Electroweak Unification

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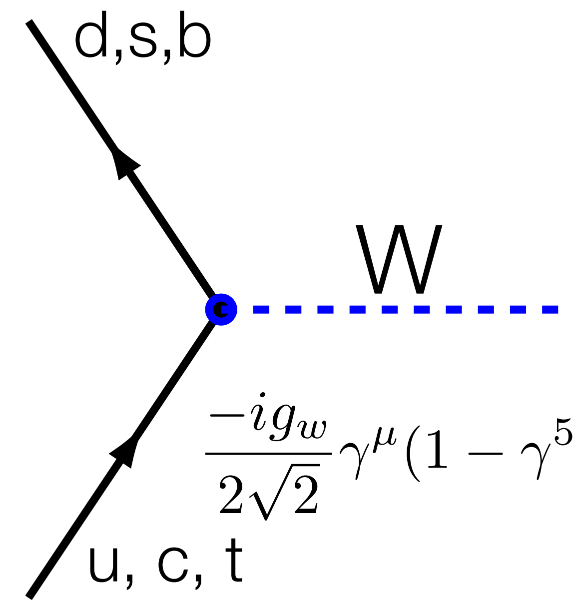
Outlook

- PS 4 due next Tuesday
- No class on 8 December
- Will have extended office hours thereafter (will keep updated on website)
    - very helpful if you can let me know in advance if you plan to come

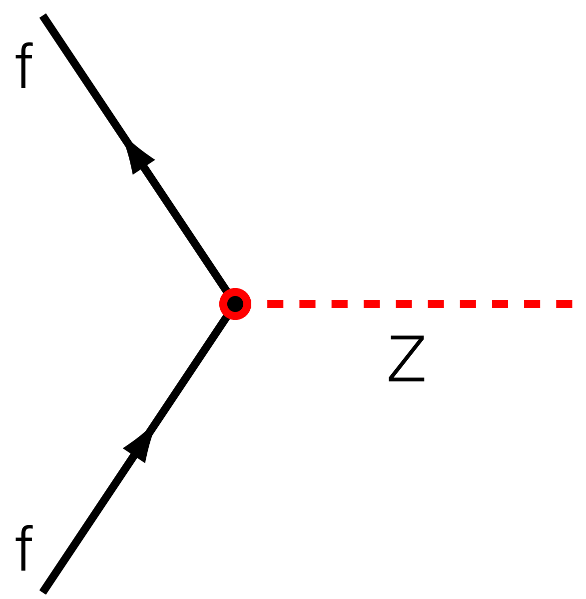
Sun	Mon	Tues	Wed	Thu	Fri	Sat
		1	2 1300-1600	3 1500-1600	4 1100-1600	5
6	7	8 No Class	9 1000-1600	10	11	12
13	14 1300-1700	15 1300-1600	16	17 1300-1600	18 1300-1600	19
20	21 FINAL EXAM (1400)					

Last time:

- We completed our discussion of the weak CC interaction
  - GIM mechanism
  - considered CKM matrix elements (quark transitions)
    - numeric factors depending on quark transition
- We started talking about the weak neutral current
  - varying "vector"/"axial-vector" components depending on particle type
  - no new calculations elements; we know how to calculate relevant traces
- Explore the weak interaction further today . . .



The Weak Neutral Current



$$\frac{-ig_Z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)$$

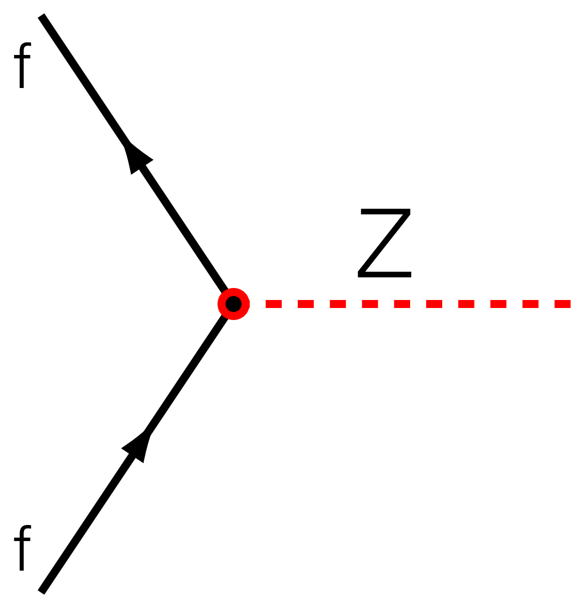
	c_V	c_A
$\nu_e\ \nu_\mu\ \nu_\tau$	1/2	1/2
$e\ \mu\ \tau$	$-1/2 + 2\sin^2\theta_W$	-1/2
$u\ c\ t$	$1/2 - 4/3\sin^2\theta_W$	1/2
$d\ s\ b$	$-1/2 + 2/3\sin^2\theta_W$	-1/2

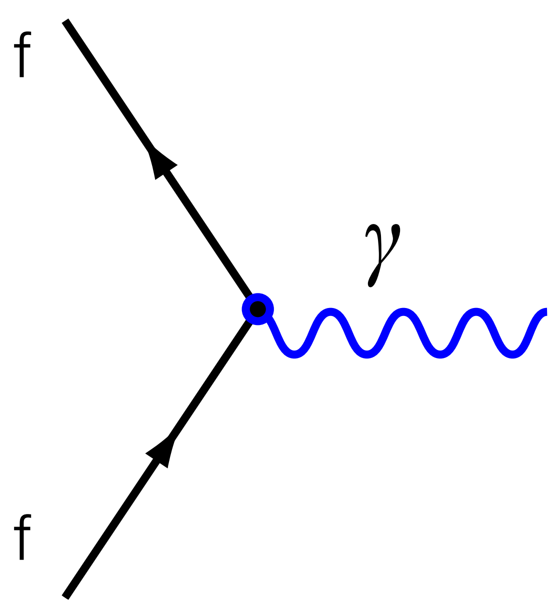
- The weak neutral current is mediated by the Z boson (M_Z=91 GeV/c^2)
- It also shows the parity-violating structure of having both vector and axial-vector couplings
- However, it is a bit more complicated than the case of the W (weak charged current)
    - The vector and axial vector components depend on the type of particle
    - $\sin^2\theta_W = 0.23126 \pm 0.00005$

ν_e	ν_μ	ν_τ
e	μ	τ

u	c	t
d	s	b

Z vs. γ

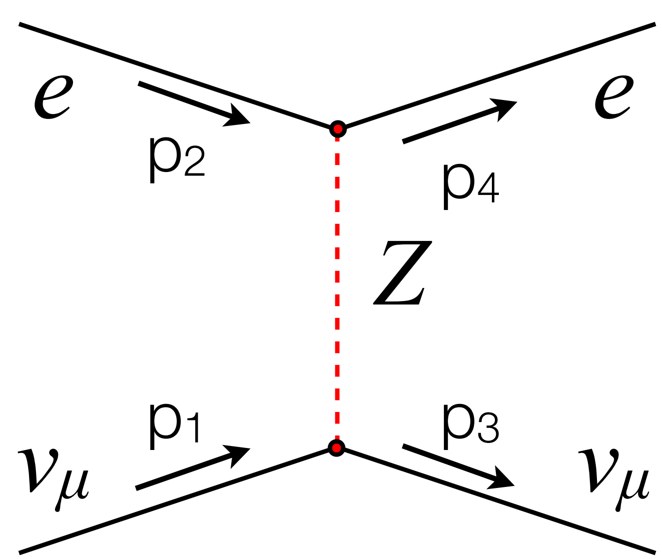




- Note that (almost) every interaction that can occur via the Z can happen via the photon
- At low energies (E << $M_Z c^2$)

