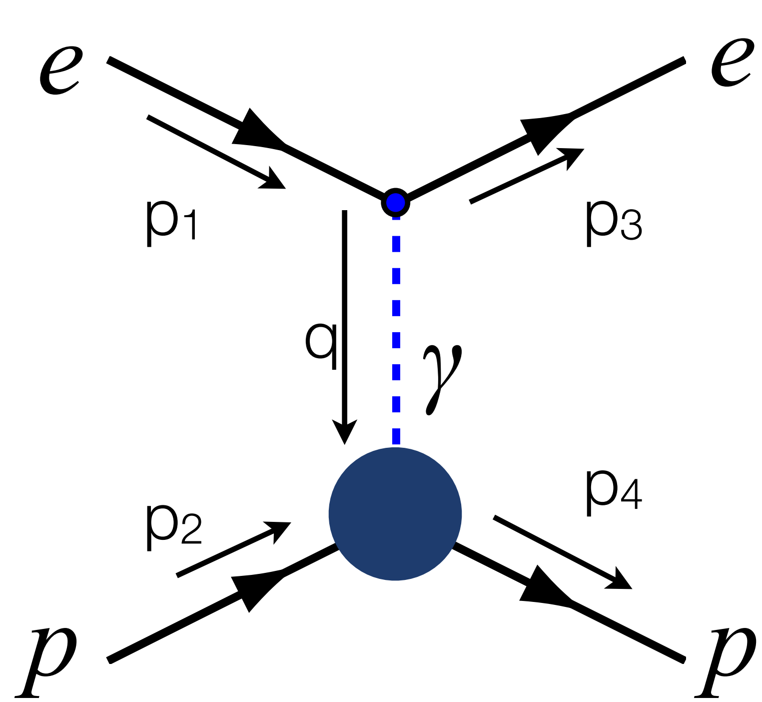
$$= \frac{e^4}{64\pi^2}\frac{1}{E_1^2\sin^4\frac{\theta}{2}}\frac{E_3}{E_1}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2M}\sin^2\frac{\theta}{2}\right] = \frac{\alpha^2}{4}\frac{1}{E_1^2\sin^4\frac{\theta}{2}}\frac{E_3}{E_1}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2M}\sin^2\frac{\theta}{2}\right]$$

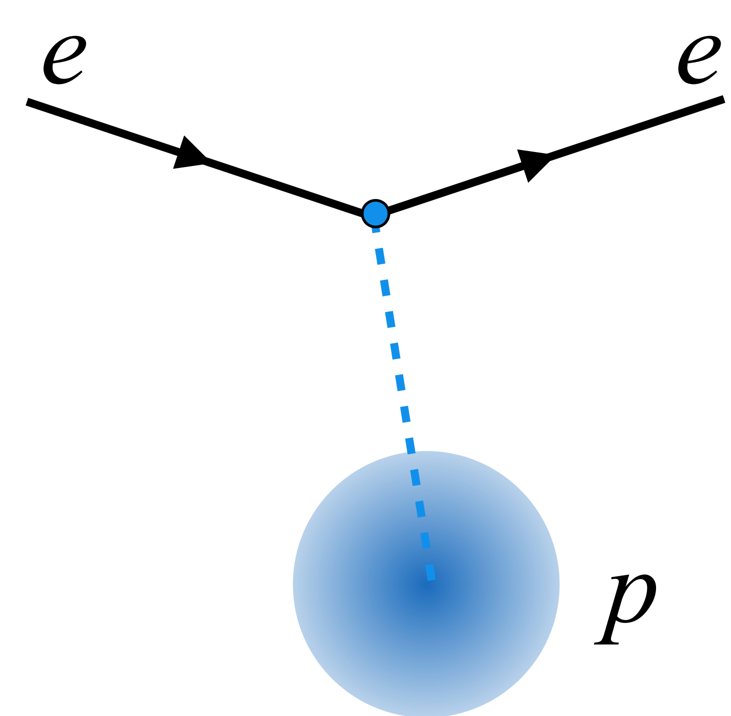
- Note that there is only one degree of freedom in the scattering apart from the incident electron energy E_1:
  - $E_3 = \dfrac{E_1 M}{M + E_1(1-\cos\theta)}$
  - $Q^2 = \dfrac{2ME_1^2(1-\cos\theta)}{M + E_1(1-\cos\theta)}$
  - i.e. if we know one of E_3, θ, or Q^2, the others are determined.

