Discrete Symmetries

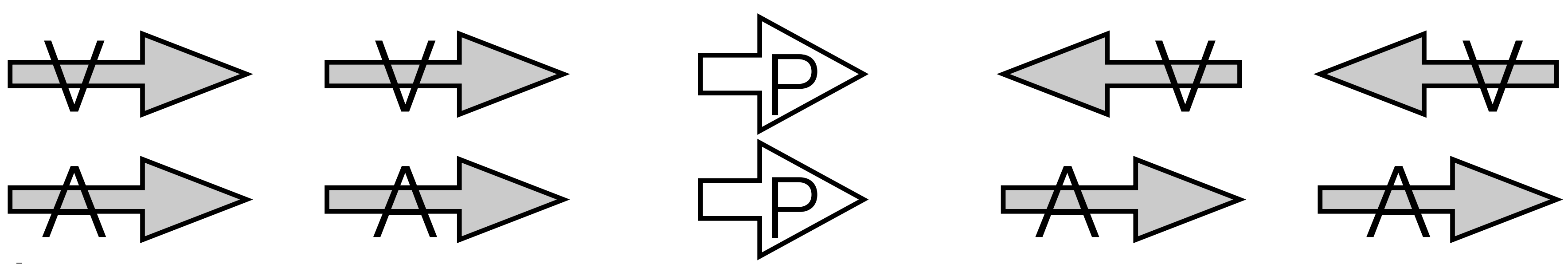
H. A. Tanaka

Overview

- Thus far we have talked about “continuous symmetries”
- In particular:
  - angular momentum conservation results from symmetry of rotations (isotropy)
  - isospin symmetry results from symmetry of rotations in isospin space
- Today we will talk about two “discrete” operations
  - parity (spacial inversion)
  - charge conjugation (particle/anti-particle exchange)

Parity:

- Parity: the operation by which we reverse the spatial coordinates: **x** → -**x**
- Consider a vector **V**: under parity, it “reverses” to -**V**
  - P(**V**) = -**V**
- There is another “vector” quantity called an “axial” or “pseudo” vector
  - consider **c** = **a** x **b**, where **a, b** are vectors
    - P(**c**) = P(**a** x **b**) = -**a** x -**b** = **a** x **b** = **c**
    - Axial/pseudo-vectors don’t change sign under parity!
- In respecting parity as a symmetry, we are okay if quantities are constructed entirely as vectors or as axial vectors:



  - Examples:

$$\vec{F} = m\vec{a} \qquad \vec{L} = \vec{r} \times \vec{p}$$

Parity Eigenvalues

- If a state is an eigenvector of the parity operator we have:

$$P|A\rangle = p|A\rangle$$

  - where p is the eigenvalue. If we apply p again, we have

$$P(P|A\rangle) = P(p|A\rangle) = pP|A\rangle = p^2|A\rangle$$

  - but applying P twice brings us back to the initial state

$$PP = P^2 = I \qquad p = \pm 1$$

Parity of a Bound State:

- Consider a bound state of quarks and anti-quarks (i.e. meson or baryon)
- Two components of parity:
  - "intrinsic parity" of the particles:
    - quarks have intrinsic parity 1, anti-quarks have intrinsic parity -1
  - "Orbital" due to the wavefunction of the quarks:
    - $(-1)^l$ where l is the orbital angular momental

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\,e^{-i\varphi}$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta$$

$$Y_1^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\,e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\,e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\,e^{-i\varphi}$$

$$Y_2^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\,e^{i\varphi}$$

$$Y_2^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\,(3\cos^2\theta - 1)$$

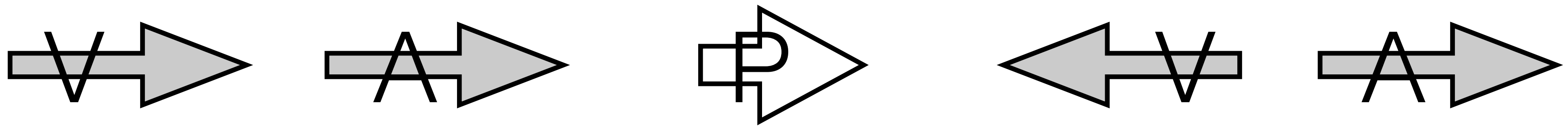
$$Y_2^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\,e^{2i\varphi}$$

  - The total parity is the product of intrinsic and orbital
  - Parity eigenvalue is the same before and after if interaction conserves parity

Examples

- The pion: quark, anti-quark in s = 0, l = 0 state:
  - $P = 1 \times -1 \times (-1)^0 = -1$
- ρ: quark, anti-quark in s = 1, l=0 state
  - $P= 1 \times -1 \times (-1)^0 = -1$
- Two pions in l=0 state:
  - $P= -1 \times -1 \times (-1)^0 = 1$
- Two pions in l=1 state
  - $P= -1 \times -1 \times (-1)^1 = -1$
- Decay of ρ: since s=1 (J=1), we must have l=1 in final state
  - two pions is OK (parity is the same) $P = -1 \times -1 \times (-1)^1 = -1$
  - three pions violates parity (opposite parities) $-1 \times -1 \times -1 \times (-1)^1 = 1$

