The Z boson

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Final examination

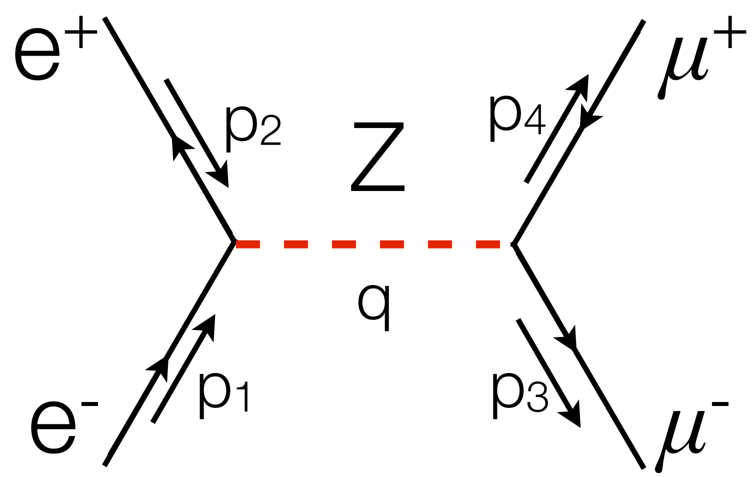
- Feynman diagram calculation:
  - electromagnetic or weak interaction process (scattering/decay)
  - use Feynman rules and get to a cross section or decay rate
  - there are many steps, so I will try to provide the result after each step
    - if you get stuck on any step, take the result and go to the next step
- Structure of the weak interaction
  - parity violation and its consequences (e.g. what decays are suppressed)
  - CKM matrix elements (how to apply them)
  - electroweak unification:
    - what is it?
    - what is θ_W? where does it appear?
- You can use your textbook and a non-programmable calculator.

Averaging with projection operators

- suppose we use P_L to project out the left chiral component of an incoming particle. The other incoming particle is unpolarized.
  - In summing the spins, we get: $|M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{RR}|^2$
  - The projection operator will kill $|M_{RL}|^2$ and $|M_{RR}|^2$, leaving $|M_{LL}|^2 + |M_{LR}|^2$
  - The average amplitude is 1/2 x $(|M_{LL}|^2 + |M_{LR}|^2)$
    - so we divide by 2
- If we use P_L to project out the left chiral compotes of both particles:
  - In summing the spins, we get: $|M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{RR}|^2$
  - The projection operator will kill $|M_{RL}|^2$ and $|M_{RR}|^2$, $|M_{LR}|^2$
    - only leaving $|M_{LL}|^2$
  - The “average” amplitude is $|M_{LL}|^2$
    - so we leave it alone.

$e^+ e^- \rightarrow \mu^+ + \mu^-$

- Let's consider the pair production of muons via the Z:



Note: different convention from text book (p_3 assigned here to μ^-, not μ^+)

Assume e, μ masses can be ignored

- Feynman rules:
  - incoming leg: $\bar{v}(2)\dfrac{-ig_Z}{2}\gamma^\mu(c_V^e - c_A^e\gamma^5)u(1) \quad (2\pi)^4\ \delta^4(p_1+p_2-q)$
  - outgoing leg: $\bar{u}(3)\dfrac{-ig_Z}{2}\gamma^\nu(c_V^\mu - c_A^\mu\gamma^5)v(4) \quad (2\pi)^4\ \delta^4(q-p_3+p_4)$
  - Propagator: $\displaystyle\int \frac{d^4q}{(2\pi)^4}\ \frac{-i(g_{\mu\nu} - q_\mu q_\nu/m_Z^2c^2)}{q^2 - m_Z^2c^2}$

$$\mathcal{M} = -\frac{g_Z^2}{4(q^2 - m_Z^2c^2)}\left[\bar{v}(2)\gamma^\mu(c_V^e - c_A^e\gamma^5)u(1)\right]\ \left[\bar{u}(3)\gamma^\nu(c_V^\mu - c_A^\mu\gamma^5)v(4)\right]$$
$$\left[g_{\mu\nu} - q_\mu q_\nu/(M_z^2c^2)\right]$$

