Weak Interaction of Hadrons and Neutral Current

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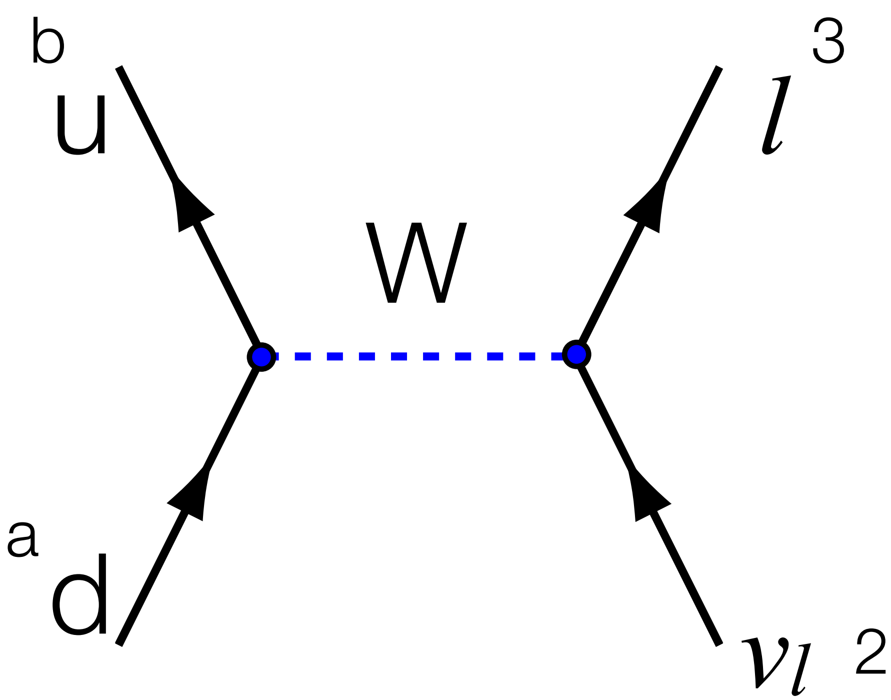
Midterm:

- Replace the midterm grade with final grade is higher
  - i.e. if final grade > midterm grade, final is worth 60% of your grade.
  - otherwise, midterm is 20%, final is 40% (as before).

So far:

- Examined the weak charged current interaction of leptons (muon decay, etc.)
- We saw how the coupling includes a vector and axial-vector piece
  - parity violation is built into the weak interaction
- From a calculation standpoint, the new element is g5:
  - we learned how to evaluate traces with g5
- Now we move on to quarks. Two issues arise:
  - Quarks are in bound states that we don't know how to describe
    - we'll need to make some guesses/ansatz
  - Quarks can transition between "generations"
    - Leptons always stay within their generation

Pion Decay:



- Lepton fermion leg

$$\left[\bar{u}_3 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) v_2\right]$$

- Quark Fermion leg

$$\left[\bar{v}_b \frac{-ig_w}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) u_a\right]$$

$$\left[\bar{v}_b \frac{-ig_w}{2\sqrt{2}} \gamma^\nu (1-\gamma^5) u_a\right] \Rightarrow F^\nu = f_\pi p^\nu$$

- Propagator

$$\int \frac{1}{(2\pi)^4} d^4 q \qquad \frac{i g_{\mu\nu}}{M_W^2 c^2}$$

$$\mathcal{M} = \frac{g_W^2}{8 M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) v_2\right] f_\pi p_\mu$$

Summing over spins:

$$\mathcal{M}\mathcal{M}^* = \left(\frac{g^2}{8M_W^2c^2}\right)^2 \left[\bar{u}_3\gamma^\mu(1-\gamma^5)v_2\right] \left[\bar{u}_3\gamma^\nu(1-\gamma^5)v_2\right]^* f_\pi^2 p_\mu p_\nu$$

$$\sum_{\text{a, b spins}} \left[\bar{u}(a)\Gamma_1 u(b)\right] \left[\bar{u}(a)\bar{\Gamma}_2 u(b)\right]^* = \text{Tr}\left[\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)\right]$$

$$\langle|\mathcal{M}|^2\rangle = \frac{g_W^4}{64M_W^4c^4} f_\pi^2 p_\mu p_\nu \, \text{Tr}\left[\gamma^\mu(1-\gamma^5)(\not{p}_2)\gamma^\nu(1-\gamma^5)(\not{p}_3 + m_l c)\right]$$

- We've done this trace already:

$$\text{Tr} \Rightarrow 8 \times \left[p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - g^{\mu\nu} p_2 \cdot p_3 - i\epsilon^{\mu\nu\lambda\sigma} p_{2\lambda} p_{3\sigma}\right]$$

- So:

$$\langle|\mathcal{M}|^2\rangle = \frac{f_\pi^2 g_W^4}{8M_W^4c^4}\left[2 \times (p\cdot p_2)(p\cdot p_3) - p^2(p_2\cdot p_3)\right]$$