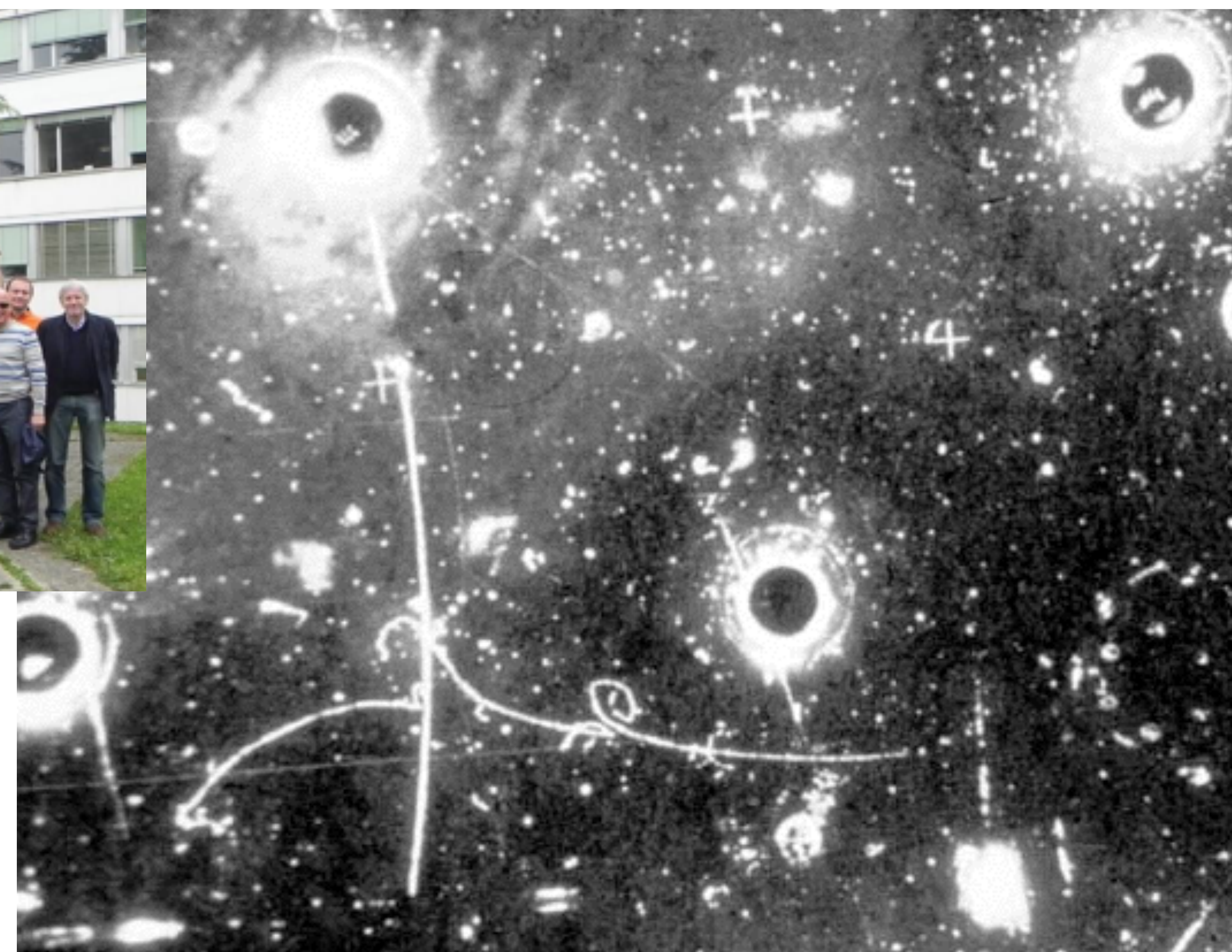
$$\frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_Z^2 c^2)}{q^2 - M_Z^2 c^2} \Rightarrow \frac{i g_{\mu\nu}}{M_Z^2 c^2}$$

  - Z propagator suppressed by Z mass
  - EM interaction masks weak interaction
- The exception is the neutrino
  - No electric charge, no EM interaction