Spin Summation

$$\mathcal{M} = -\frac{g_Z^2}{4(q^2 - m_Z^2 c^2)} \left[\bar{v}(2)\gamma^\mu (c_V^e - c_A^e \gamma^5) u(1)\right] \left[\bar{u}(3)\gamma_\mu (c_V^\mu - c_A^\mu \gamma^5) v(4)\right]$$

$$\mathcal{M}\mathcal{M}^* = \left[\frac{g_Z^2}{4(q^2 - m_Z^2 c^2)}\right]^2 \underline{\left[\bar{v}(2)\gamma^\mu (c_V^e - c_A^e \gamma^5) u(1)\right] \left[\bar{v}(2)\gamma^\nu (c_V^e - c_A^e \gamma^5) u(1)\right]^*}$$

$$\underline{\left[\bar{u}(3)\gamma_\mu (c_V^\mu - c_A^\mu \gamma^5) v(4)\right] \left[\bar{u}(3)\gamma_\nu (c_V^\mu - c_A^\mu \gamma^5) v(4)\right]^*}$$

$$\sum_{\text{a, b spins}} \left[\bar{u}(a)\Gamma_1 u(b)\right] \left[\bar{u}(a)\bar{\Gamma}_2 u(b)\right]^* = \mathrm{Tr}\left[\Gamma_1 (\not{p}_b + m_b c)\bar{\Gamma}_2 (\not{p}_a + m_a c)\right]$$

$$\langle \mathcal{M}\mathcal{M}^* \rangle = \frac{1}{4}\left[\frac{g_Z^2}{4(q^2 - m_Z^2 c^2)}\right]^2$$

$$\underline{\mathrm{Tr}\left[\gamma^\mu (c_V^e - c_A^e \gamma^5)\ \not{p}_1 \gamma^\nu (c_V^e - c_A^e \gamma^5)\ \not{p}_2\right]}$$

$$\underline{\mathrm{Tr}\left[\gamma_\mu (c_V^\mu - c_A^\mu \gamma^5) \not{p}_4 \gamma_\nu (c_V^\mu - c_A^\mu \gamma^5) \not{p}_3\right]}$$

Breaking up the Terms:

- The first trace:

$$\mathrm{Tr}\left[\gamma^\mu(c_V^e - c_A^e\gamma^5)\ \not{p}_1\gamma^\nu(c_V^e - c_A^e\gamma^5)\ \not{p}_2\right]$$

$$\mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_2] \Rightarrow 4\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(p_1 \cdot p_2)\right]$$

$$\mathrm{Tr}[\gamma^5 \not{p}_1 \gamma^\nu \not{p}_2 \gamma^\mu] \Rightarrow 4i\epsilon^{\alpha\nu\beta\mu} p_{1\mu} p_{2\nu}$$

$$\boxed{4(c_V^{e2} + c_A^{e2})\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(p_1 \cdot p_2)\right]} - \boxed{8ic_V^e c_A^e \times \epsilon^{\alpha\nu\beta\mu} p_{1\mu} p_{2\nu}}$$

- Second trace:

$$\mathrm{Tr}\left[\gamma_\mu(c_V^\mu - c_A^\mu\gamma^5)\ \not{p}_4\gamma_\nu(c_V^e - c_A^e\gamma^5)\ \not{p}_3\right]$$

$$\boxed{4(c_V^{\mu 2} + c_A^{\mu 2})\left[p_{3\mu} p_{4\nu} + p_{3\nu} p_{4\mu} - g_{\mu\nu}(p_3 \cdot p_4)\right]} - \boxed{8ic_V^\mu c_A^\mu \times \epsilon_{\rho\nu\sigma\mu} p_3^\rho p_4^\sigma}$$

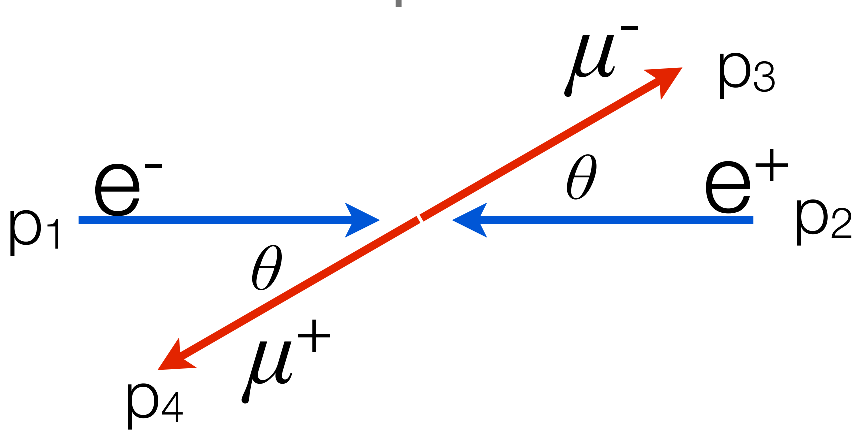
Our final expression for the amplitude:

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{2}\left[\frac{g_Z^2}{(q^2 - m_Z^2 c^2)}\right]^2$$

$$\{(c_V^{e2} + c_A^{e2})(c_V^{\mu 2} + c_A^{\mu 2}) \times [(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4)]$$

$$+4c_V^e c_A^e c_V^\mu c_A^\mu \left[(p_1 \cdot p_4)(p_2 \cdot p_3) - (p_1 \cdot p_3)(p_2 \cdot p_4)\right]\}$$