Decay Rate:

$$\langle|\mathcal{M}|^2\rangle = \frac{f_\pi^2 g_W^4}{8M_W^4 c^4}\left[2\times(p\cdot p_2)(p\cdot p_3) - p^2(p_2\cdot p_3)\right]$$

- Going into the rest frame of the decay, we can work out the kinematics:
  - Recall that "2" is the outgoing neutrino which we take to be massless

$$p = p_2 + p_3$$

$$p\cdot p_2 = (p_2+p_3)\cdot p_2 = p_2\cdot p_3 \qquad p\cdot p_3 = (p_2+p_3)\cdot p_3 = p_2\cdot p_3 + m_l^2c^2$$

$$p^2 = p_2^2 + p_3^2 + 2p_2\cdot p_3 \qquad 2p_2\cdot p_3 = m_\pi^2c^2 - m_l^2c^2$$

$$\langle|\mathcal{M}|^2\rangle = \frac{f_\pi^2 g_W^4}{16M_W^4 c^4} m_l^2(m_\pi^2 - m_l^2)$$

$$\Gamma = \frac{|\mathbf{p}_2|}{8\pi\hbar m_\pi^2 c}\langle|\mathcal{M}|^2\rangle \qquad |\mathbf{p}_2| = \frac{c}{2m_\pi}(m_\pi^2 - m_l^2)$$

$$\Gamma = A\times m_l^2(m_\pi^2 - m_l^2)^2$$

Now Consider the l=μ/e

$$\Gamma_l = A \times m_l^2(m_\pi^2 - m_l^2)^2$$

- We take the ratio of the decay rates:

$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} = 1.28 \times 10^{-4}$$

$$\pi^- \rightarrow e^- + \bar{\nu}_e$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

- using the known masses of e/μ/π
- Experiments can measure this and obtain (1.230±0.004) x10^{-4}
- The PIENU experiment at TRIUMF will use this to test the universality of the lepton coupling to the W.

m_e = 0.511 MeV/c^2
m_μ = 105.66 MeV/c^2
m_π = 139.57 MeV/c^2

The PIENU Experiment at TRIUMF

Improved Measurement of the $\pi \to e\nu$ Branching Ratio

A. Aguilar-Arevalo,[1] M. Aoki,[2] M. Blecher,[3] D. I. Britton,[4] D. A. Bryman,[5] D. vom Bruch,[5] S. Chen,[6] J. Comfort,[7] M. Ding,[6] L. Doria,[8] S. Cuen-Rochin,[5] P. Gumplinger,[8] A. Hussein,[9] Y. Igarashi,[10] S. Ito,[2] S. H. Kettell,[11] L. Kurchaninov,[8] L. S. Littenberg,[11] C. Malbrunot,[5,*] R. E. Mischke,[8] T. Numao,[8] D. Protopopescu,[4] A. Sher,[8] T. Sullivan,[5] D. Vavilov,[8] and K. Yamada[2]

(PIENU Collaboration)