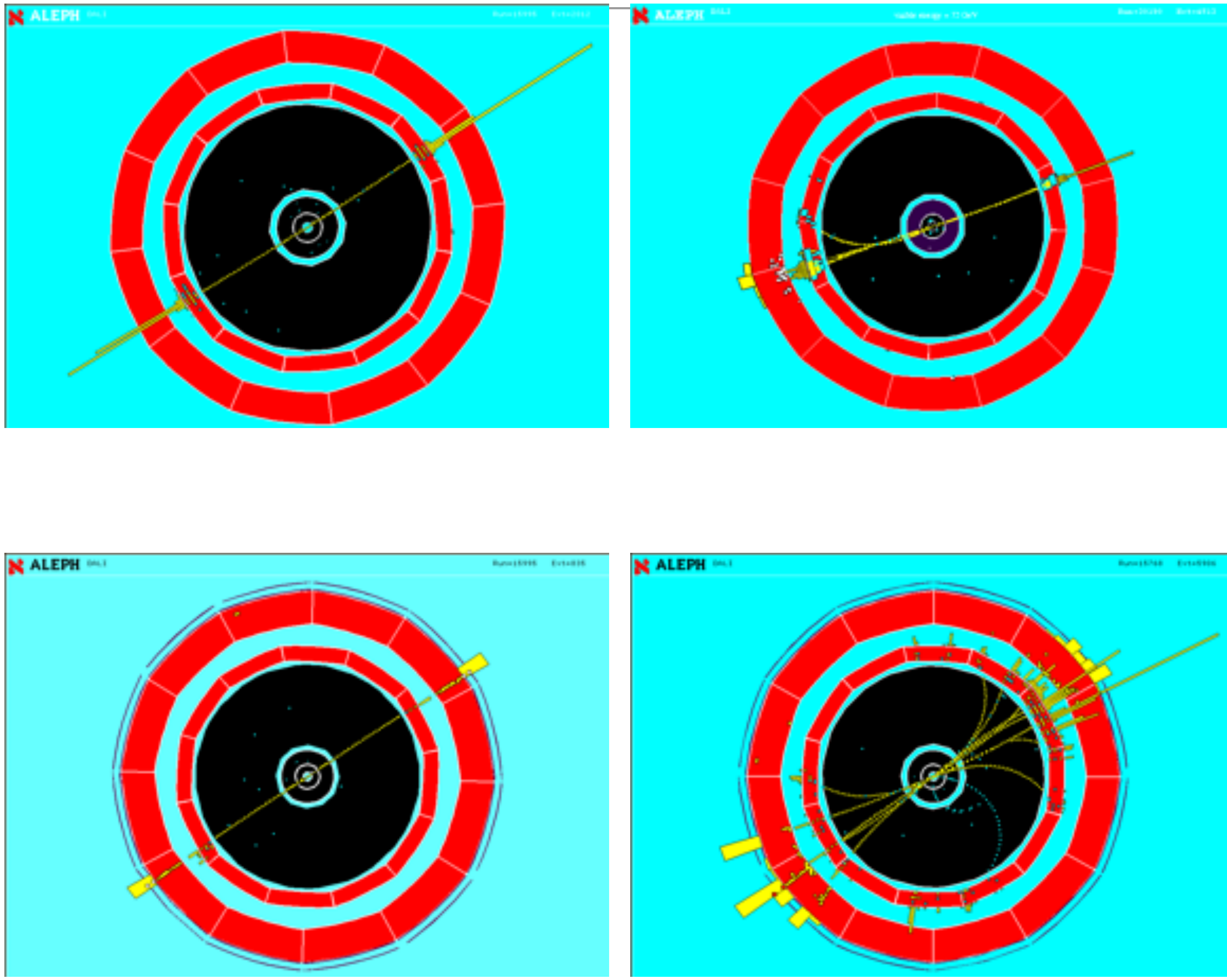


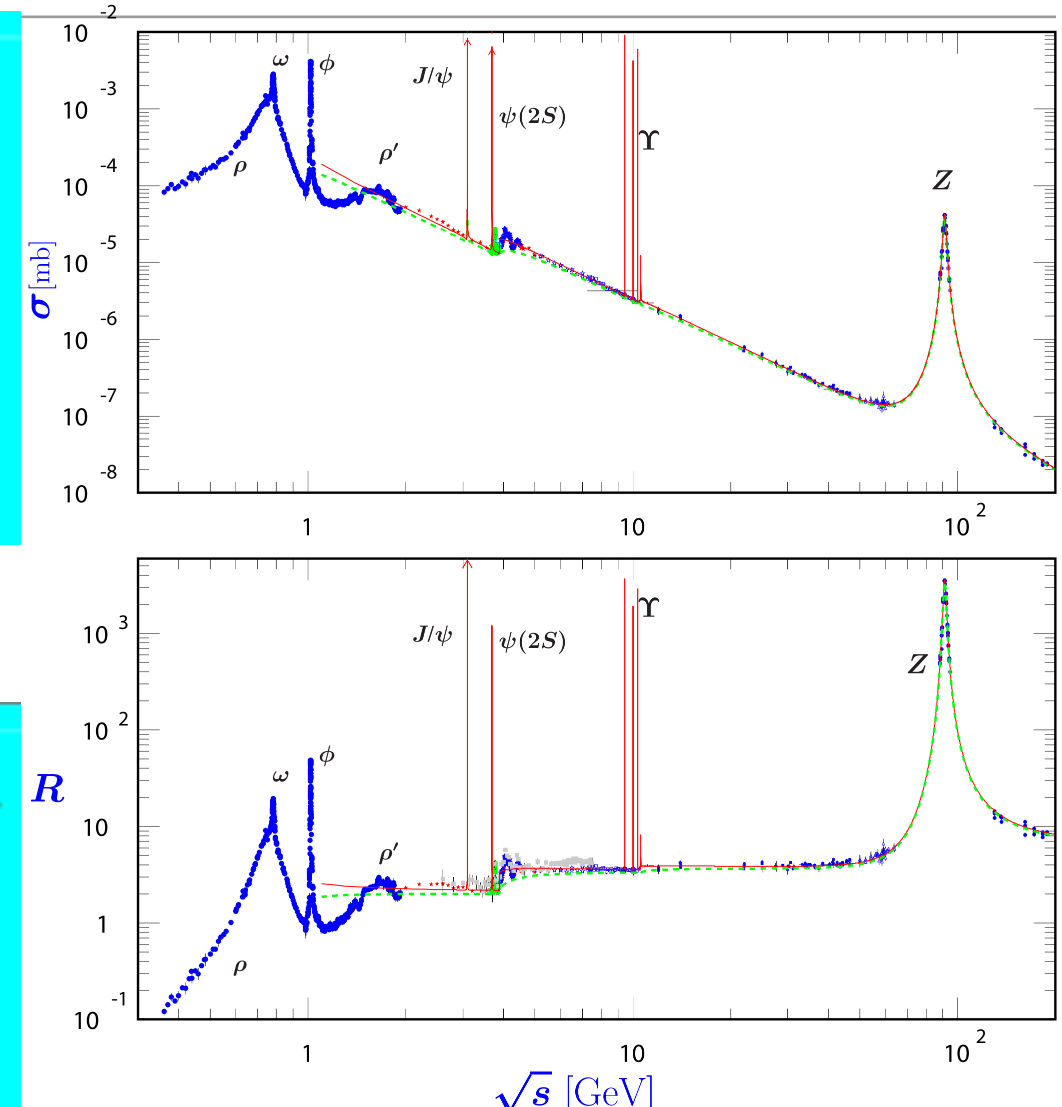
Observation:

- Intense anti neutrino beam produced
    - Scattering of atomic electron out of nowhere observed



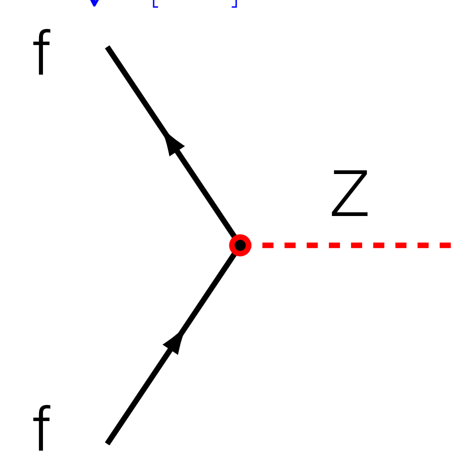
Z production in $e^+ + e^-$ collisions





- f = e, μ, τ, q

$$\frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_Z^2 c^2)}{q^2 - M_Z^2 c^2}$$



Electroweak unification

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

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Received 9 September 1960

Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

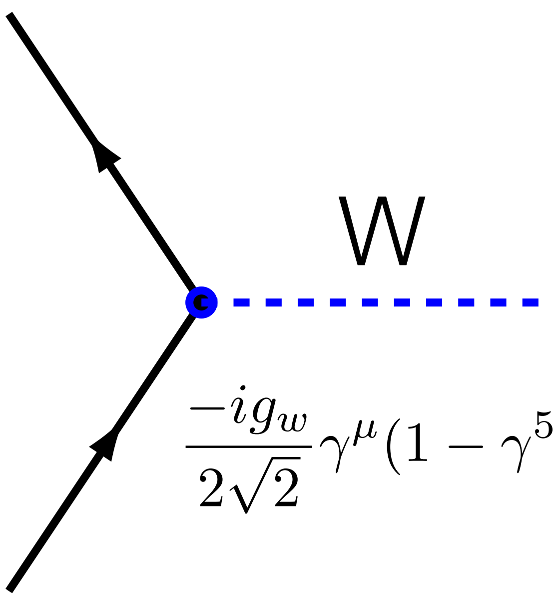
1. Introduction

At first sight there may be little or no similarity between electromagnetic effects and the phenomena associated with weak interactions. Yet certain remarkable parallels emerge with the supposition that the weak interactions are mediated by unstable bosons. Both interactions are universal, for only a single coupling constant suffices to describe a wide class of phenomena: both interactions are generated by vectorial Yukawa couplings of spin-one fields ††. Schwinger first suggested the existence of an "isotopic" triplet of vector fields whose universal couplings would generate both the weak interactions and electromagnetism — the two oppositely charged fields mediate weak interactions and the neutral field is light [2]). A certain ambiguity beclouds the self-interactions among the three vector bosons; these can equivalently be interpreted as weak or electromagnetic couplings. The more recent accumulation of experimental evidence supporting the $\Delta I = \frac{1}{2}$ rule characterizing the non-leptonic decay modes of strange particles indicates a need for at least one additional neutral intermediary [3]).