- Let’s put in the kinematics: $q^2 = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 \Rightarrow 4E^2$



$$p_1 \cdot p_3 = E^2/c^2 - \mathbf{p}^2 \cos\theta \Rightarrow E^2(1 - \cos\theta)$$
$$p_1 \cdot p_4 = E^2/c^2 + \mathbf{p}^2 \cos\theta \Rightarrow E^2(1 + \cos\theta)$$
$$p_2 \cdot p_3 = E^2/c^2 + \mathbf{p}^2 \cos\theta \Rightarrow E^2(1 + \cos\theta)$$
$$p_2 \cdot p_4 = E^2/c^2 - \mathbf{p}^2 \cos\theta \Rightarrow E^2(1 - \cos\theta)$$

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{2}\left[\frac{g_Z^2 E^2}{(4E^2 - m_Z^2 c^2)}\right]^2$$

$$\{\left[(c_V^e)^2 + (c_A^e)^2\right]\left[(c_V^\mu)^2 + (c_A^\mu)^2\right] \times \left[2(1 + \cos^2\theta)\right] + 16\, c_V^e c_A^e c_V^\mu c_A^\mu \;\cos\theta\}$$

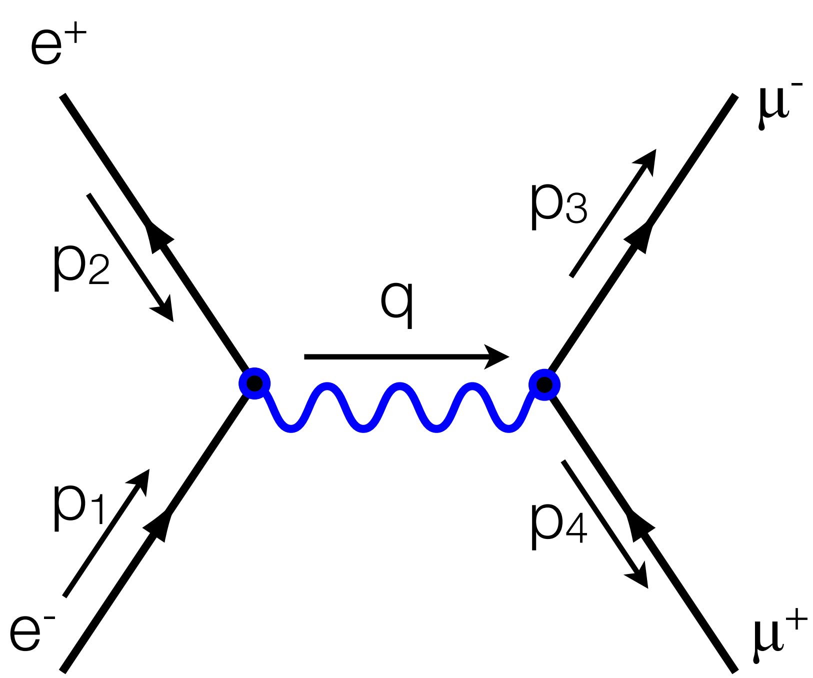
The End:

- The differential cross section in the CM frame

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2}\frac{|p_f|}{|p_i|}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{16\pi}\right)^2 \left[\frac{g_Z^2 E}{(4E^2 - m_Z^2 c^2)}\right]^2$$

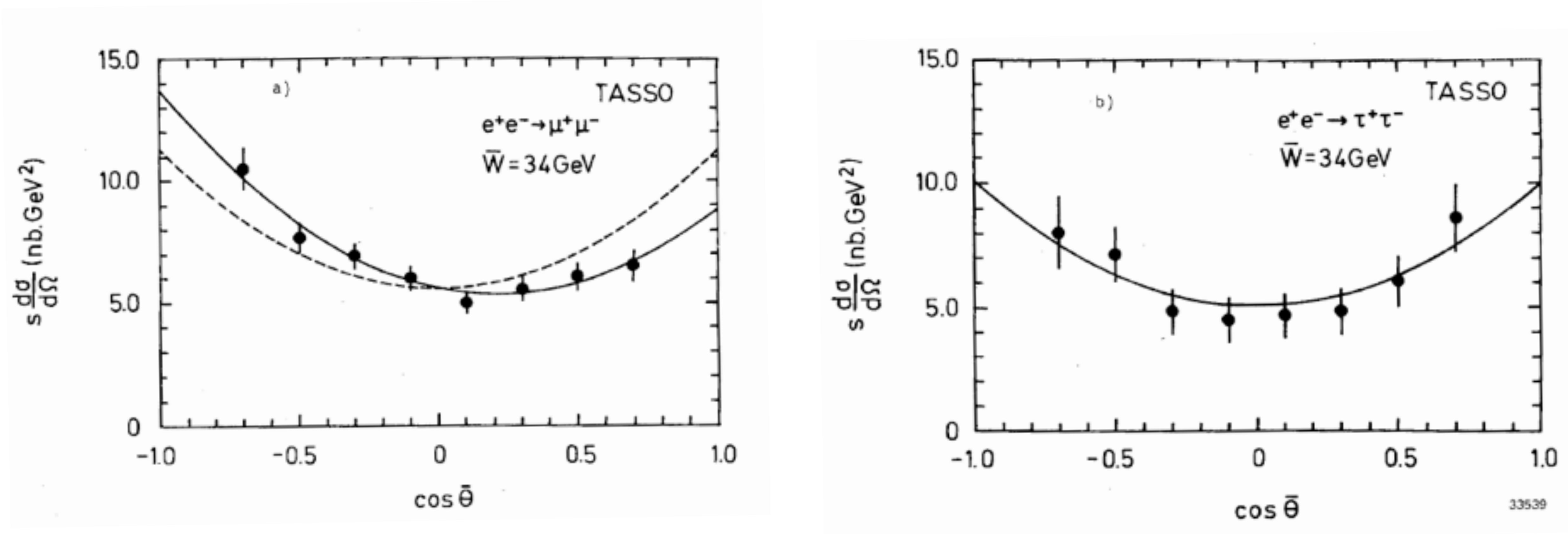
$$\left\{\left[(c_V^e)^2 + (c_A^e)^2\right]\left[(c_V^\mu)^2 + (c_A^\mu)^2\right] \times (1+\cos^2\theta) + 8\, c_V^e c_A^e c_V^\mu c_A^\mu \; \cos\theta\right\}$$