POINT-LIKE PARTICLES

- So far, we have focussed on point particles
    - as far as we know, electrons and quarks do not have any spatial extent
- What if the particle we are dealing with is not fundamental?



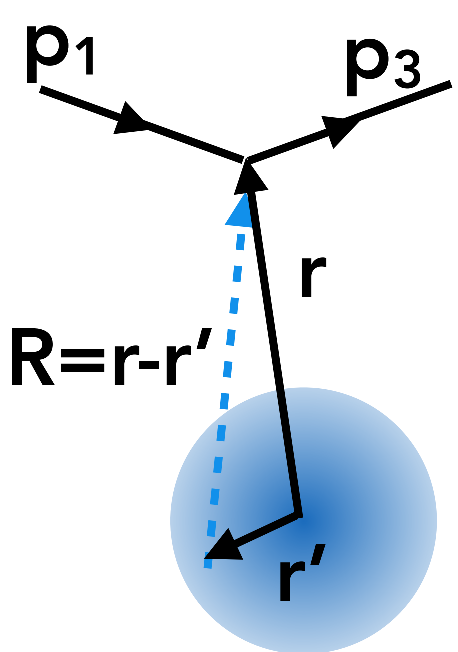




FORM FACTORS: APPROACH 1

- Accounting for spatial distribution of an arbitrary charge distribution:

$$\psi_3(\mathbf{r}) = e^{i(\mathbf{p}_3\cdot\mathbf{x} - E_3 t)} \qquad \psi_1(\mathbf{r}) = e^{i(\mathbf{p}_1\cdot\mathbf{x} - E_1 t)}$$



$$\mathcal{M} = \int d^3\mathbf{r}\; \psi_3^*(\mathbf{r})\; V(\mathbf{r})\; \psi_1(\mathbf{r})$$

$$V(\mathbf{r}) = Q\int d^3\mathbf{r}'\; \frac{\rho(\mathbf{r}')}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_3$$

$$\mathcal{M} = Q\int d^3\mathbf{r}\; \int d^3\mathbf{r}'\; e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} e^{i\mathbf{q}\cdot(\mathbf{r}')} \frac{\rho(\mathbf{r}')}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

$$= Q\int d^3\mathbf{R}\; \frac{e^{i\mathbf{q}\cdot\mathbf{R}}}{4\pi|\mathbf{R}|} \int d^3\mathbf{r}'\; \rho(\mathbf{r}') e^{i\mathbf{q}\cdot(\mathbf{r}')}$$

ANALYSIS

$$\mathcal{M} = Q \boxed{\int d^3\mathbf{R}\ \frac{e^{i\mathbf{q}\cdot\mathbf{R}}}{4\pi|\mathbf{R}|}} \boxed{\int d^3\mathbf{r}'\ \rho(\mathbf{r}')e^{i\mathbf{q}\cdot(\mathbf{r}')}}$$

- First term is matrix element for a point charge

$$\mathcal{M} = Q\int d^3\mathbf{r}\ \int d^3\mathbf{r}'\ e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}e^{i\mathbf{q}\cdot(\mathbf{r}')}\frac{\rho(\mathbf{r}')}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

$$= Q\int d^3\mathbf{r}\ e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{4\pi|\mathbf{r}|} \qquad \rho(\mathbf{r}') = \delta(\mathbf{0})$$

- Express as a modification of the matrix element of a point charge.