Mixing Vectors and Pseudovectors



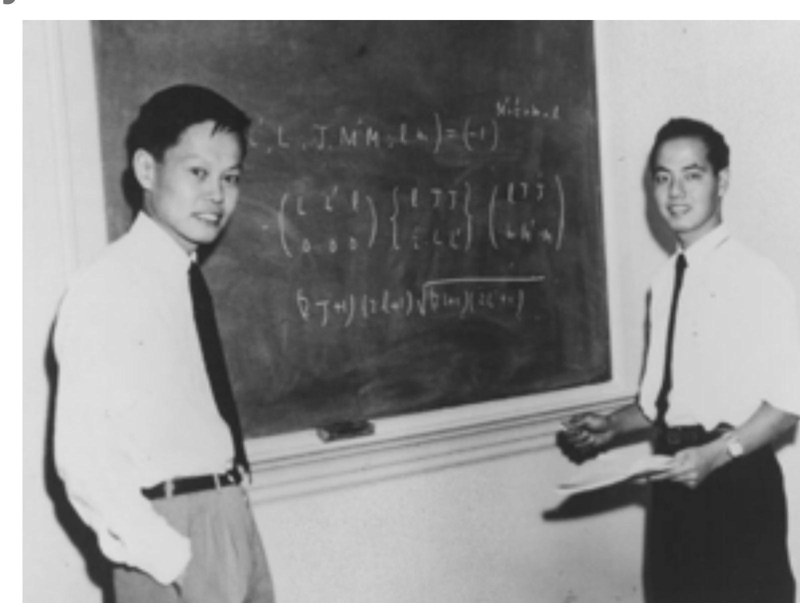
- two things which were point in the same direction are now pointing in opposite directions!
- One way to formalize this is to take the dot product of a vector, pseudovector
  - $P(\mathbf{V}\cdot\mathbf{A}) = -\mathbf{V}\cdot\mathbf{A} \neq \mathbf{V}\cdot\mathbf{A}$
- Another thing we can do is add vectors and pseudo-vectors.
  - Magnitude is conserved under parity so long as we add only vectors or only pseudovectors:
    - $P(|\mathbf{V_1}+\mathbf{V_2}|) = |-\mathbf{V_1}-\mathbf{V_2}| = |\mathbf{V_1}+\mathbf{V_2}|$
    - $P(|\mathbf{A_1}+\mathbf{A_2}|)= |\mathbf{A_1}+\mathbf{A_2}|$
  - But $P(|\mathbf{V}+\mathbf{A}|)= |-\mathbf{V}+\mathbf{A}| \neq |\mathbf{V}+\mathbf{A}|$
- An observable that correlates a vector and an pseudovector or an interaction that involves the sum of a vector and an pseudovector violates parity!

θ/τ Puzzle

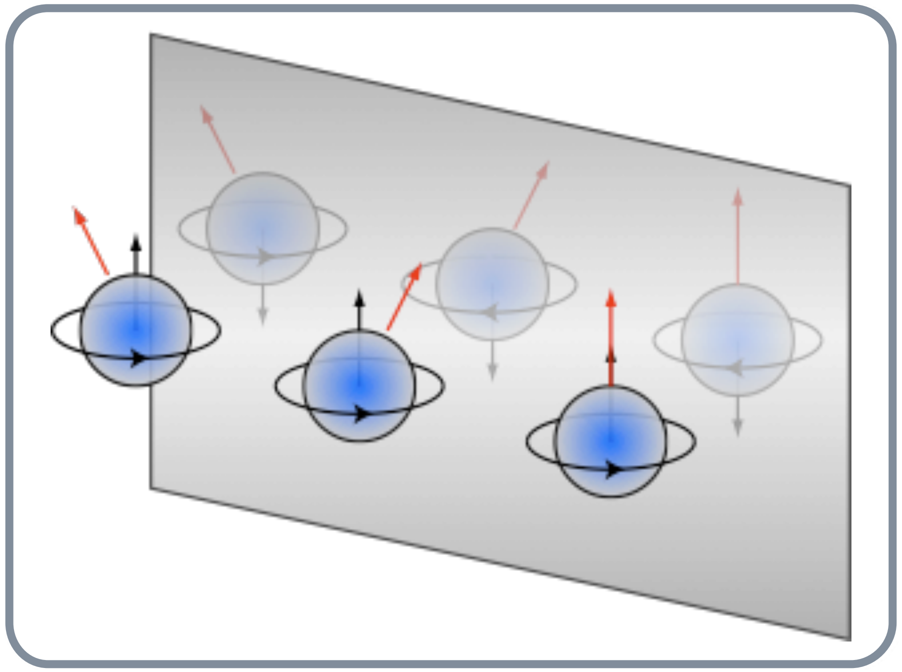
- There were two particles that were identical in every way (charge, mass, spin, etc.) but decayed to different parity eigenstates:

$$
\begin{array}{lcll}
\theta^+ & \rightarrow & \pi^+ + \pi^0 & (P = (-1)^2 = 1) \\
\tau^+ & \rightarrow & \pi^+ + \pi^0 + \pi^0 & (P = (-1)^3 = -1) \\
 & \rightarrow & \pi^+ + \pi^+ + \pi^- & (P = (-1)^3 = -1)
\end{array}
$$