Compare to electrodynamics

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g_e^4}{4E^2}\left[1+\cos^2\theta\right]$$

Comparison to measurements



How did the asymmetry come about?

- We can break up the Z interaction into two parts:

$$c_V\gamma^\mu + c_A\gamma^\mu\gamma^5 \Rightarrow \tfrac{1}{2}(c_V + c_A)\gamma^\mu(1+\gamma^5) + \tfrac{1}{2}(c_V - c_A)\gamma^\mu(1-\gamma^5)$$

- One couples to left chiral and the other to right chiral particles
  - But the coupling is not the same!
- Within the incident e^+e^- beams, we have assumed there is no polarization
  - equal components of each chirality
- Recall from last time that:

$$\gamma^\mu\left(\frac{1-\gamma^5}{2}\right) = \left(\frac{1+\gamma^5}{2}\right)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)$$

- First term above couples right chiral electrons with left chiral positrons

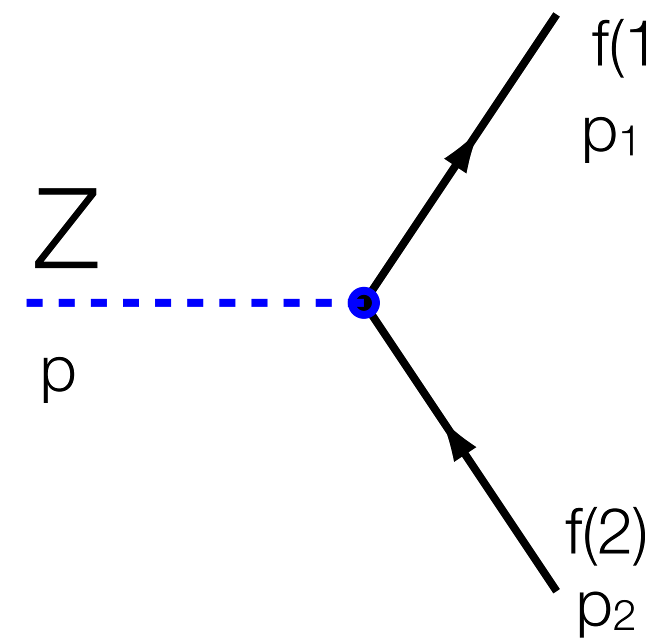
$$u_L \equiv \tfrac{1}{2}(1-\gamma^5)u \qquad \bar{u}_R \equiv \bar{u}\tfrac{1}{2}(1+\gamma^5)$$

- vice versa for the second term

Z has net polarization

Z decays: (9.23)

- Consider the decay of the Z to two fermions (quark/lepton)
  - let's ignore the fermion masses (i.e. set to 0)
- Treat all possible fermion pairs by not specifying c_V, c_A
  - putting in the appropriate values at the end, case-by-case



Vertex $\bar{u}_1 \; \frac{-ig_Z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5) \; v_2 \; \times \; (2\pi)^4 \; \delta^4(p - p_1 - p_2)$