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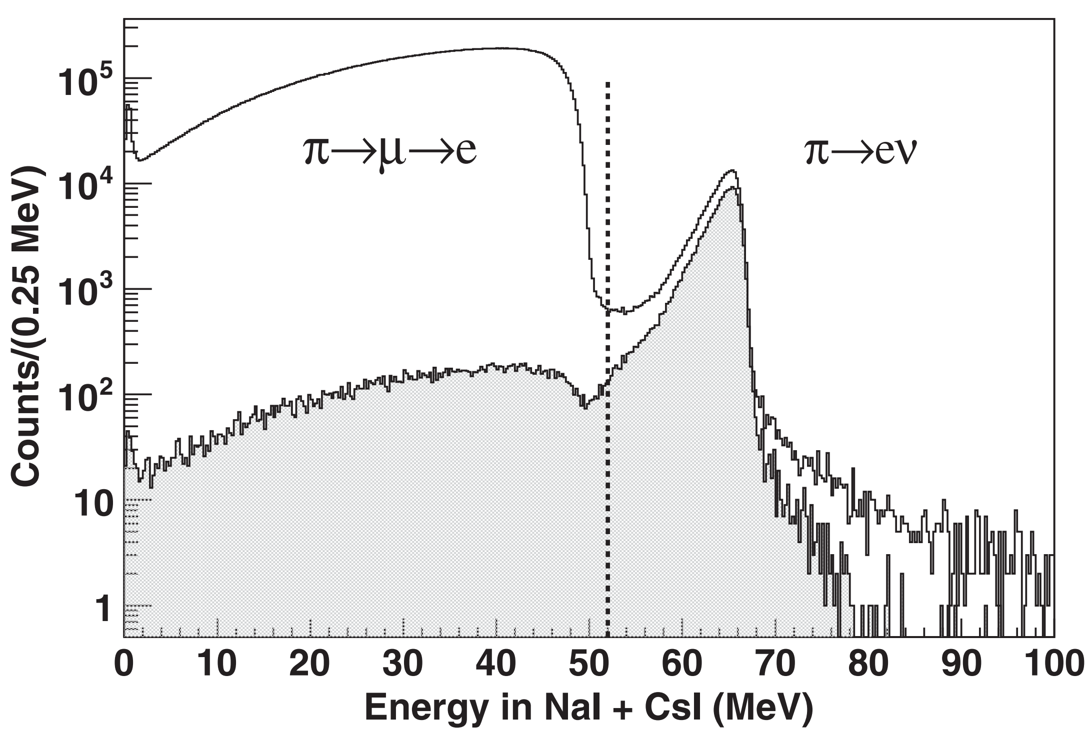
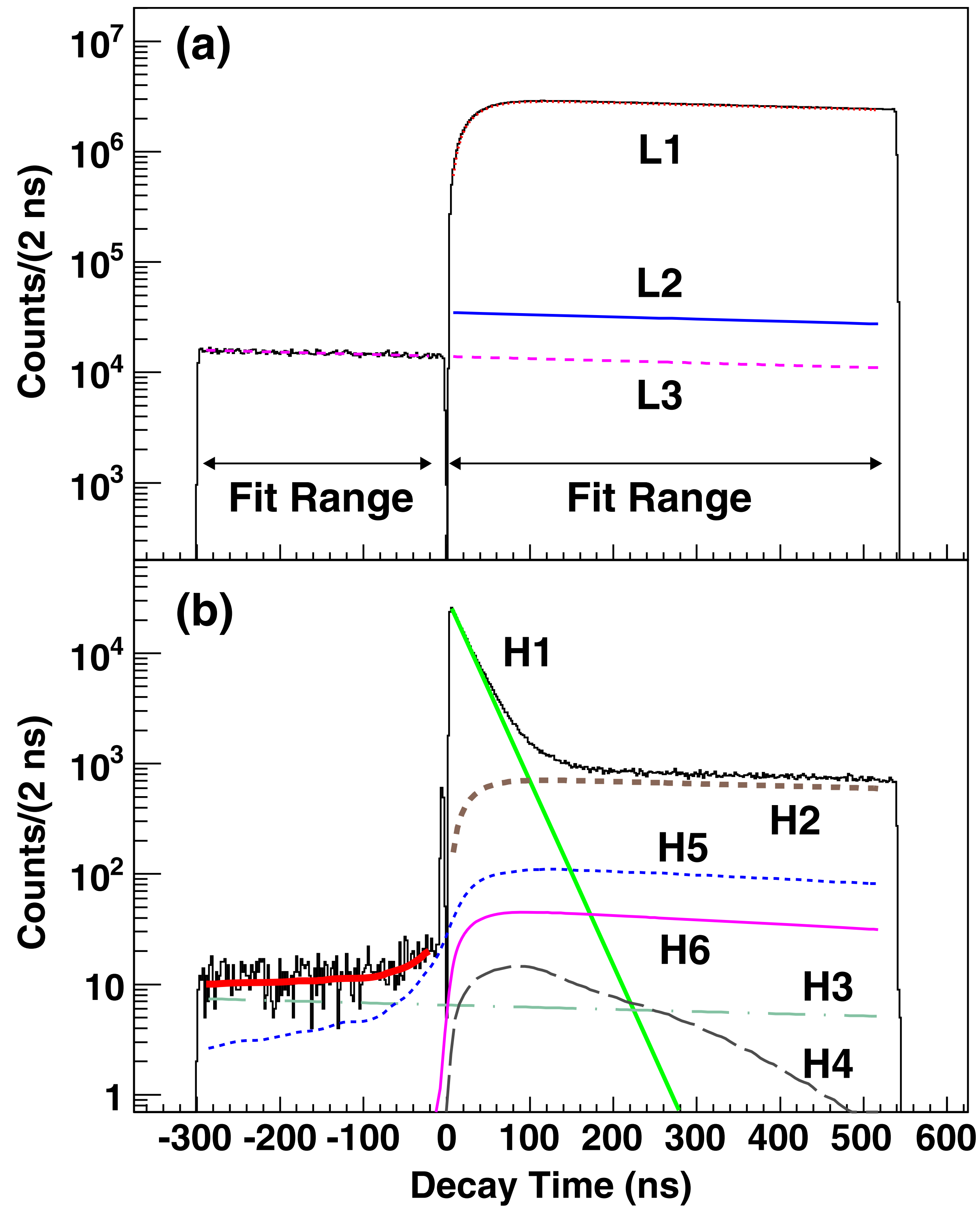
A new measurement of the branching ratio $R_{e/\mu} = \Gamma(\pi^+ \to e^+\nu + \pi^+ \to e^+\nu\gamma)/\Gamma(\pi^+ \to \mu^+\nu + \pi^+ \to \mu^+\nu\gamma)$ resulted in $R^{\mathrm{exp}}_{e/\mu} = [1.2344 \pm 0.0023(\mathrm{stat}) \pm 0.0019(\mathrm{syst})] \times 10^{-4}$. This is in agreement with the standard model prediction and improves the test of electron-muon universality to the level of 0.1%.