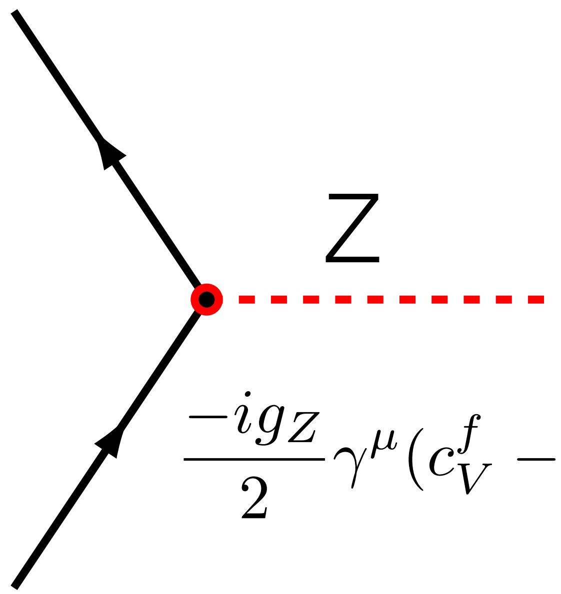
The mass of the charged intermediaries must be greater than the K-meson mass, but the photon mass is zero — surely this is the principal stumbling block in any pursuit of the analogy between hypothetical vector mesons and photons. It is a stumbling block we must overlook. To say that the decay intermediaries

The challenges of unification

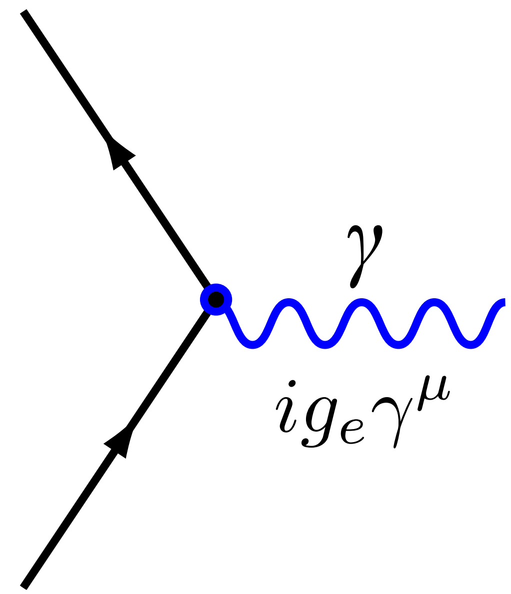
- (electroweak) unification seeks to show that electromagnetic and weak interactions have a common “origin”:
- The structure of the interactions are different



$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



$$\frac{-ig_Z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)$$



$$ig_e\gamma^\mu$$

- The intermediaries are (very) massive in one case, massless in the other

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2} \qquad \frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_Z^2c^2)}{q^2-M_Z^2c^2} \qquad \frac{-ig_{\mu\nu}}{q^2}$$

- We will deal with the first of these issues (mass is a “stumbling block”)

Chirality:

- What is chirality or a chiral state?
  - Let's look at some of the mathematical features
- First, the definitions:

$$u_L = P_L u = \frac{1}{2}(1-\gamma^5)u \qquad u_R = P_R u = \frac{1}{2}(1+\gamma^5)u$$

- Note that P_L and P_R satisfy the conditions of "projection operators"
  - $P_L + P_R = 1$ — Projection operators are "complete"
  - $P_L P_R = P_R P_L = 0$ — Projection operators are orthogonal
  - $P_L P_L = P_L$, $P_R P_R = P_R$ — Once you project out a component, projecting further has no effect
- Other examples of projection operators?

More explicitly:

- In our chosen representation, this is what P_L and P_R look like

$$P_L = \frac{1}{2}(1-\gamma^5) = \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$P_R = \frac{1}{2}(1+\gamma^5) = \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

- In full 4x4 form, we have:

$$P_L = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \qquad P_R = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

- Of course, all the properties from before hold for these explicit matrices

What do they do to Dirac spinors?

- Recall our plane wave states:

$$u^{(1)} = \sqrt{E+mc^2}\begin{pmatrix} 1 \\ 0 \\ \frac{cp_z}{E+mc^2} \\ \frac{c(p_x+ip_y)}{E+mc^2} \end{pmatrix} \qquad u^{(2)} = \sqrt{E+mc^2}\begin{pmatrix} 0 \\ 1 \\ \frac{c(p_x-ip_y)}{E+mc^2} \\ \frac{-cp_z}{E+mc^2} \end{pmatrix}$$

- When the momentum is along the z-axis, we found that these are helicity states along the z-axis:

$$u^{(1)} = \sqrt{E+mc^2}\begin{pmatrix} 1 \\ 0 \\ \frac{cp_z}{E+mc^2} \\ 0 \end{pmatrix} \qquad u^{(2)} = \sqrt{E+mc^2}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-cp_z}{E+mc^2} \end{pmatrix}$$

- i.e. these are eigenstates of $S_z = \hbar/2\ \Sigma_z$

$$\Sigma_z = \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Apply the chiral projections on u$^{(1)}$:

- Chiral projections of the “right” helicity state:

$$P_L u^{(1)} = \frac{1}{2}\sqrt{E+mc^2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ \frac{cp_z}{E+mc^2} \\ 0 \end{pmatrix} = \frac{1}{2}\sqrt{E+mc^2}\begin{pmatrix} 1-\frac{cp_z}{E+mc^2} \\ 0 \\ -(1-\frac{cp_z}{E+mc^2}) \\ 0 \end{pmatrix}$$

$$P_R u^{(1)} = \frac{1}{2}\sqrt{E+mc^2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ \frac{cp_z}{E+mc^2} \\ 0 \end{pmatrix} = \frac{1}{2}\sqrt{E+mc^2}\begin{pmatrix} 1+\frac{cp_z}{E+mc^2} \\ 0 \\ 1+\frac{cp_z}{E+mc^2} \\ 0 \end{pmatrix}$$

- if m = 0 (pc=E), u$^{(1)}$ is a right-chiral (and helicity state) state
- If p = 0 (E=mc^2), u$^{(1)}$ has equal parts of left- and right-chiral states

Chiral projections of $u^{(2)}$

- Applying our projection matrices:

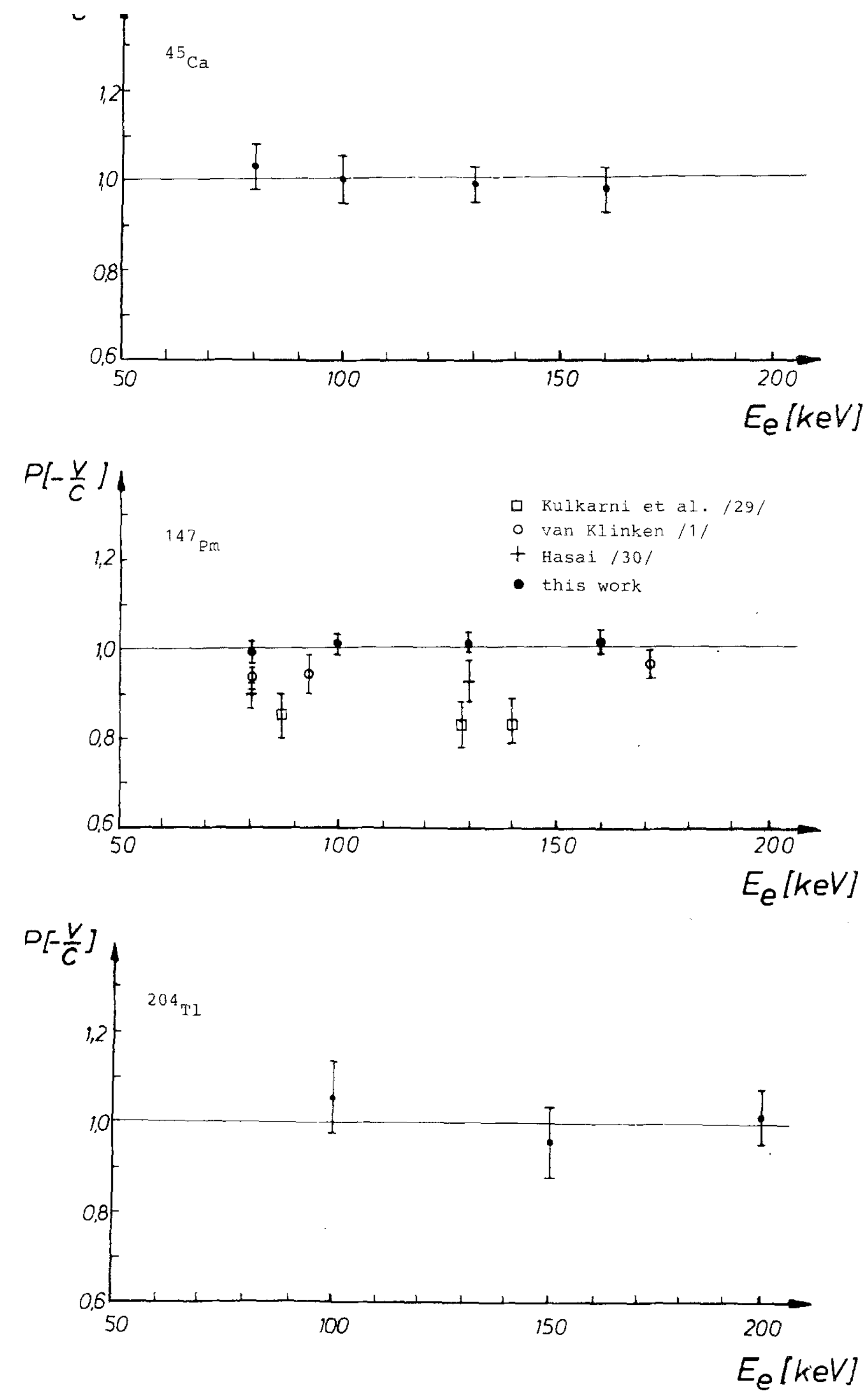
$$P_L u^{(2)} = \frac{1}{2}\sqrt{E+mc^2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-cp_z}{E+mc^2} \end{pmatrix} = \frac{1}{2}\sqrt{E+mc^2}\begin{pmatrix} 0 \\ 1+\frac{cp_z}{E+mc^2} \\ 0 \\ -(1+\frac{cp_z}{E+mc^2}) \end{pmatrix}$$