Incoming Z ϵ_μ

$$\mathcal{M} = \frac{g_Z}{2}\epsilon_\mu \times \bar{u}_1\gamma^\mu(c_V^f - c_A^f\gamma^5)v_2$$

$$|\mathcal{M}|^2 = \frac{g_Z^2}{4}\epsilon_\mu\epsilon_\nu^* \times [\bar{u}_1\gamma^\mu(c_V^f - c_A^f\gamma^5)v_2][\bar{u}_1\gamma^\nu(c_V^f - c_A^f\gamma^5)v_2]^*$$

Spin/polarization summation

$$|\mathcal{M}|^2 = \frac{g_Z^2}{4}\epsilon_\mu \epsilon_\nu^* \times [\bar{u}_1 \gamma^\mu (c_V^f - c_A^f \gamma^5) v_2][\bar{u}_1 \gamma^\nu (c_V^f - c_A^f \gamma^5) v_2]^*$$

- Hopefully the fermion part is familiar to you now

$$\sum_{s_1, s_2} [\bar{u}_1 \gamma^\mu (c_V^f - c_A^f \gamma^5) v_2][\bar{u}_1 \gamma^\mu (c_V^f - c_A^f \gamma^5) v_2]^*$$

$$\Rightarrow \mathrm{Tr}[\not{p}_2 \gamma^\mu (c_V^f - c_A^f \gamma^5) \not{p}_1 \gamma^\nu (c_V^f - c_A^f \gamma^5)]$$

- what about the Z?

$$\sum_{s_Z} \epsilon_\mu \epsilon_\nu^* \quad \Rightarrow -g_{\mu\nu} + \frac{p_\mu p_\nu}{M_Z^2 c^2}$$

from Problem 9.1 in PS 4

Trace algebra

$$\mathrm{Tr}[\not{p}_2\gamma^\mu(c_V^f - c_A^f\gamma^5)\not{p}_1\gamma^\nu(c_V^f - c_A^f\gamma^5)]$$

- As we discussed just before, break this up into parts we know how to manage

$$\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu] = 4 \times [p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - g^{\mu\nu}(p_1 \cdot p_2)]$$

$$\Rightarrow 4(c_V^{f\,2} + c_A^{f\,2}) \times [p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - g^{\mu\nu}(p_1 \cdot p_2)]$$

$$\mathrm{Tr}[\gamma^5\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu] = 4i\epsilon^{\alpha\nu\beta\mu}p_{1\alpha}p_{2\beta}$$

$$\Rightarrow -8i\; c_V^f c_A^f \times \epsilon^{\alpha\nu\beta\mu}p_{1\alpha}p_{2\beta}$$

$$\Rightarrow -8i\; c_V^f c_A^f \times \epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}$$

$$\Rightarrow 4(c_V^2 + c_A^2) \times [p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - g^{\mu\nu}(p_1 \cdot p_2)] - 8i\; c_V^f c_A^f \times \epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}$$

Put it together:

$$|\mathcal{M}|^2 = g_Z^2 \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{M_Z^2 c^2}\right]$$

$$\times[(c_V^2 + c_A^2) \times [p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - g^{\mu\nu}(p_1 \cdot p_2)]$$

$$-2i\, c_V^f c_A^f \times \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}]$$

$$\begin{aligned}|\mathcal{M}|^2 = g_Z^2(c_V^2 + c_A^2)\times \quad & [-(p_1 \cdot p_2) - (p_1 \cdot p_2) + 4(p_1 \cdot p_2) \\ & + \frac{1}{m_Z^2 c^2}\left((p \cdot p_2)(p \cdot p_1) + (p \cdot p_2)(p \cdot p_1) - p^2(p_1 \cdot p_2)\right)]\end{aligned}$$

$$|\mathcal{M}|^2 = g_Z^2(c_V^2 + c_A^2) \times \left[2(p_1 \cdot p_2) + \frac{1}{m_Z^2 c^2}\left((2(p \cdot p_2)(p \cdot p_1) - p^2(p_1 \cdot p_2)\right)\right]$$

$$p^2 = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2(p_1 \cdot p_2) = 2(p_1 \cdot p_2) = m_Z^2 c^2$$

$$p \cdot p_{1,2} = (p_1 + p_2) \cdot p_{1,2} = p_{1,2}^2 + (p_1 \cdot p_2)$$

$$|\mathcal{M}|^2 = g_Z^2(c_V^2 + c_A^2) m_Z^2 c^2$$

Towards a decay rate:

- Averaging over initial spins:

$$|\mathcal{M}|^2 = g_Z^2(c_V^2 + c_A^2)m_Z^2c^2$$

$$\langle|\mathcal{M}|^2\rangle \;\Rightarrow \frac{1}{3}g_Z^2(c_V^2 + c_A^2)m_Z^2c^2$$

$$\Gamma = \frac{S|\mathbf{p}|}{8\pi\hbar m_Z^2 c}|\mathcal{M}|^2$$

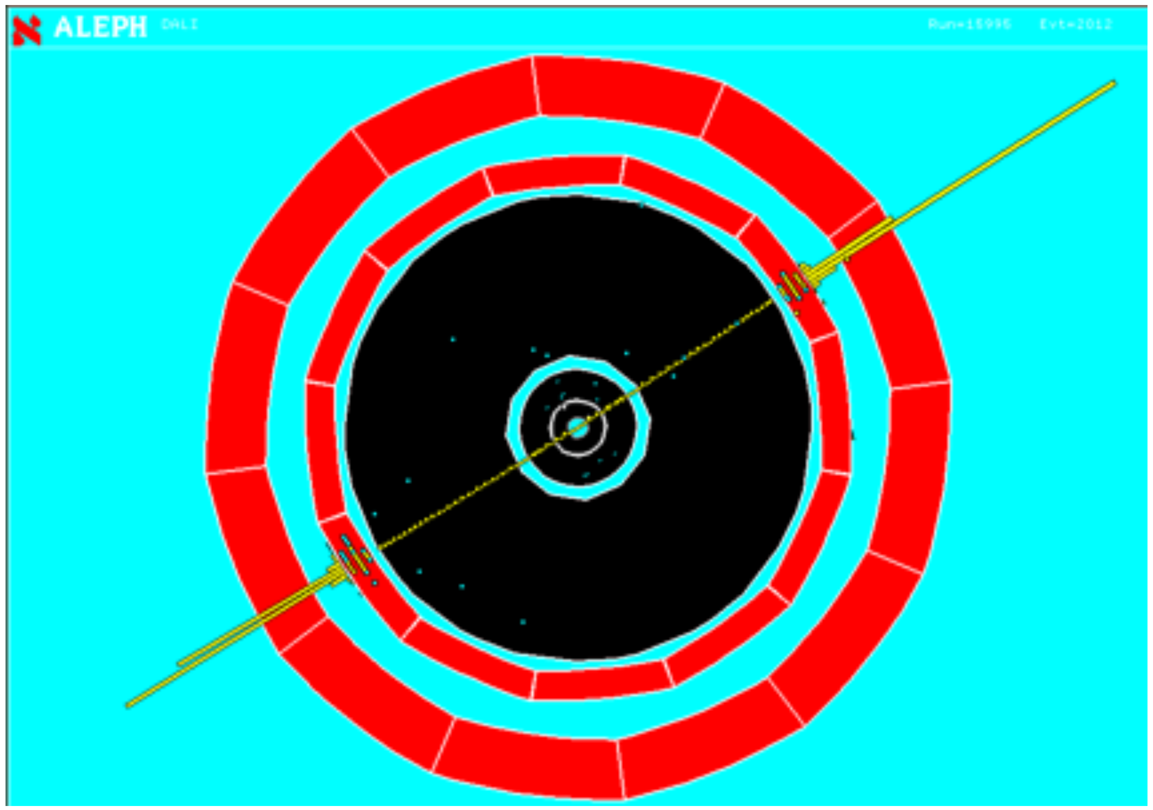
$$|\mathbf{p}| = m_Z c/2$$

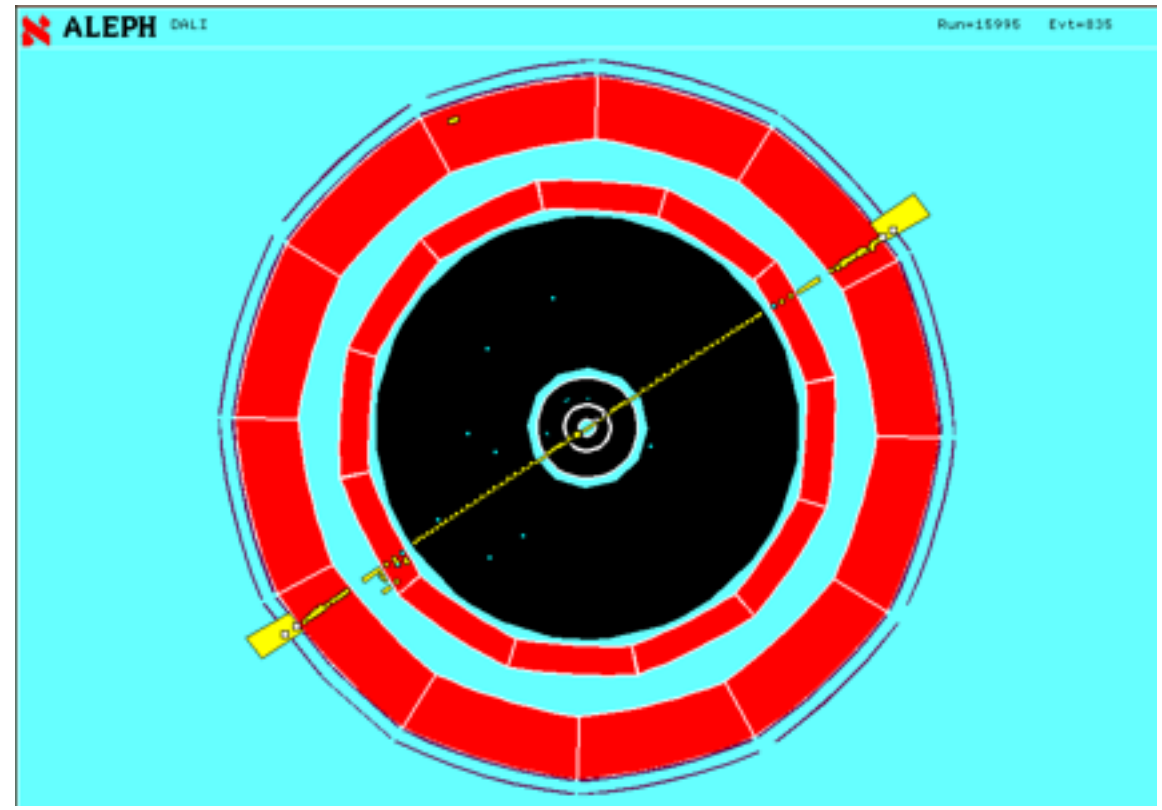
$$S = 1$$

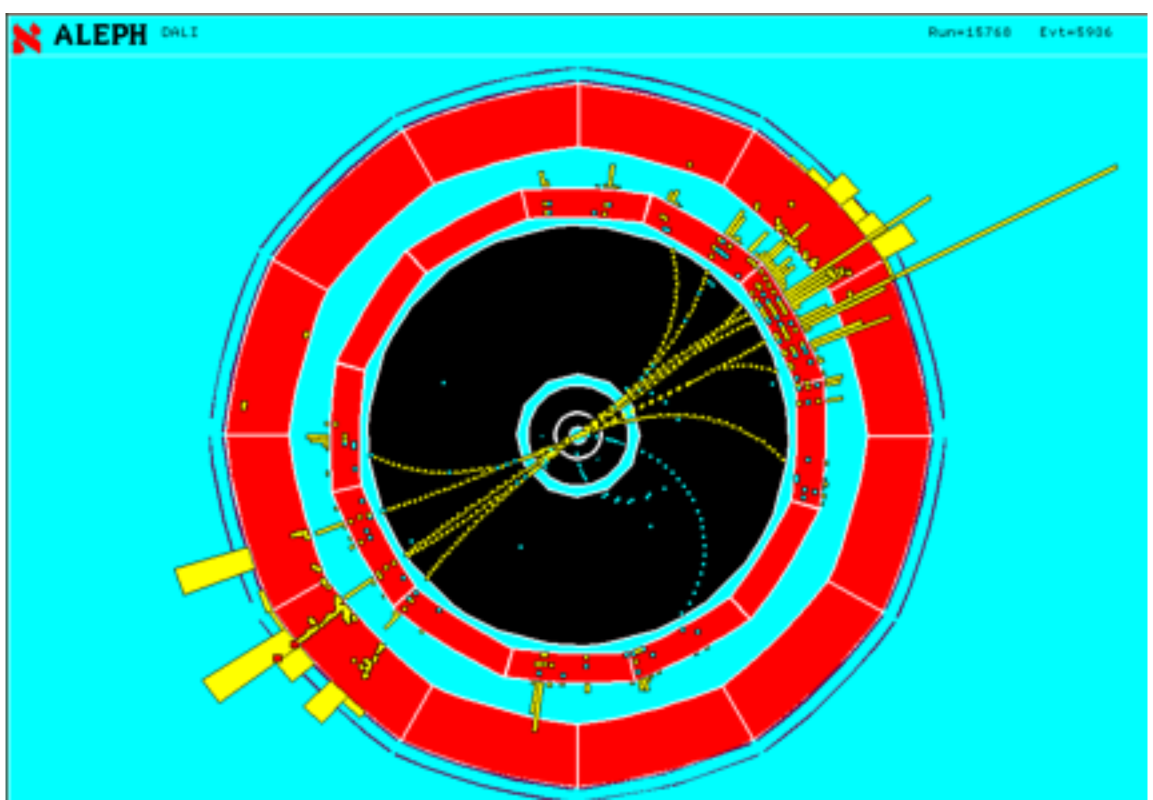
$$\Rightarrow \frac{1}{3}g_Z^2(c_V^2 + c_A^2)m_Z^2c^2\frac{m_Z c}{2\times 8\pi m_Z^2 c}$$