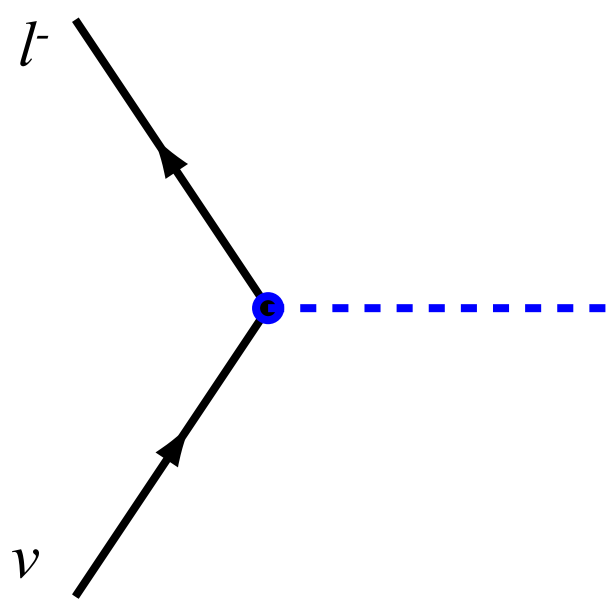


Kaons:

$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2(m_K^2 - m_e^2)}{m_e^2(m_K^2 - m_\mu^2)} = 2.57 \times 10^{-5}$$

- Branching fractions:
    - $K^+ \rightarrow e^+ + \nu_e$ = $(1.582 \pm 0.007) \times 10^{-5}$
    - $K^+ \rightarrow \mu^+ + \nu_\mu$ = 0.6356 ± 0.0011
    - Ratio = 2.49×10^{-5}
- Can also apply to D^+ and B^+, but:
    - electronic decay mode for D^+ not observed yet ($BR < 8.8 \times 10^{-6}$)
    - electronic/muonic decay mode for B^+ not observed yet ($BR < 10^{-6}$)

Weak interactions of leptons

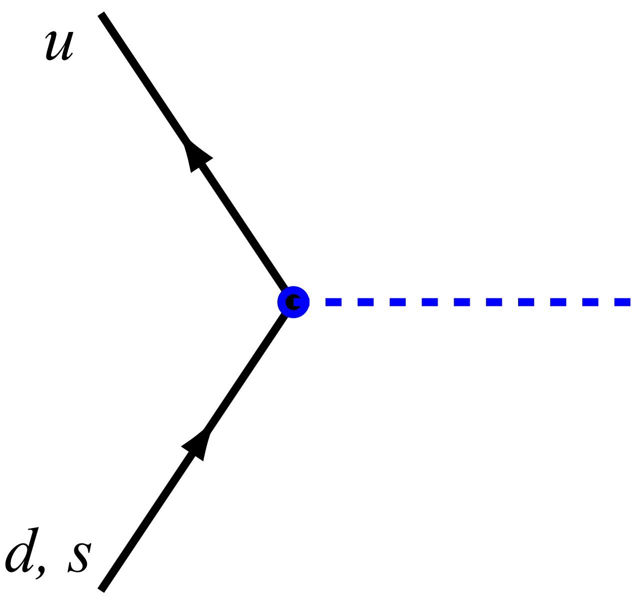


$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

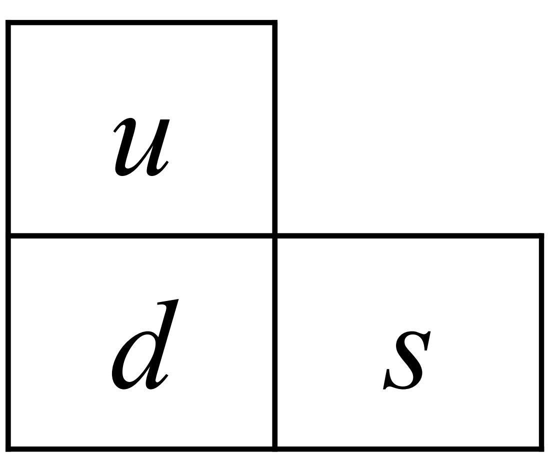
ν_e	ν_μ	ν_τ
e	μ	τ

- We have used the following Feynman rule for the vertex of a leptonic weak interaction
- This had two properties:
  - the neutrino and lepton must “match”
  - the coupling is the same for each lepton type
- We say that the interaction is “diagonal” with respect to lepton flavor and that the coupling is “universal”