$$P_R u^{(2)} = \frac{1}{2}\sqrt{E+mc^2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-cp_z}{E+mc^2} \end{pmatrix} = \frac{1}{2}\sqrt{E+mc^2}\begin{pmatrix} 0 \\ 1-\frac{cp_z}{E+mc^2} \\ 0 \\ 1-\frac{cp_z}{E+mc^2} \end{pmatrix}$$

- if m = 0 (pc=E), $u^{(2)}$ is a left chiral state
- if p = 0 ($E=mc^2$), $u^{(2)}$ has equal parts of left- and right-chiral states

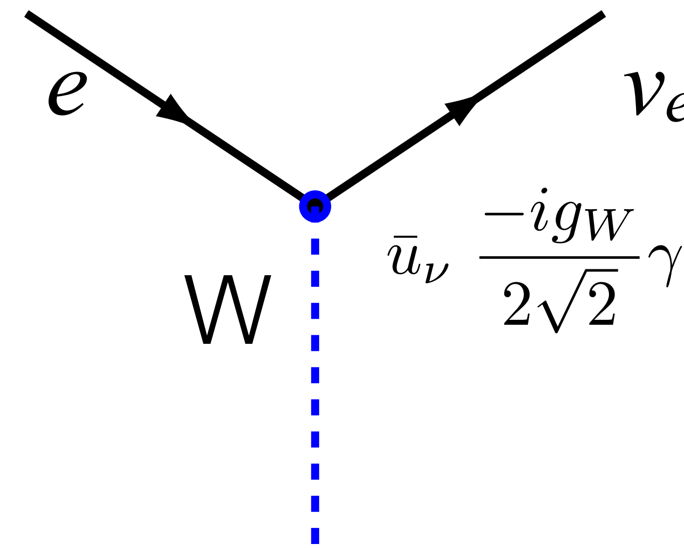
In other words:

- For a particle at rest:
  - Helicity eigenstates consists of equal parts of chiral states
  - Chiral states consist of equal parts of each helicity
- For a massless particle (or E>>mc^2)
  - Left/right helicity states correspond to left/right chiral states.
- In general, a left chiral particle has:
  - probability $(1 - \beta)/2$ of being right-helicity
  - probability $(1 + \beta)/2$ of being left-helicity
- Since weak interactions couple only to left-chiral states, polarization (right-left helicity) goes as $-\beta$



Chiral States:

- Consider a weak interaction “current”:
  - Redefine the vertex so that the γ^5 structure is part of the fermion



$$\bar{u}_\nu \; \frac{-ig_W}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) \; u_e$$

$$\bar{u}_\nu \; \frac{-ig_W}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) \; u_e \Rightarrow \frac{-ig_W}{\sqrt{2}} \bar{u}_\nu \gamma^\nu u_{eL}$$

$$u_{eL} \equiv \frac{1}{2}(1-\gamma^5) u_e \qquad \bar{u}_{eL} = \bar{u}_e \frac{1+\gamma^5}{2}$$

- We can go further:

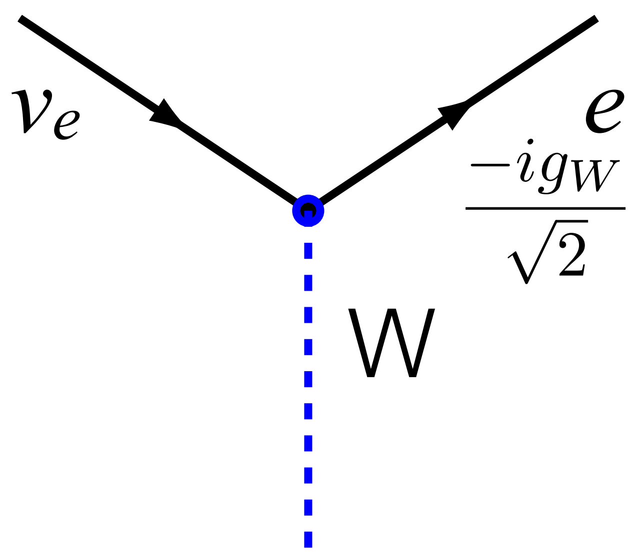
$$\left(\frac{1-\gamma^5}{2}\right)^2 = \frac{1}{4}\left(1 - 2\gamma^5 + (\gamma^5)^2\right)$$
$$= \left(\frac{1-\gamma^5}{2}\right)$$

$$\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) = \left(\frac{1+\gamma^5}{2}\right) \gamma^\mu$$

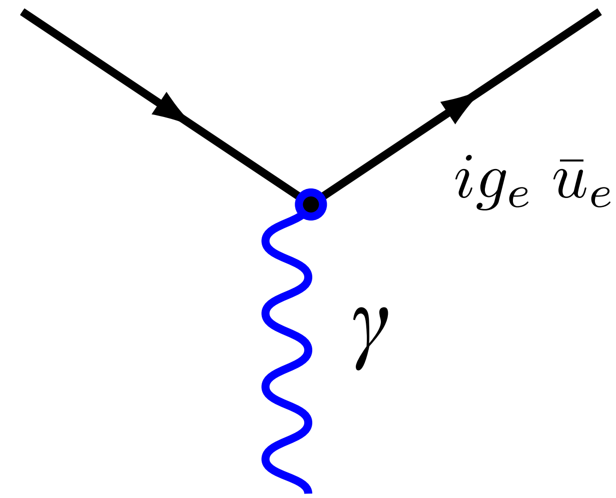
$$\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) = \left(\frac{1+\gamma^5}{2}\right) \gamma^\mu \left(\frac{1-\gamma^5}{2}\right)$$

$$\bar{u}_\nu \; \frac{1}{2} \gamma^\nu (1-\gamma^5) \; u_e = u_\nu^\dagger \gamma^0 \frac{1+\gamma^5}{2} \gamma^\nu \frac{1-\gamma^5}{2} u_{eL}$$
$$= u_\nu^\dagger \frac{1-\gamma^5}{2} \gamma^0 \gamma^\nu u_{eL} = u_{\nu L}^\dagger \gamma^0 \gamma^\nu u_{eL}$$
$$= \bar{u}_{\nu L} \gamma^\nu u_{eL}$$