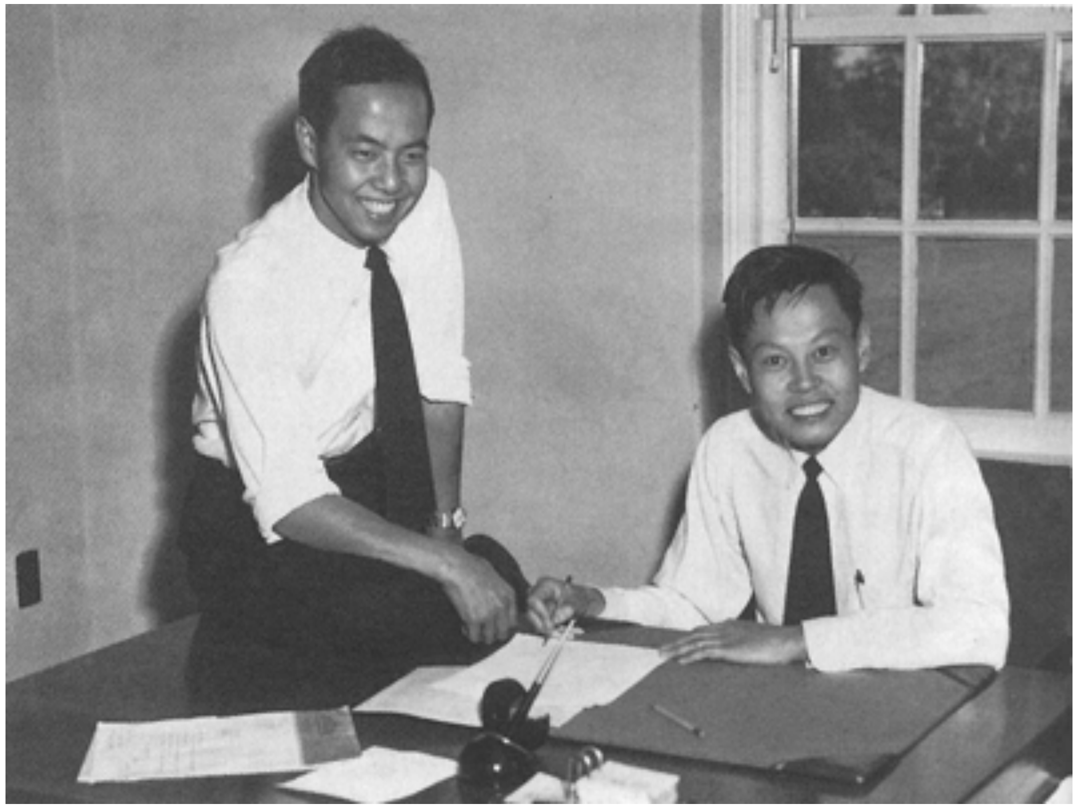
- Is it just a coincidence that two such particles exist?
- Modern understanding:
    - Parity violation is an inherent feature of weak decays
    - θ/τ are the same particle (K^+)



Demonstration of Parity Violation:



- β decay in polarized Cobalt 60
  - Cool Co^{60} atoms and polarize their angular momentum.
  - Angular momentum is a pseudo vector
  - Momentum of positrons emitted in β decay is a vector
- Correlation between the two demonstrates parity violation



The Neutrino:

- Ultimate manifestation of parity violation:
  - (assume for now that the neutrino is massless)
  - Neutrinos can be classified as “left-handed” and “right-handed” based on the orientation of their spin with their motion
    - Same as “left-handed” and “right-handed” polarization of light
- Left-handed neutrinos interact in the weak interaction, right-handed neutrinos do not.
  - Since neutrinos interact only via the weak interaction, the right-handed neutrino doesn’t interact at all! (except maybe via gravity)
  - actually:
    - we will see that a different quantity called “chirality” is actually what determines weak interaction coupling, but for ~massless particles, chirality ~ helicity).

Pion Decay and Parity Violation:

- Define the "helicity" of a neutrino: $\vec{s} \cdot \vec{v}$
  - The following assumes that the neutrino is massless:
    - helicity = -1 ("left-handed"): spin and velocity are antiparallel
    - helicity = +1 ("right-handed")

$$\nu_{\mu(L/R)} \longleftarrow \pi^+ \longrightarrow \mu^+_{(L/R)}$$

- Consider the decay of a π^+ at rest:
  - Since the π^+ is spinless, the μ^+ and ν must have the same helicity
  - Like ^{60}Co, the direction of the e^+ in μ^+ decay is correlated with the spin of the muon:

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

  - this itself is a manifestation of parity violation
- if neutrinos are emitted with both helicity, the μ^+ will not be polarized and there will be no correlation in the e^+ direction with the muon direction
- if neutrino are emitted with only one helicity, the μ^+ will be polarized

“Charge Conjugation”:

- The operation of “charge conjugation” flips all the “internal” quantum numbers of the state:
  - electric charge, color, lepton number
  - turns a particle into its corresponding anti-particle
  - “Kinematic” properties such as energy/momentum, angular momentum (including spin) are unchanged.
- Examples:
  - C(electron) → positron
  - C(proton) → antiproton
  - C(photon) → photon
  - $C(\pi^+) \rightarrow \pi^-$
  - $C(C(\pi^+)) \rightarrow C(\pi^-) = \pi^+ : C^2=1$

C Eigenvalue:

- Since $C^2 = 1$, the eigenvalues of the C operator must be ±1
  - The assignment of eigenvalues to particular states is convention-dependent.
  - It is customary to assign $C|\gamma\rangle = -|\gamma\rangle$
    - This is because charges create electromagnetic fields and these (along with the fields) switch sign with C
  - By considering $\pi^0 \rightarrow \gamma + \gamma$ we can assign $C|\pi^0\rangle = +|\pi^0\rangle$
  - But once we assign this, we can't have: $\pi^0 \rightarrow \gamma + \gamma + \gamma$

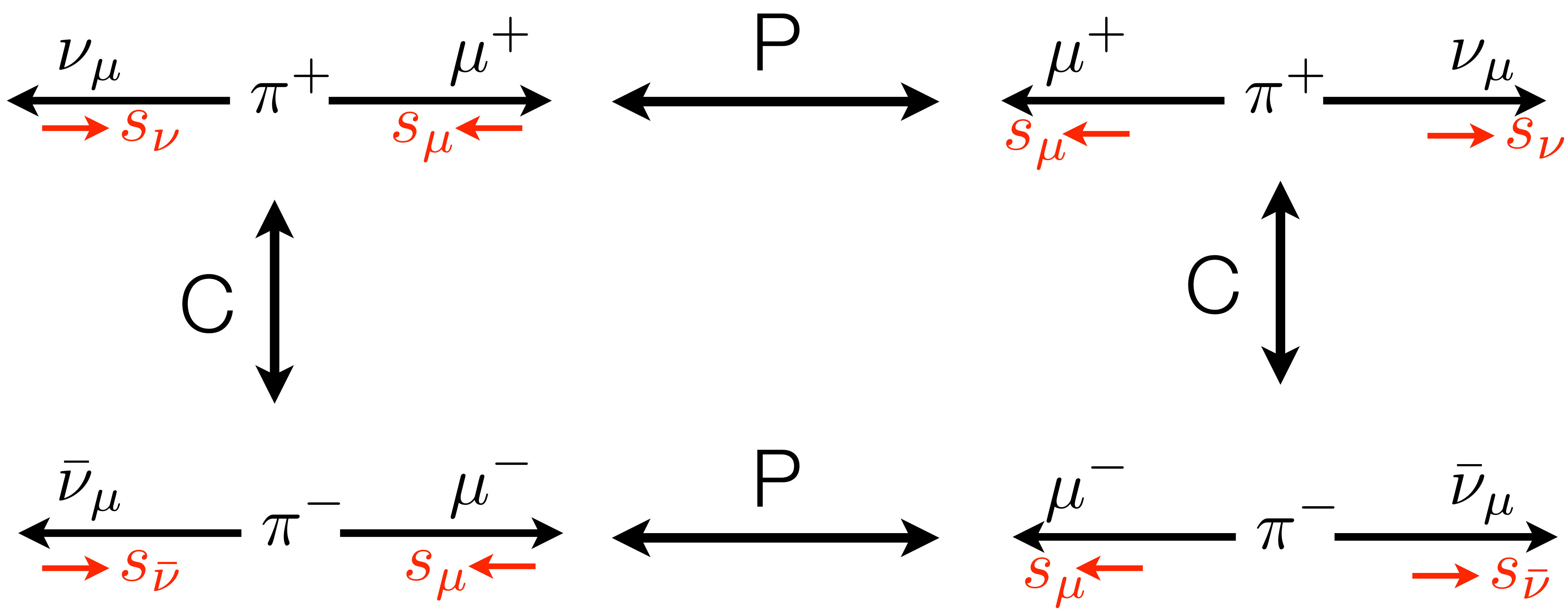
CP:

- C is manifestly broken in the weak interaction along with parity:

$$
\begin{tikzcd}
\nu_L \arrow[r, leftrightarrow, "P"] \arrow[d, leftrightarrow, "C"'] & \nu_R \arrow[d, leftrightarrow, "C"] \\
\bar{\nu}_L \arrow[r, leftrightarrow, "P"'] & \bar{\nu}_R
\end{tikzcd}
$$

- We wish to have a symmetry between processes and their anti-particle ("conjugate") counterparts
  - C does not allow this e.g. left-handed anti-neutrinos don't exist
  - Combined operation of C and P would restore the symmetry.
- Historically, people wanted to establish a symmetry between processes and their anti-matter counterparts. By flipping the parity along with charge conjugation, one could restore a symmetry (at least for these processes)