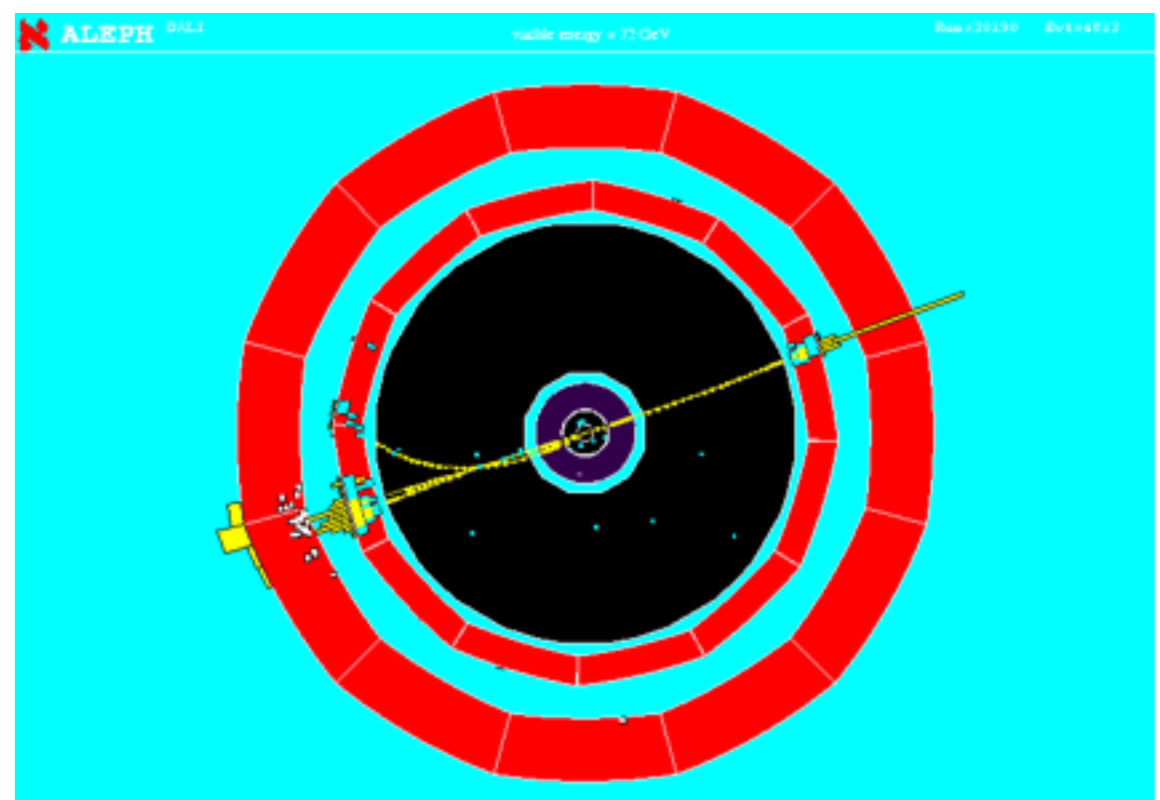
$$\Rightarrow \frac{1}{48\pi\hbar}g_Z^2(c_V^2 + c_A^2)m_Z c^2$$

Z decays at LEP









Prediction vs. Measurement

	c_V	c_A	$c_V^2 + c_A^2$	Fraction
$\nu_e\ \nu_\mu\ \nu_\tau$	1/2	1/2	0.5 (x3)	6.8%
$e\ \mu\ \tau$	-1/2+2 $\sin^2\theta_W$	-1/2	0.251 (x3)	3.4%
$u\ c\ t$	+1/2-4/3 $\sin^2\theta_W$	1/2	0.287 (x2x3)	11.6%
$d\ s\ b$	-1/2+2/3 $\sin^2\theta_W$	-1/2	0.379 (x3x3)	15.4%
		Total	7.386	

Z DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
$e^+ e^-$		(3.363 $\pm$0.004) %		45594
$\mu^+ \mu^-$		(3.366 $\pm$0.007) %		45594
$\tau^+ \tau^-$		(3.370 $\pm$0.008) %		45559
$\ell^+ \ell^-$	[b]	(3.3658$\pm$0.0023) %		–
$\ell^+ \ell^- \ell^+ \ell^-$	[h]	(3.30 $\pm$0.31) $\times 10^{-6}$	S=1.1	45594
invisible		(20.00 $\pm$0.06) %		–
hadrons		(69.91 $\pm$0.06) %		–
$(u\overline{u}+c\overline{c})/2$		(11.6 $\pm$0.6) %		–
$(d\overline{d}+s\overline{s}+b\overline{b})/3$		(15.6 $\pm$0.4) %		–
$c\overline{c}$		(12.03 $\pm$0.21) %		–
$b\overline{b}$		(15.12 $\pm$0.05) %		–

$\sin^2\theta_W = 0.23126$

don't forget the factor of 3 for color in quarks!

That's it!

- Please remember to respond to doodle poll for review session
    - I would like to set a time/place by the end of the week.
- Please send me any suggestions/topics/questions/issues for the review
- I will announce how you can collect PS 4