$$\mathcal{M} = \mathcal{M}_0 \times F(\mathbf{q}^2)$$

- also consider the distribution of magnetic moment to get another factor

BACK TO THE CROSS SECTION

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4}\frac{1}{E_1^2\sin^4\frac{\theta}{2}}\frac{E_3}{E_1}\left[\cos^2\frac{\theta}{2}+\frac{Q^2}{2M}\sin^2\frac{\theta}{2}\right]$$

$$\tau = \frac{Q^2}{4M^2}$$

$$\Rightarrow \frac{\alpha^2}{4E_1^2\sin^4\frac{\theta}{2}}\frac{E_3}{E_1}\left[\frac{G_E^2(Q^2)+\tau G_M^2(q^2)}{1+\tau}\cos^2\frac{\theta}{2}+2\tau G_M^2(Q^2)\sin^2\frac{\theta}{2}\right]$$

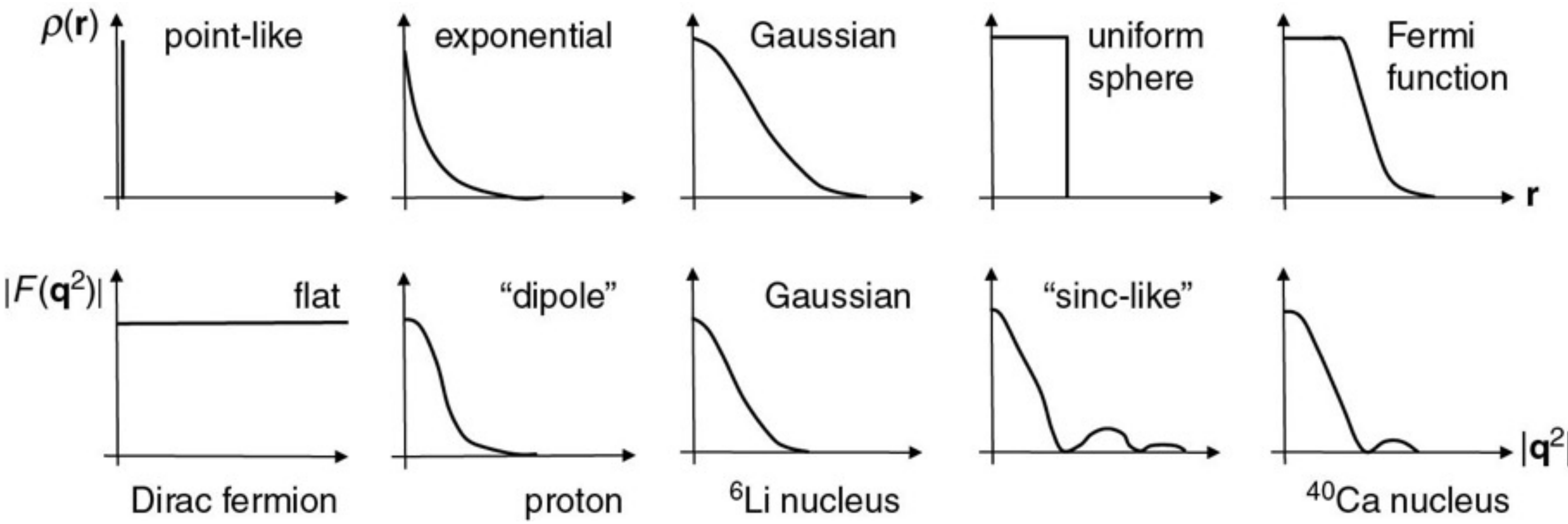
- Connecting to the previous discussion
- If $Q^2 \ll M^2$ then to a good approximation $Q^2 \sim q^2$

$$Q^2 = \mathbf{q}^2 - (E_1 - E_3)^2$$

$$E_1 - E_3 = \frac{Q^2}{2M}$$

$$\Rightarrow Q^2\left(1+\frac{Q^2}{4M^2}\right) = \mathbf{q}^2$$

PARMETRIZATIONS



- Measure the form factors as a function of q^2 or Q^2
  - effectively measuring the Fourier transform of the charge/magnetic moment of the proton
  - most results use the "dipole" parametrization