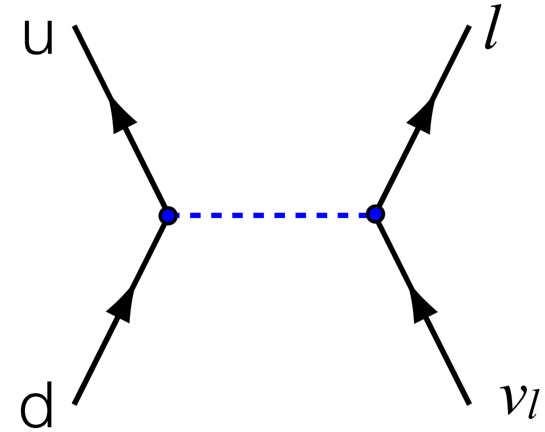
Weak Interaction of the quarks

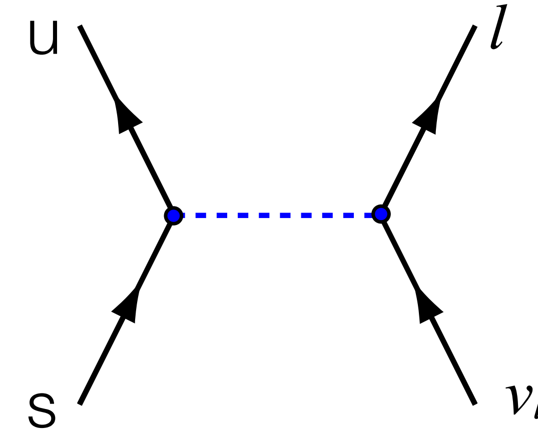


$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



- We'll step back several decades to 1963 when we only knew of three quarks (sort of)
- People noticed that decays of strange particles to non-strange particles were "slower" than expected
- We can compare pion and kaon decays:

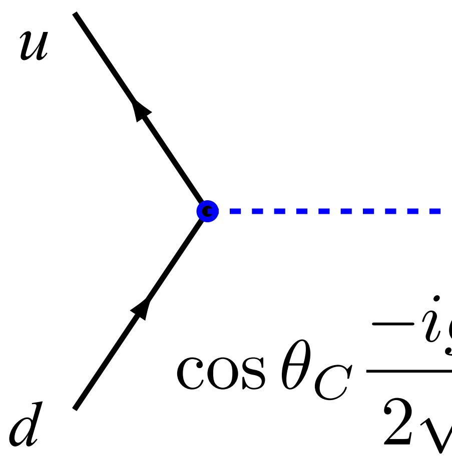




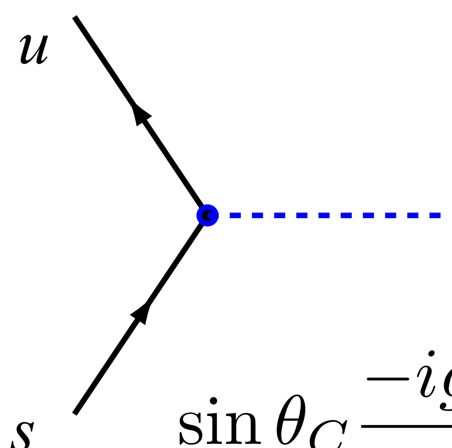
$$\Gamma = \frac{f_\pi^2}{\pi\hbar m_\pi^3}\left(\frac{g_w}{4M_W}\right)^4 m_l^2(m_\pi^2 - m_l^2)^2$$

$$\frac{\Gamma(K^- \to \mu^- + \nu_\mu)}{\Gamma(\pi^- \to \mu^- + \nu_\mu)} = \left(\frac{m_\pi}{m_K}\right)^3 \left(\frac{m_K^2 - m_\mu^2}{m_\pi^2 - m_\mu^2}\right)^2 \sim 18$$

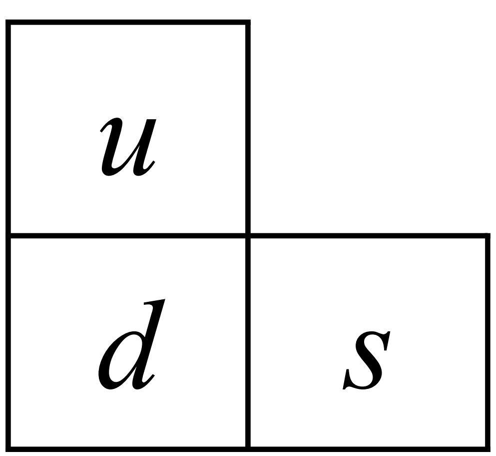
Cabibbo Angle:



$$\cos\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

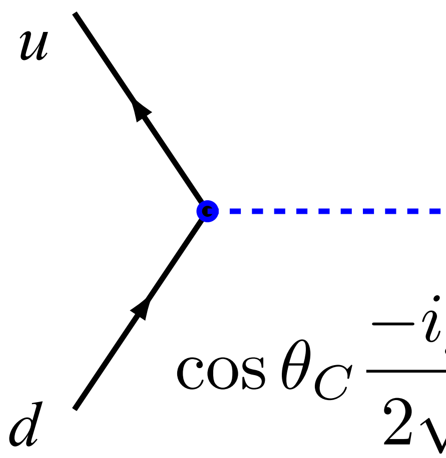


$$\sin\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

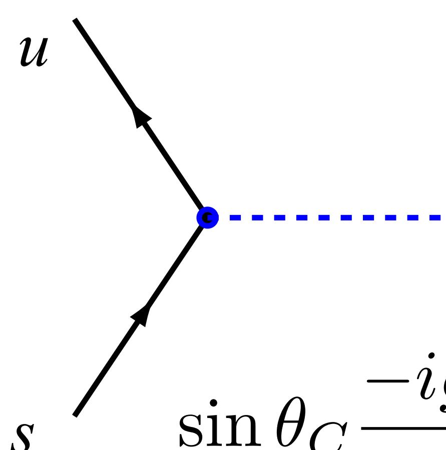


- Experimentally, the ratio is more like 1.3, indicating that something is missing from the above analysis.
- Cabibbo introduced the "Cabibbo angle" θ_C
  - $u \leftrightarrow s$ transitions have a factor sin θ_C in the vertex
  - $u \leftrightarrow d$ transitions have a factor cos θ_C in the vertex
  - Experimentally, θ_C~13.15°
- With this, Cabibbo was able to relate a host of decay of strange and non-strange particles
  - Overall, it shows that $u \leftrightarrow s$ are disfavored while are $u \leftrightarrow d$ are favored