Pion Decay in terms of C, P and CP:



- Through the combined operation of C and P (aka CP), the π^+ decay process can be related to π^- decay

The Neutral Kaon System:

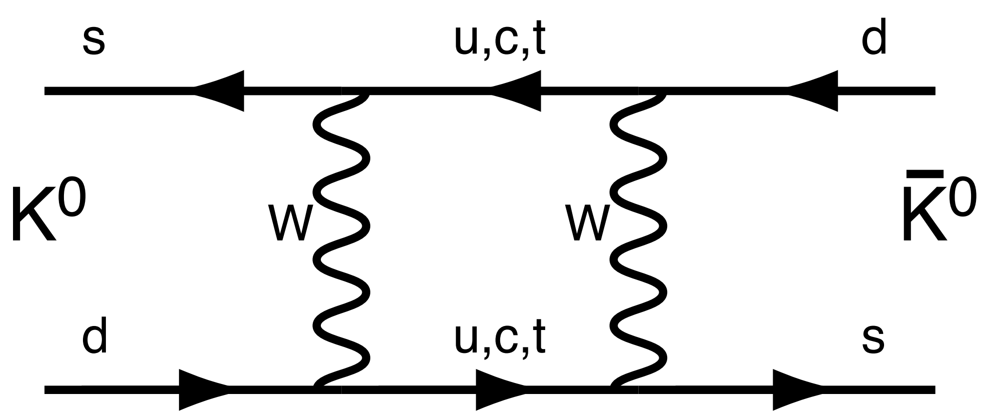
- There are two types of neutral kaons produced in strong interactions

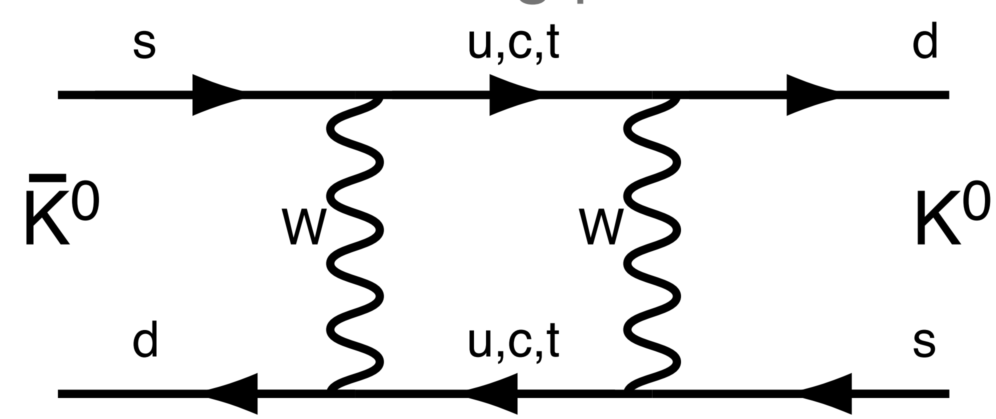
$$|K^0\rangle \rightarrow |\bar{s}d\rangle \qquad |\bar{K}^0\rangle \rightarrow |s\bar{d}\rangle$$

- As flavor eigenstates (definite strangeness), we can produce them as follows:

$$\begin{aligned}
\pi^- + p &\rightarrow \Lambda + K^0 \\
\pi^- + p &\rightarrow n + n + \bar{\Lambda} + \bar{K}^0 \\
\pi^+ + p &\rightarrow p + K^+ + \bar{K}^0
\end{aligned}$$

- After production, the two states can “mix” via the following processes





Consequences of Mixing:

- After some time, kaons are no longer in a state of definite flavor
  - (i.e. they are no longer purely K^0 or $\bar{K^0}$)
  - Being the lightest strange particle, they can decay only weakly
  - Assuming that CP is conserved in the weak interaction, construct CP eigenstates (i.e. states with CP eigenvalues)
- Consider the transformation properties of the states

$$C|K^0\rangle = |\bar{K^0}\rangle \qquad P|K^0\rangle = -|K^0\rangle$$

$$C|\bar{K^0}\rangle = |K^0\rangle \qquad P|\bar{K^0}\rangle = -|\bar{K^0}\rangle$$

- We can then construct the following CP eigenstates:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\bar{K^0}\rangle\right] \qquad CP|K_1\rangle = \frac{1}{\sqrt{2}}\left[-|\bar{K^0}\rangle + |K^0\rangle\right] = |K_1\rangle$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\bar{K^0}\rangle\right] \qquad CP|K_2\rangle = \frac{1}{\sqrt{2}}\left[-|\bar{K^0}\rangle - |K^0\rangle\right] = -|K_2\rangle$$