$$F(q^2) = \left(\frac{1}{1 + \frac{q^2}{M^2}}\right)^2$$

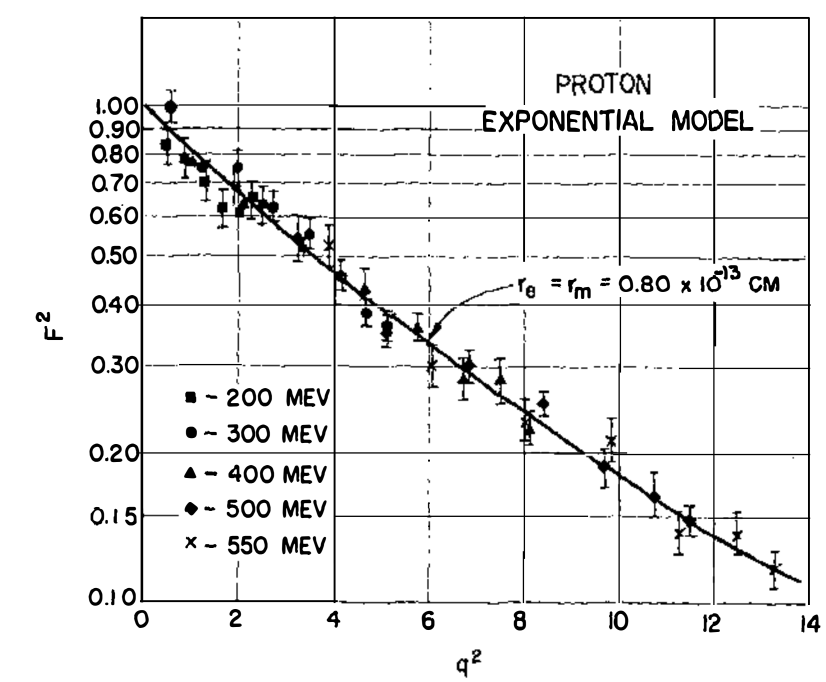


FIG. 8. An example of a model which fits the experimental values of F^2. This model gives the proton an exponential charge density and an exponential magnetic-moment density with rms radii 0.80×10^{-13} cm. Other close models fit equally well.

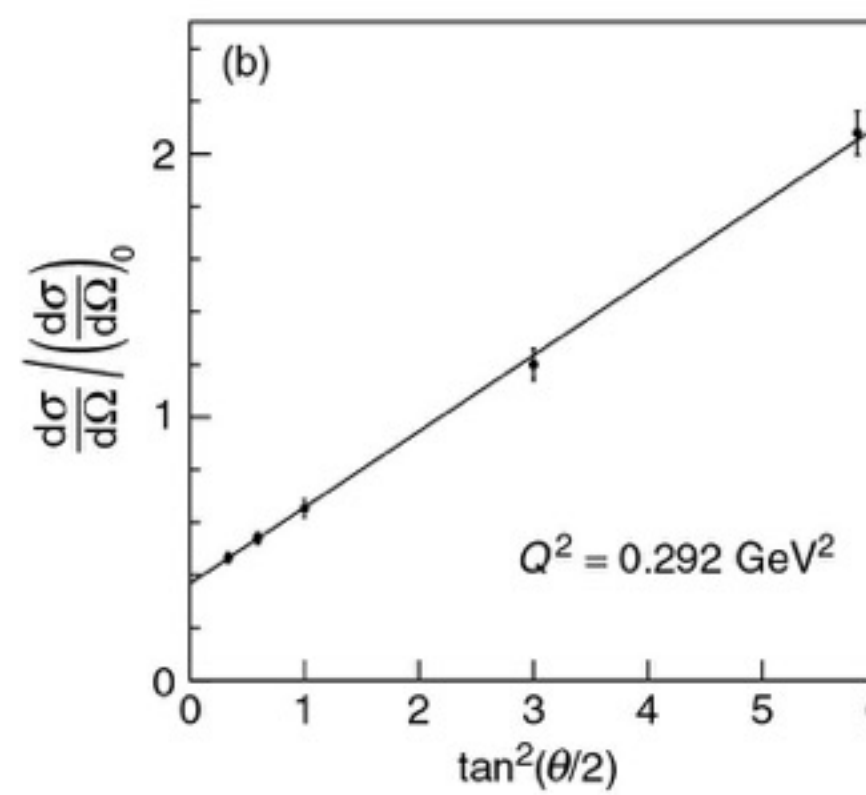
SEPARATING THE FORM FACTORS

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \frac{E_3}{E_1} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} \cos^2\frac{\theta}{2} + 2\tau G_M^2(Q^2) \sin^2\frac{\theta}{2} \right]$$