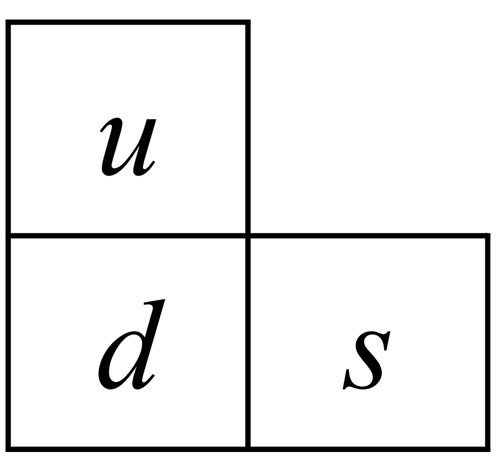
A Problem:

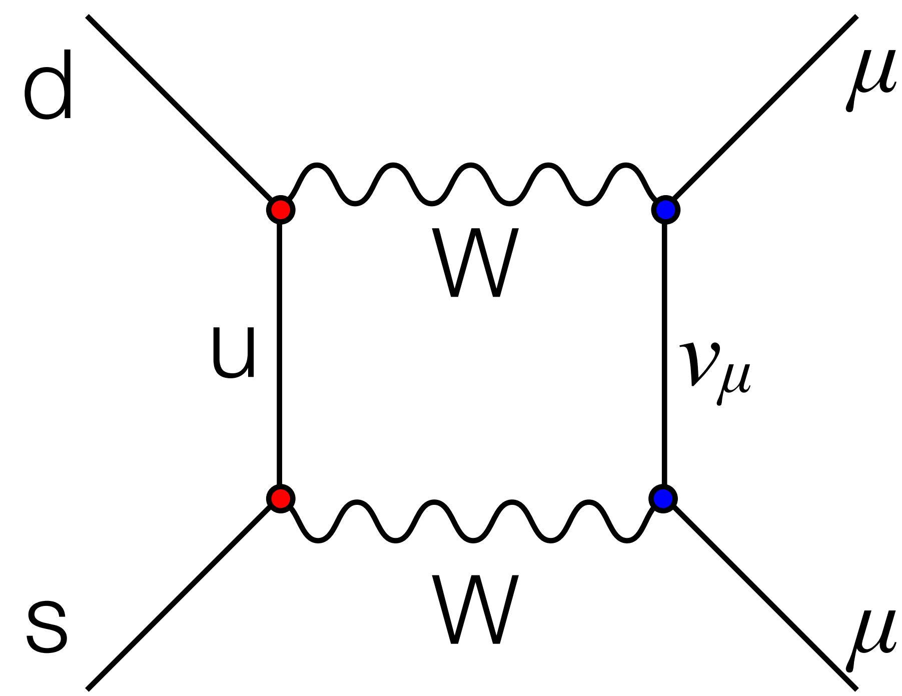


$$\cos\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



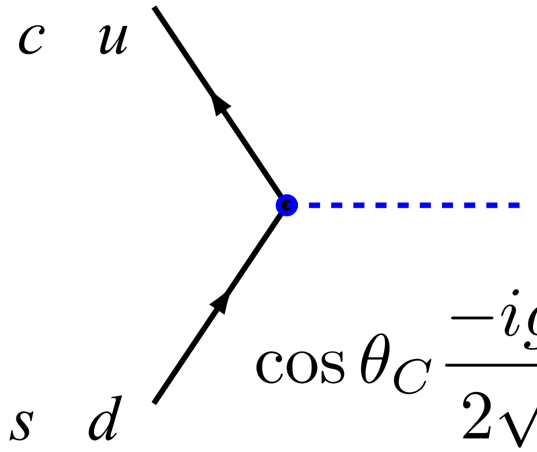
$$\sin\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



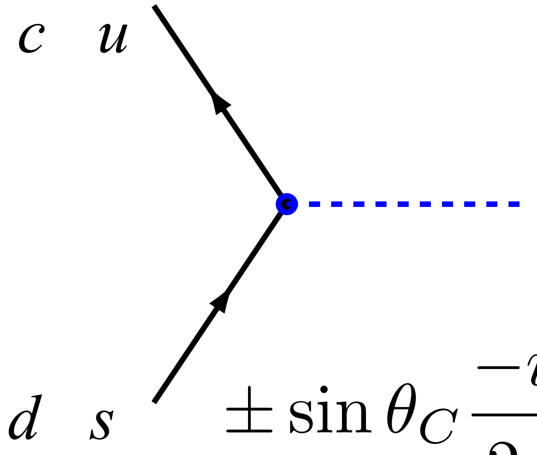


- The above process should occur as $K^0 \to \mu + \mu$
  - But it doesn't (or at least a rate much less than expected even after taking into account the Cabibbo factors)