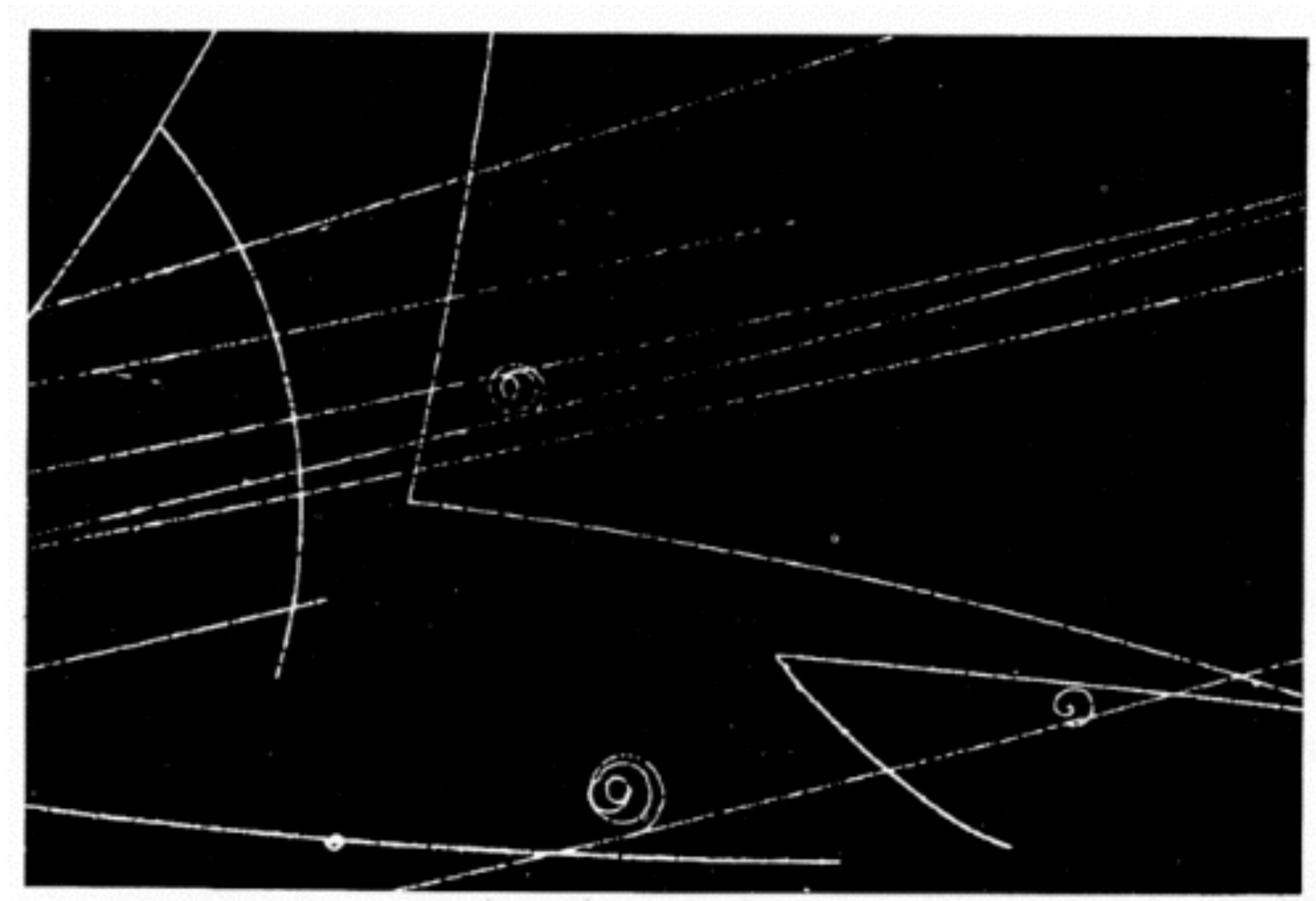
Decay Modes:

- Consider the final state of two pions:

$$
\begin{array}{llll}
K \to \pi\pi & C: 1 \times 1 & P: (-1) \times (-1) & CP: 1 \times 1 = 1 \\
K \to \pi\pi\pi & C: 1 \times 1 & P: (-1) \times (-1) \times (-1) & CP: 1 \times (-1) = -1
\end{array}
$$

- CP conservation:
    - K_1 can decay to $\pi\pi$, but not $\pi\pi\pi$,
    - K_2 can decay to $\pi\pi\pi$, but not $\pi\pi$
- Due to the reduced phase space,

  K_2 has a much longer lifetime than K_1



- Experimentally: $\tau_1 = 5.11 \times 10^{-8}$ s

  $\tau_2 = 8.95 \times 10^{-11}$ s
- If we wait long enough, K_1 will decay away and leave only K_2

Time Evolution and Matter Effects

- After some time, we are left with only K_2

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\bar{K^0}\rangle\right]$$

  - despite starting with strange state (K^0) we now have an equal mixture of strange and anti-strange.
  - These behave differently in strong interactions (just like the strange interactions which produced the kaons in the first place).
  - For instance, the following preserves strangeness

$$\bar{K^0} + p \to \Lambda + \pi^+$$

    - there is no corresponding process for K^0, one must produce additional strange particles that increases the threshold; below this, the K^0 passes through without attenuating.
- By passing a beam of K_2 through material, one can eliminate the $\overline{K}^0$ part of the wavefunction leaving only K^0.