$$\tau = \frac{Q^2}{4M^2}$$

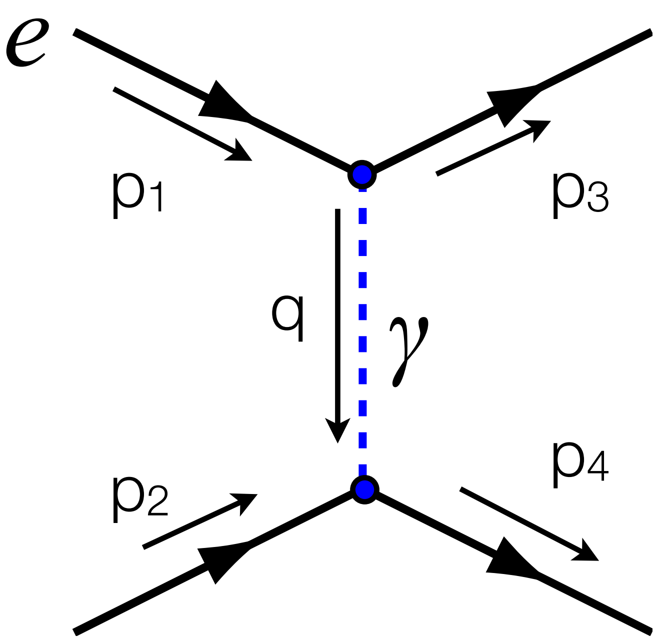
- Electric form factor (G_E) is dominant at low Q^2
- Magnetic form factor (G_M) is dominant at high Q^2
- General strategy:
  - Vary beam energy
  - Measure cross section vs. angle at a fixed Q^2
  - compare relative to "Mott" Cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \frac{E_3}{E_1} \cos^2\frac{\theta}{2}$$



FORM FACTORS: APPROACH II

- Another approach to form factors:



Lepton current $\mathrm{L}^{\mu\nu}$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{(p_1 - p_3)^4} \boxed{\mathrm{Tr}\left[\gamma^\mu(\not{p}_1 + mc)\gamma^\nu(\not{p}_3 + mc)\right]} \boxed{\mathrm{Tr}\left[\gamma_\mu(\not{p}_2 + Mc)\gamma_\nu(\not{p}_4 + Mc)\right]}$$