Relation to EM interactions



$$\frac{-ig_W}{\sqrt{2}}\bar{u}_{\nu L}\gamma^\nu u_{eL}$$



$$ig_e\ \bar{u}_e\gamma^\nu u_e$$

- We can work on the electromagnetic side now
  - Having defined "uL", we can also define "uR" $u_R \equiv \frac{1+\gamma^5}{2}u$ $\quad \bar{u}_R = \bar{u}\frac{1-\gamma^5}{2}$
  - We then have $u = u_L + u_R$

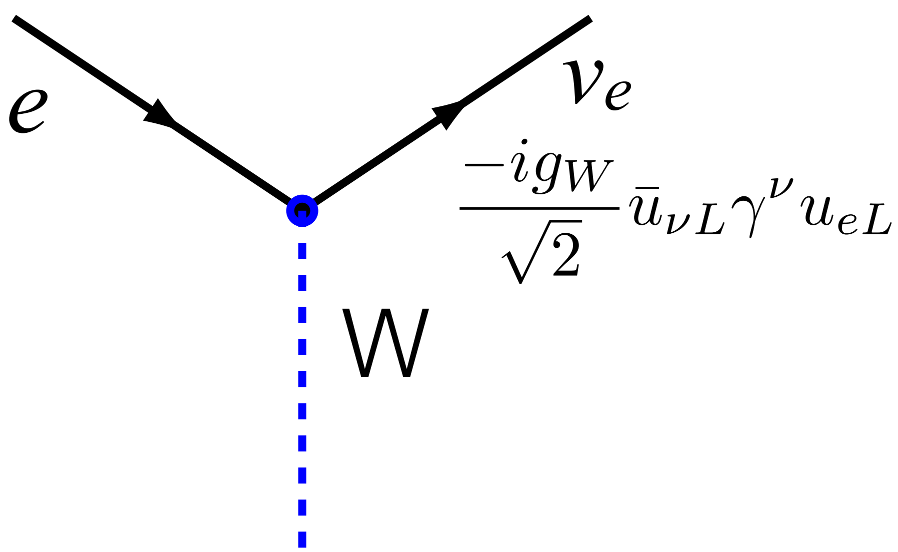
$$\bar{u}\gamma^\nu u = (\bar{u}_L + \bar{u}_R)\gamma^\nu(u_L + u_R) = \bar{u}_L\gamma^\nu u_L + \bar{u}_R\gamma^\nu u_R + \bar{u}_L\gamma^\nu u_R + \bar{u}_R\gamma^\nu u_L$$

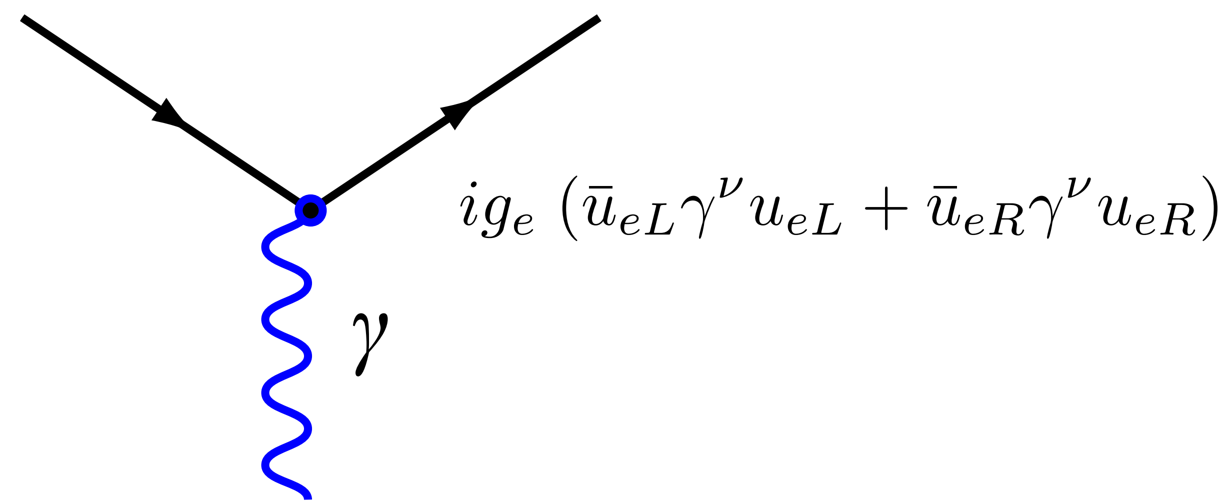
$$(1\pm\gamma^5)\gamma^\mu(1\pm\gamma^5) = \gamma^\mu(1\mp\gamma^5)(1\pm\gamma^5) = \gamma^\mu(1-(\gamma^5)^2) = 0$$

$$\bar{u}\gamma^\nu u = \bar{u}_L\gamma^\nu u_L + \bar{u}_R\gamma^\nu u_R$$

$$ig_e\ (\bar{u}_{eL}\gamma^\nu u_{eL} + \bar{u}_{eR}\gamma^\nu u_{eR})$$

Now Compare Weak CC and EM:





- Electromagnetic and weak interactions have the same structure if we specify the chirality of the fermions. Weak interactions are missing a coupling to right-chirality particles that are present in the electromagnetic interactions.

- We'll start labeling spinor states by the particle species:

$$e \equiv u_e \qquad \nu \equiv u_\nu$$

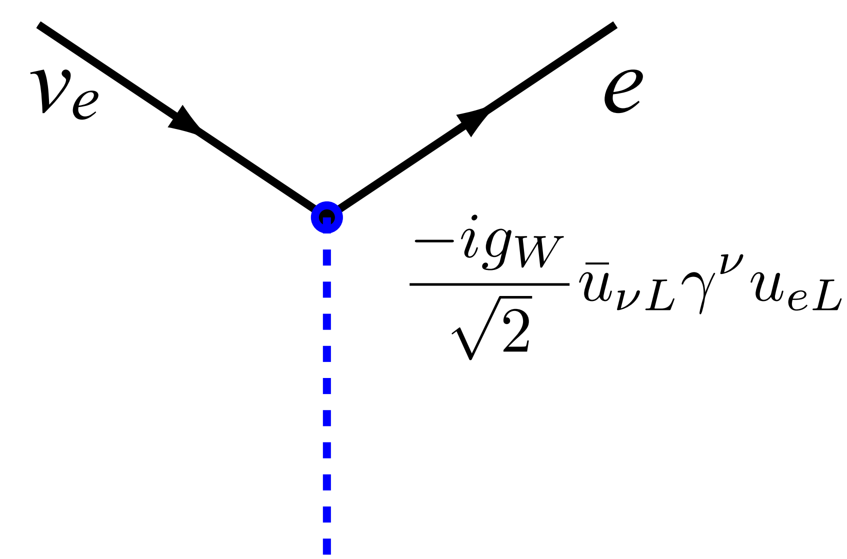
- Define "currents":

$$j_\mu^{em} = -\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R \qquad j_\mu^- = \bar{\nu}_L\gamma_\mu e_L$$

$$j_\mu^+ = \bar{e}_L\gamma_\mu\nu_L$$

Building the structure of the weak interaction

- Up until now, the transition from $\nu_e \leftrightarrow e$ has been somewhat ad-hoc
- We now formalize this transition by defining a doublet structure to contain the fermions



$$\chi_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

- and defining two operators to enact the transitions:

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\tau_+ e_L = \nu_e \qquad \tau_- \nu_{eL} = 0$$

$$\tau_- \nu_e = e_L \qquad \tau_- e_L = 0$$

- Then we can write the currents as:

$$j^+_\mu = \bar{e}_L \gamma_\mu \nu_L = \bar{\chi}_L \gamma_\mu \tau^+ \chi_L$$

$$j^-_\mu = \bar{e}_L \gamma_\mu \nu_L = \bar{\chi}_L \gamma_\mu \tau^- \chi_L$$

A third current:

- Where did τ^+ and τ^- come from?
  - Recall that the raising/lowering operators

    $$\tau^{\pm} = \tau^1 \pm i\tau^2$$

  - where τ are Pauli matrices (relabeled to avoid confusion with spin)
  - Evidently, the weak charged currents are effected in a two-dimensional spinor space with SU(2) matrices (yet another space!)
  - This means there should be a third transformation associated with τ^3
- The third current is given by:

$$\bar{\chi}_L \gamma_\mu \tau^3 \chi_L = (\bar{\nu}_L,\ \bar{e}_L)\,\gamma_\mu \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \tau^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2}\bar{e}_L \gamma_\mu e_L$$

EM as the Weak NC?