Regeneration

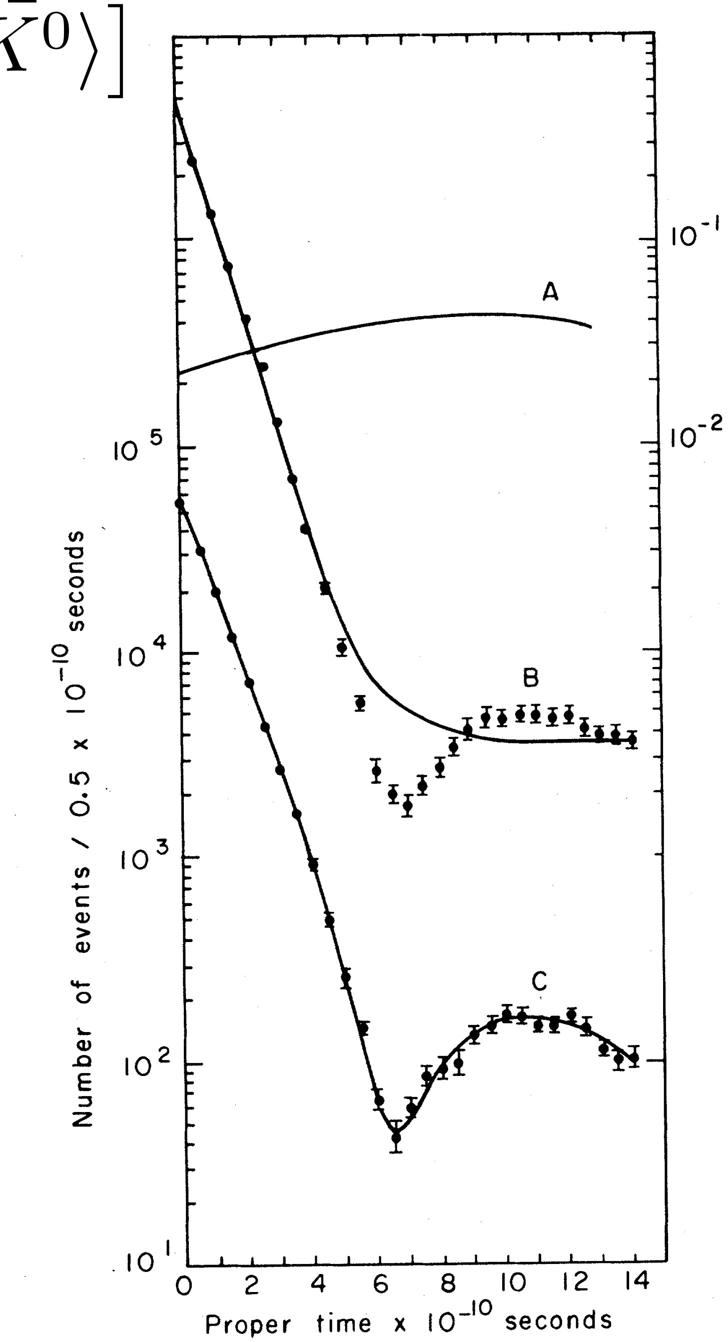
- Recall that:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\bar{K}^0\rangle\right] \qquad |K_2\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\bar{K}^0\rangle\right]$$

  - so that

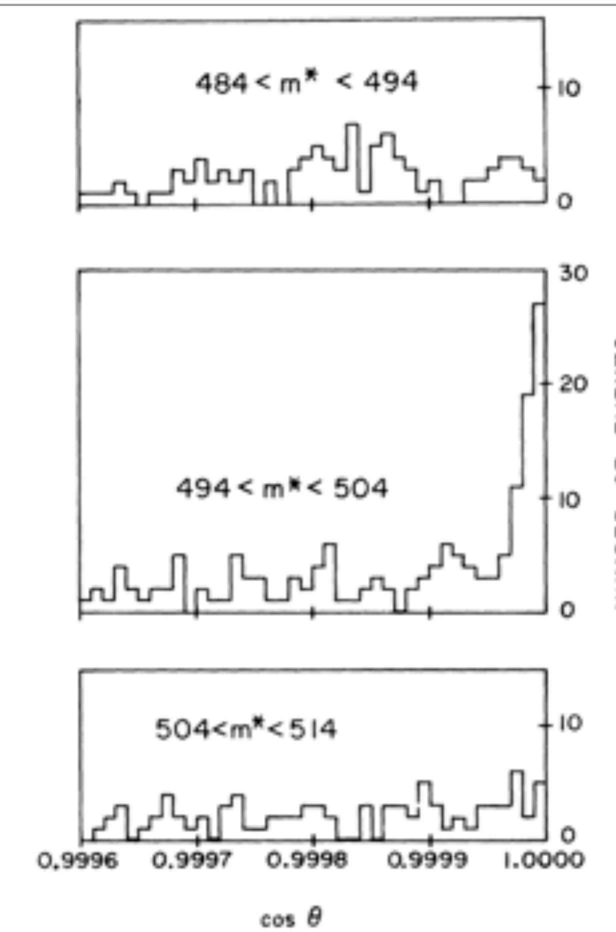
$$|K^0\rangle = \frac{1}{\sqrt{2}}\left[|K_1\rangle + |K_2\rangle\right]$$

- the short-lived K_1 has reappeared!
  - By waiting, we can make K_1 go away and leave K_2
  - By passing K_2 through material, we can absorb the $\bar{K}^0$ component and make K_1 come back!