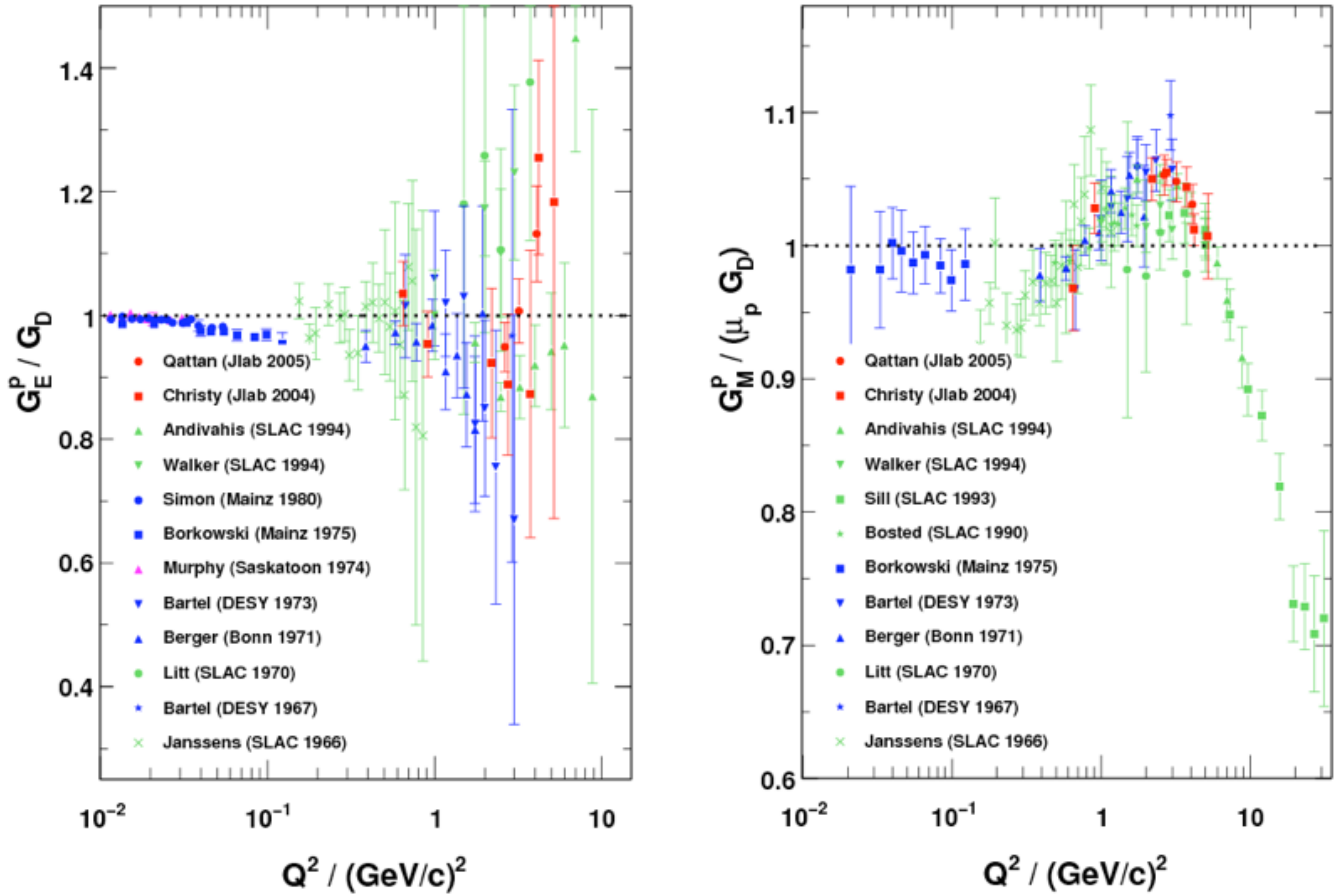
quark current $\mathrm{H}_{\mu\nu}$

- What form can the hadronic current take?
  - it must carry the Lorentz indices $\mu\nu$
  - Lorentz quantities with indices: $g_{\mu\nu}$, p_2, p_4, q
    - $p_4 = p_2 + q$
    - can be a function of Lorentz scale quantities (masses, q^2)

$$H_{\mu\nu} = H_1(q^2)g_{\mu\nu} + H_2(q^2)p_{2\mu}p_{2\nu} + H_3(q^2)q_\mu q_\nu + H_4(q^2)(p_{2\mu}q_\nu + p_{2\nu}q_\mu)$$

- Other considerations (charge conservation) lead to relations that reduce the number of independent functions to two

CONTEMPORARY TOPICS



Deviations for dipole for.

Can we obtain a more fundamental understanding of the proton form

"PARTONS"