- Why can't the photon be the neutral current?

$$j_\mu^{em} = -\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R$$

- Note that there is an "orthogonal" current:

$$-\left(\bar{\nu}_L\gamma_\mu\nu_L + \bar{e}_L\gamma_\mu e_L\right)$$

- Mathematically, two left-chiral weak isospin 1/2 objects have produced a
  - spin 1 triplet $j_\mu^+ = \bar{e}_L\gamma_\mu\nu_L = \bar{\chi}_L\gamma_\mu\tau^+\chi_L$ spin 0 singlet $-\left(\bar{\nu}_L\gamma_\mu\nu_L + \bar{e}_L\gamma_\mu e_L\right)$

$$j_\mu^- = \bar{e}_L\gamma_\mu\nu_L = \bar{\chi}_L\gamma_\mu\tau^-\chi_L$$

$$j_\mu^3 = \bar{\chi}_L\gamma_\mu\tau^3\chi_L$$

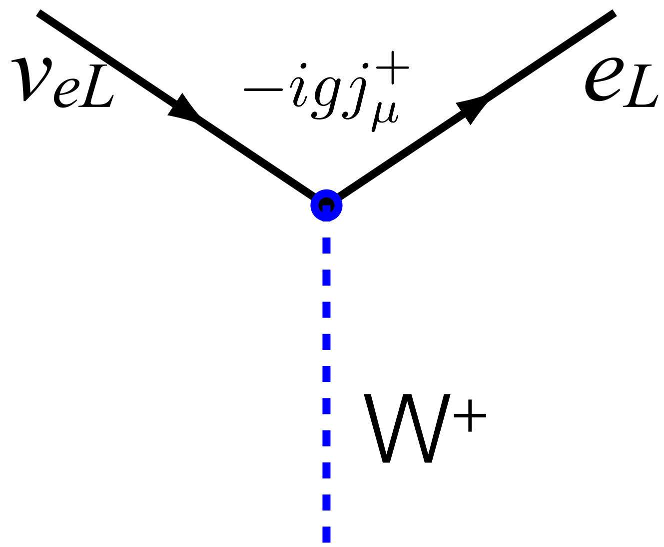
  - We can also add in a right handed component

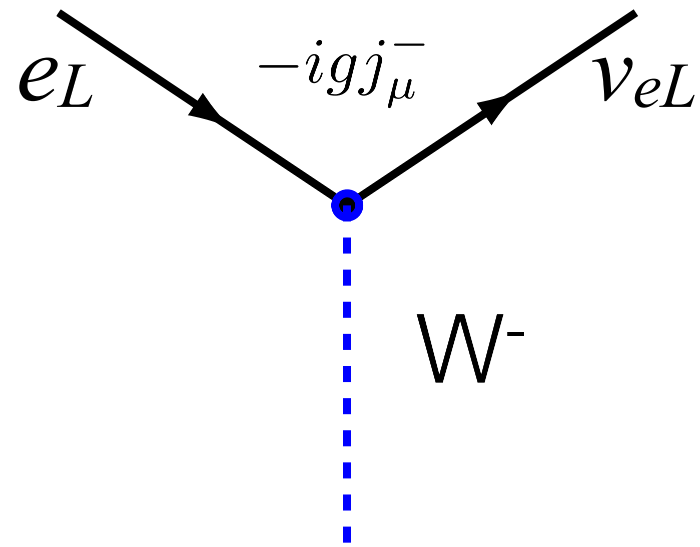
$$-\left(\bar{e}_R\gamma_\mu e_R\right)$$

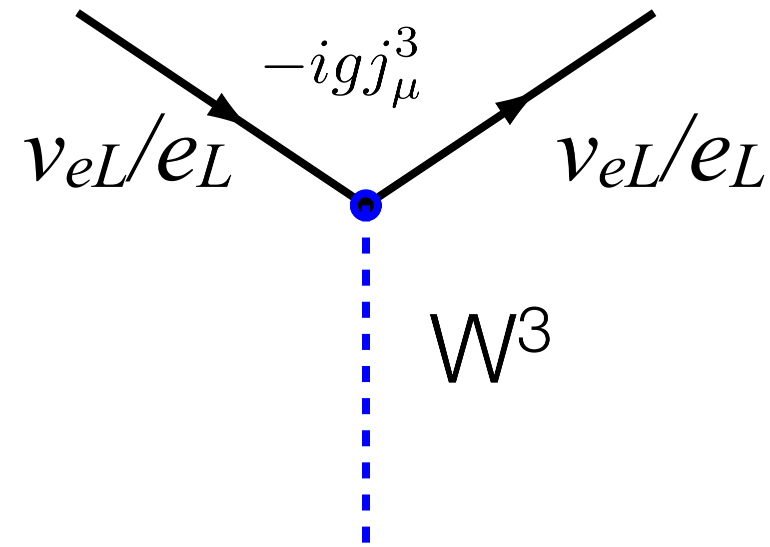
    - which is also spin 0 singlet.

Diagramatically:

- Thus far, we have three interactions associated with the three τ matrices:

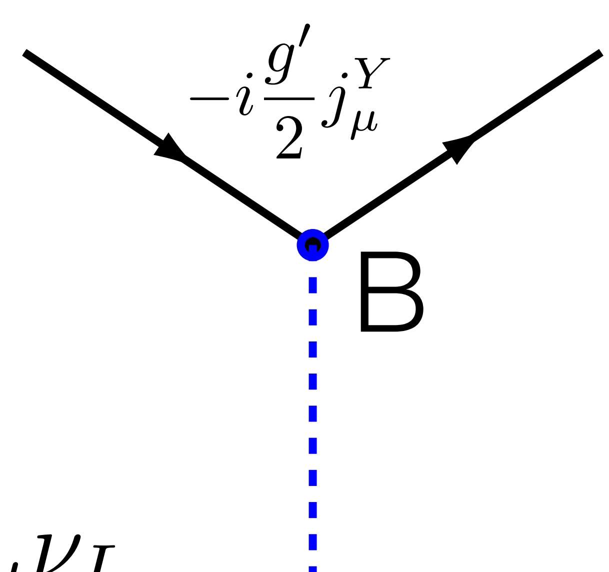






$$j_\mu^+ = \bar{\chi}_L \gamma_\mu \tau^+ \chi_L \qquad j_\mu^- = \bar{\chi}_L \gamma_\mu \tau^- \chi_L \qquad j_\mu^3 = \frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2}\bar{e}_L \gamma_\mu e_L$$

  - characterized by a single coupling g
- We introduced one more interaction
  - historically called "weak hypercharge"
  - separate coupling strength (g'/2)



$$j_\mu^Y = -2\bar{e}_R \gamma_\mu e_R - \bar{e}_L \gamma_\mu e_L - \bar{\nu}_L \gamma_\mu \nu_L$$