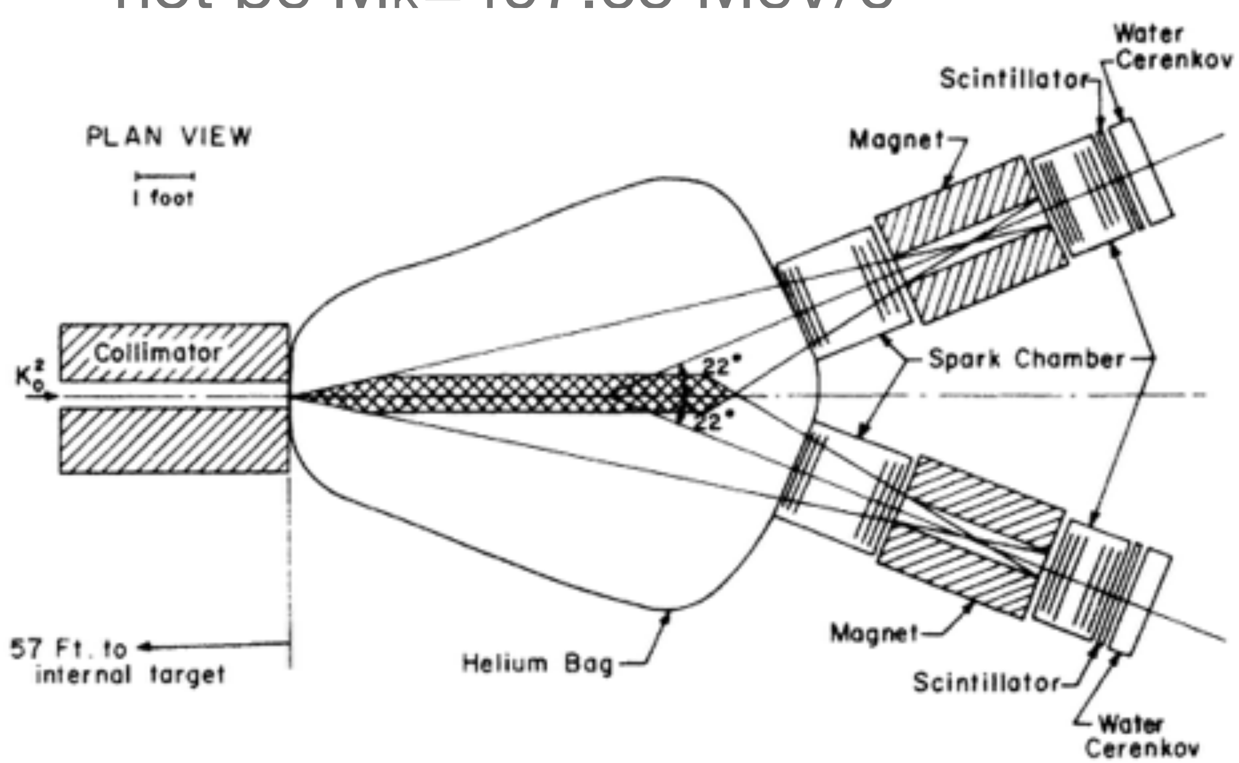


CP Violation:

- Recall K_2 cannot decay to $\pi^+\pi^-$ if CP is conserved
- We can create a pure beam of K_2 by "waiting" long enough. If we see $K_2 \rightarrow \pi^+\pi^-$ decay, we have evidence for CP violation.
- Cronin + Fitch: produce beam of K_2 and look for $\pi\pi$ decay by reconstructing $\pi^+\pi^-$. If there is a third pion, momentum will unbalanced and mass will not be M_K=497.65 MeV/c^2



θ = angle of $p_{\pi\pi}$ relative to direction of K_2 beam



45±10 events consistent with $\pi\pi$ decay.

Estimate 2×10^{-3} of all decays are CP violating

Note presence of helium bag.

Question at conference:

Could there be a (dead) fly in your He bag?

Summary

- We introduced discrete symmetries that involve “flipping” quantities:
  - Parity: “flip” the spatial coordinates: $\mathbf{x} \rightarrow -\mathbf{x}$
  - Charge conjugation: “flip” the internal quantum numbers (charge, baryon number, etc.)
- We found two classes of objects with well-defined parity:
  - vectors: switch sign (like momentum, spatial coordinates, etc.)
  - axial/pseudovectors: do not switch sign (angular momentum, etc.)
  - correlations or anything that correlates vector and pseudo vector quantities violates parity symmetry
  - this was observed in