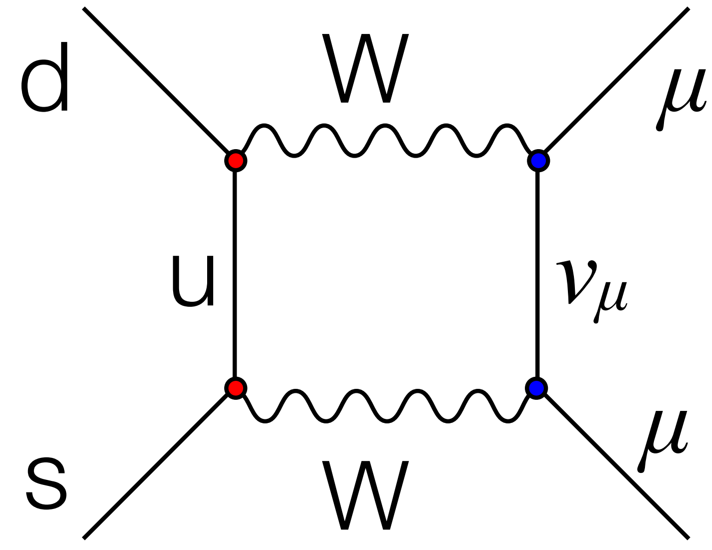
The GIM Mechanism (1970)



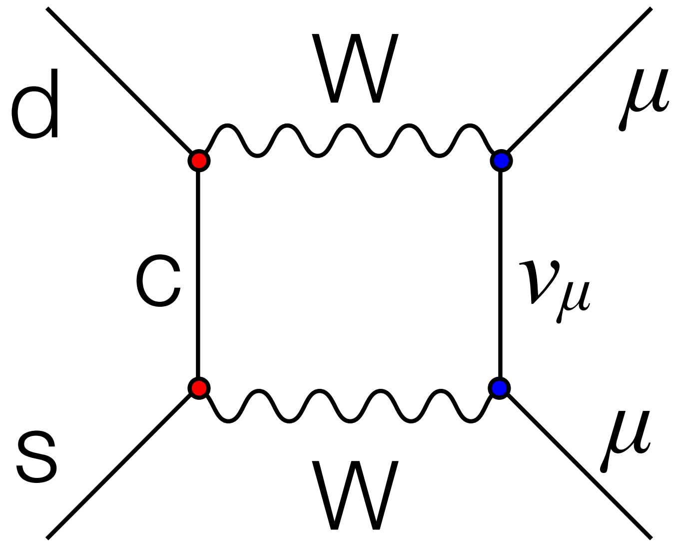
$$\cos\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



$$\pm\sin\theta_C \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



$$A_1 \sim \sin\theta_C \cos\theta_C$$



$$A_2 \sim -\sin\theta_C \cos\theta_C$$

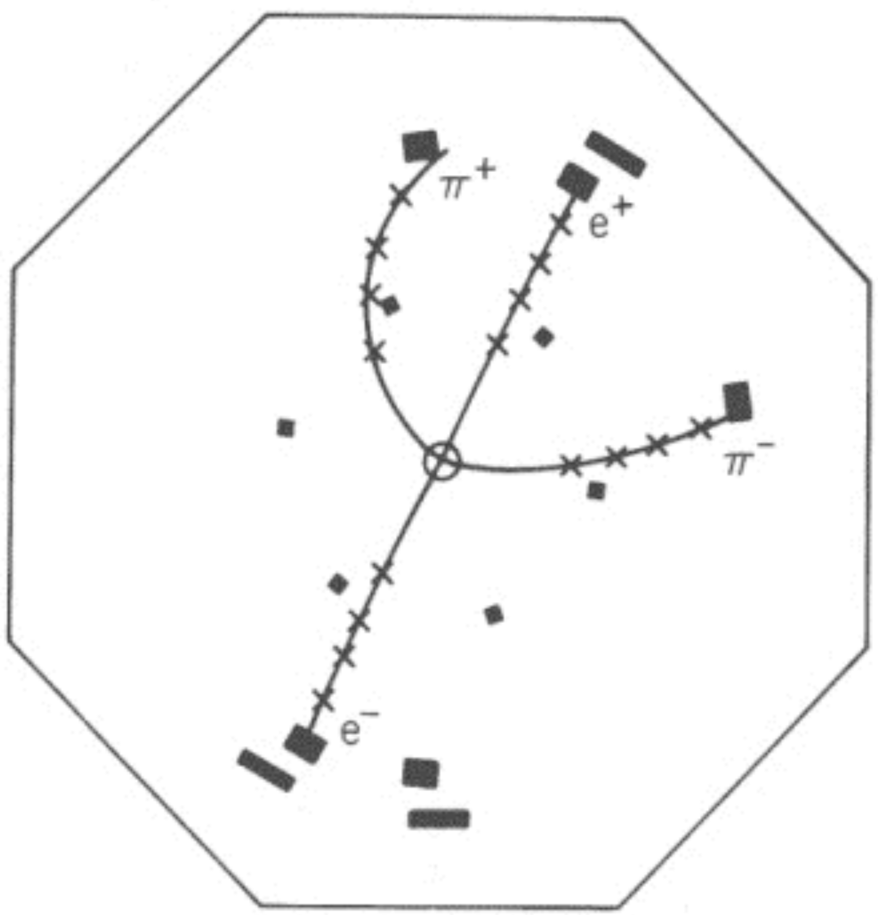
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