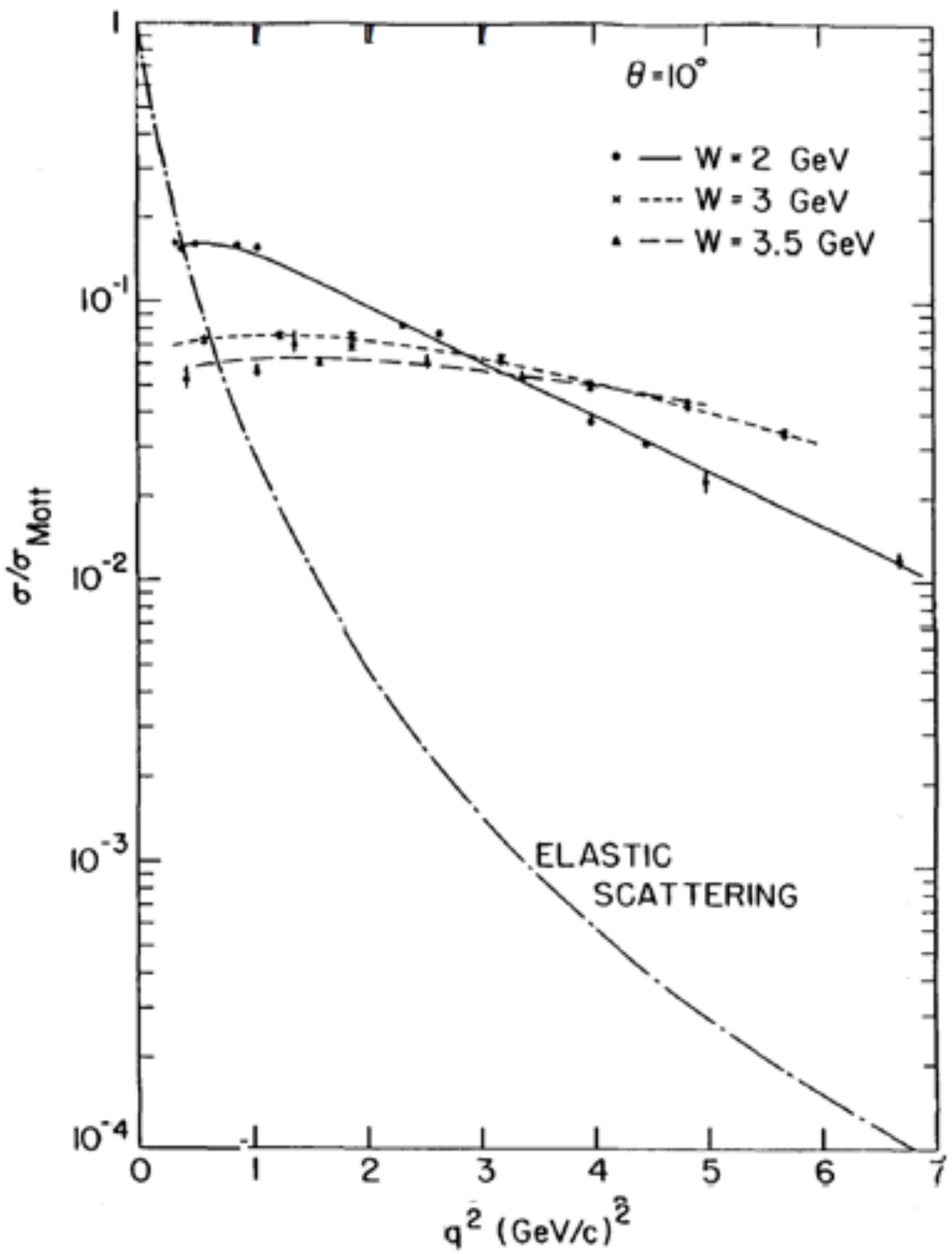
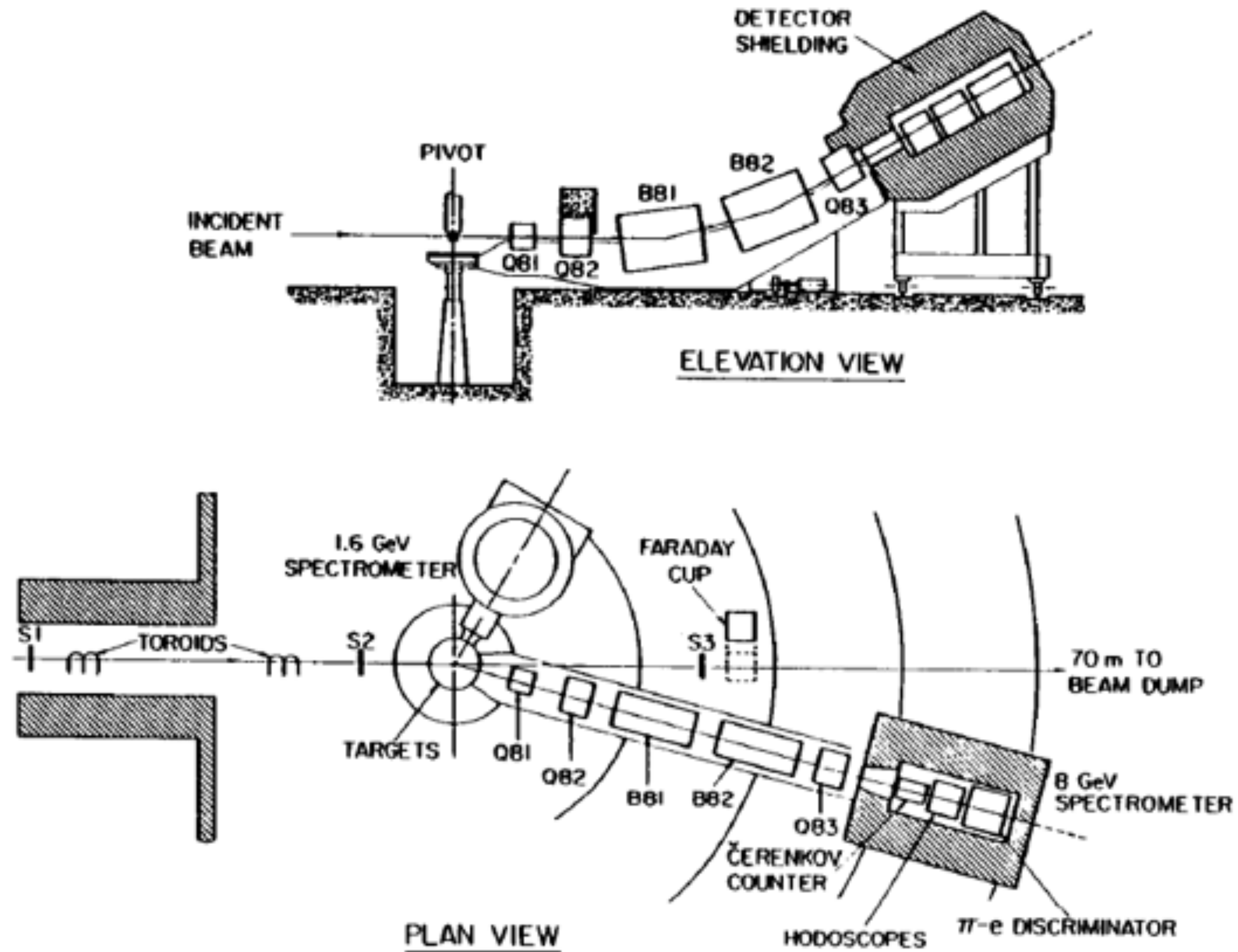


Fig. 1: $(d^2\sigma/d\Omega dE')/\sigma_{Mott}$, in GeV^{-1}, vs. q^2 for $W = 2$, 3 and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e-p scattering divided by σ_{Mott}, $(d\sigma/d\Omega)/\sigma_{Mott}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with q^2 of the inelastic cross section compared with the elastic cross section is clearly shown.



- strong high Q^2 scattering in high energy e-p scattering points to substructure in proton
  - "partons"
  - would later connect to "quarks"

IGNORANCE

- Form factors are a way to parametrize things we don't know or cannot predict fundamentally.
  - connect it to some physical picture (e.g charge distribution)
  - or appeal to things we do know (Lorentz symmetry) to provide a generalized framework for what it should be
- Typically arise when we have strongly interacting systems
  - e.g. bound quarks in baryons and mesons
    - can't predict a priori all the dynamics, etc.
  - decays of hadrons also have form factors
    - depending on the degrees of freedom, the form factor will have different kinematic dependences.
    - "blob" ↔ "form factor"

NEXT TIME

- Reading
  - 10.1-10.3
  - 10.4
  - 10.5, 10.5.2, 10.6
  - slight departure to original plan, will update.
- Problem Set 2 due next Tuesday