Electroweak mixing

- The charged currents have the structure we want:

$$j^+_\mu = \bar{\chi}_L \gamma_\mu \tau^+ \chi_L \qquad\qquad j^-_\mu = \bar{\chi}_L \gamma_\mu \tau^- \chi_L$$

- but the neutral currents do not (they don't look like Z/γ):

$$j^3_\mu = \frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2}\bar{e}_L \gamma_\mu e_L \qquad j^Y_\mu = -2\bar{e}_R \gamma_\mu e_R - \bar{e}_L \gamma_\mu e_L - \bar{\nu}_L \gamma_\nu e_L$$

- Define the A, Z particles to be linear combinations of W^3 and B

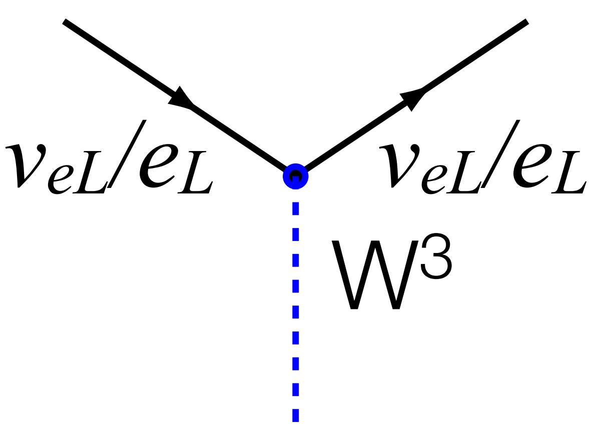
$$\begin{aligned} A_\mu &= \quad B_\mu \cos\theta_W + W^3_\mu \sin\theta_W \\ Z_\mu &= -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W \end{aligned}$$

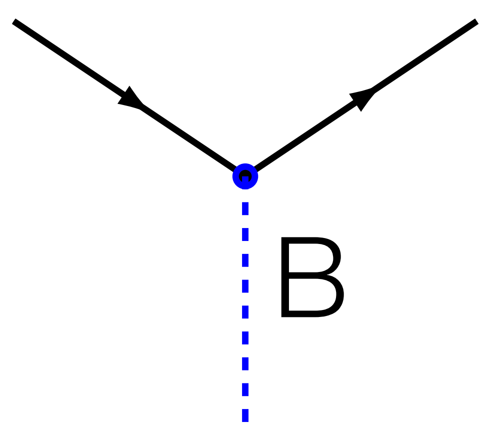
A vertex

$$-i\frac{g'}{2}\cos\theta_W(-2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L)$$
$$- ig\sin\theta_W(\frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L)$$

Z vertex

$$i\frac{g'}{2}\sin\theta_W(-2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L)$$
$$- ig\cos\theta_W(\frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L)$$





$$j^3_\mu = \frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2}\bar{e}_L \gamma_\mu e_L \qquad j^Y_\mu = -2\bar{e}_R \gamma_\mu e_R - \bar{e}_L \gamma_\mu e_L - \bar{\nu}_L \gamma_\nu e_L$$

Creating the Photon:

- The choice of “A” and “Z” was deliberate.
- Can we make “A” have the properties of the photon:
  - No coupling to neutrinos (no charge)
  - Equal coupling to e_L and e_R

A vertex

$$-i\frac{g'}{2}\cos\theta_W(-2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L)$$
$$- ig\sin\theta_W(\frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L)$$

- If g sinθ_W = g’ cosθ_W = g_e:

A vertex

$$-ig_e(-\bar{e}_R\gamma_\mu e_R - \frac{1}{2}\bar{e}_L\gamma_\mu e_L - \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L + \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L)$$

$$ig_e(\bar{e}_R\gamma_\mu e_R + \bar{e}_L\gamma_\mu e_L)$$

$$ig_e(\bar{e}\gamma_\mu e)$$

Consequences for the Z:

Z vertex

$$i\frac{g'}{2}\sin\theta_W(-2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L)$$
$$-\, ig\cos\theta_W(\frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L)$$

- if we define

$$g_Z = \frac{g_e}{\sin\theta_W\cos\theta_W} = \frac{g}{\cos\theta_W} = \frac{g'}{\sin\theta_W}$$

Z vertex

$$ig_Z\sin^2\theta_W(-\bar{e}_R\gamma_\mu e_R - \frac{1}{2}\bar{e}_L\gamma_\mu e_L - \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L)$$
$$-\, ig_Z\cos^2\theta_W(\frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L)$$

$$-ig_Z(\frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L + \sin^2\theta_W(\bar{e}_R\gamma_\mu e_R + \bar{e}_L\gamma_\mu e_L))$$

What did we do?

- We defined chiral states of our fermions
  - “pre-selects” particles that undergo the weak charged current
  - Placed “left” components of the chiral states into doublets of weak isospin
  - “right” components are on their own:
    - isospin “singlets” do not undergo the weak charged current interaction
- Based on the isospin, three ways of transforming the isospin:
  - W_1 W_2 correspond to the weak charged current.
  - W_3 is a neutral current, but it doesn’t quite have the property of the Z
- Introduce another neutral current interaction with the “B” boson
  - This has a separate coupling g’ to “weak hypercharge”
- The A/Z are linear combinations of W_3 and B appropriately chosen to reproduce the known properties of the A and Z

Some observations:

- Note that left-chiral particles are endowed with the weak charged current coupling by placing them in isospin doublets transformed by the τ matrices
  - The transformative property is the interaction. Thus we talk about how particles "transform under SU(2)" to determine their interaction with the W
  - Singlets (not in SU(2) n-plets) do not transform: no interaction
  - Generalized notion of charge
- The electroweak "unification" may seem just like bookkeeping
  - In some sense it is.
  - But it relates electromagnetic and weak interactions in a non-trivial way with a single parameter θ_W

Extending to other particles:

- We have set up a electroweak theory of a lepton and its neutrino:
  - Make three copies for the three generations of neutrinos/leptons
- For quarks, make similar isospin doublets of chiral fields:

$$\begin{pmatrix} u_L \\ d'_L \end{pmatrix}, \begin{pmatrix} c_L \\ s'_L \end{pmatrix}, \begin{pmatrix} t_L \\ b'_L \end{pmatrix}$$

- "Right" chiral fields are on their own again, don't interact via the weak CC

$$u_R, d'_R, c_R, s'_R, t_R, b'_R$$

- Assign weak isospin to get the right charge:

$$Y = 2(Q - I_3)$$

- Complete theory of the electroweak interactions of quarks and leptons

Relations:

- Recall that the muon decay rate involves the W mass and g_W:

$$\Gamma = \left(\frac{m_\mu g_W}{M_W}\right)^4 \frac{m_\mu c^2}{12\hbar(8\pi)^3}$$

- NC processes involve the Z mass (neutrino elastic scattering)

$$\sigma = \frac{2}{3\pi}(\hbar c)^2 \left(\frac{g_Z}{2M_Z c^2}\right)^4 E^2(c_V^2 + c_A^2 + c_V c_A)$$

- Now we have a relation between $g_e/g_W/g_Z$

$$g_Z = \frac{g_e}{\sin\theta_W \cos\theta_W} = \frac{g}{\cos\theta_W} = \frac{g'}{\sin\theta_W}$$

Conclusions

- The weak and electromagnetic interactions arise from a strange mixture of two interactions
  - “Weak” sector with three bosons
  - “Hypercharge” sector with one boson
- The weak charged current is a pure manifestation of the “weak” sector
- The weak neutral current,electromagnetic interaction are a hybrid of the two
- This introduces nontrivial relations between the weak CC, NC, and EM interactions called “electroweak mixing” or “electroweak unification”