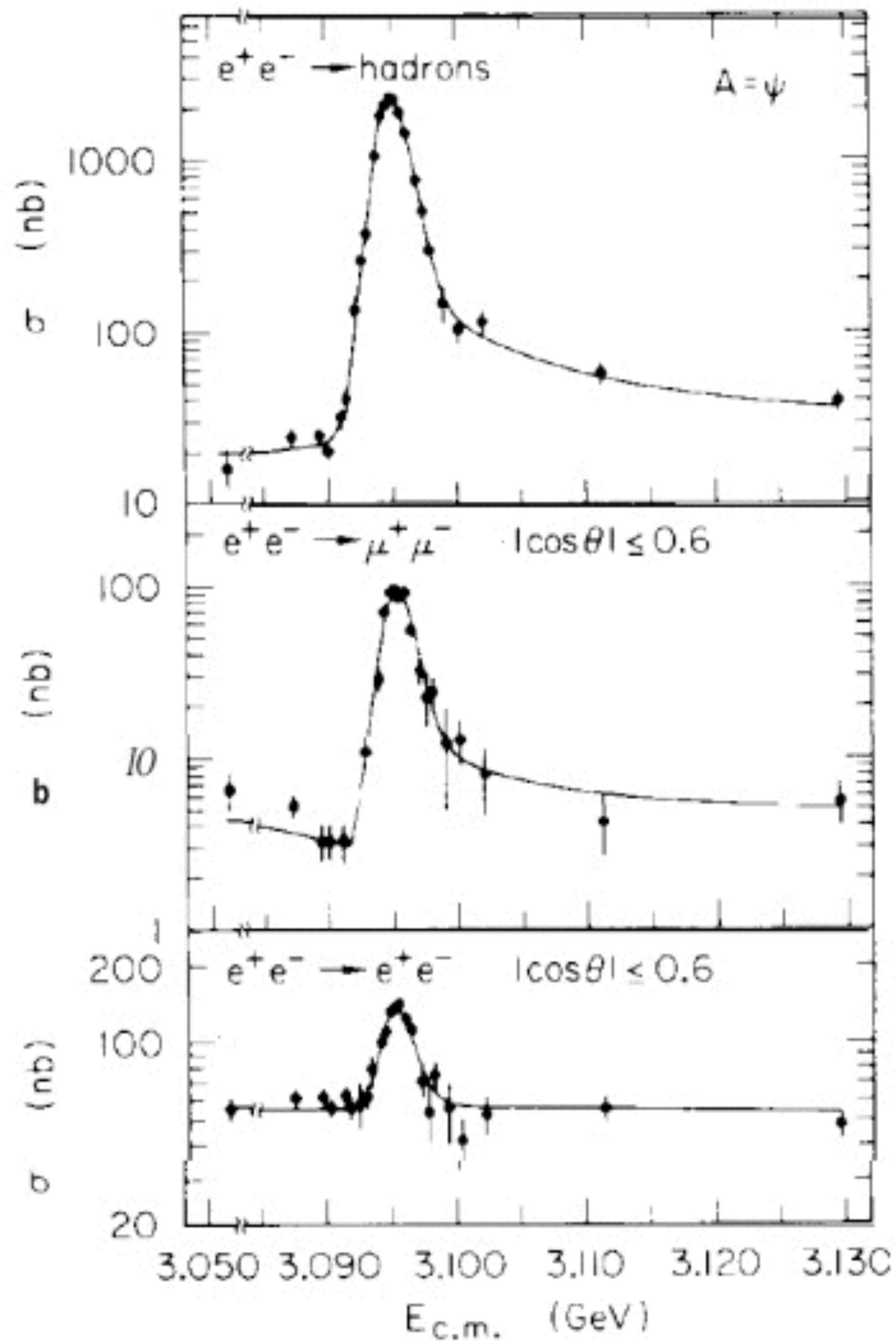
u	c
d	s

- Introduce 4th quark (charm) with coupling -sin θ_C
- Cancels contribution from u quark
- Formalizes idea of “generations”
  - Mass eigenstates “rotated” slightly from “flavour” (or weak) eigenstates

The “November” Revolution

- 1974: Discovery of the J/ψ meson at BNL, SLAC
  - e^+e^- and $q\bar{q}$ collisions produce a $c\bar{c}$ state
  - Confirmation of GIM model





1974 Nobel Prize in Physics

CP Violation and the 3rd generation

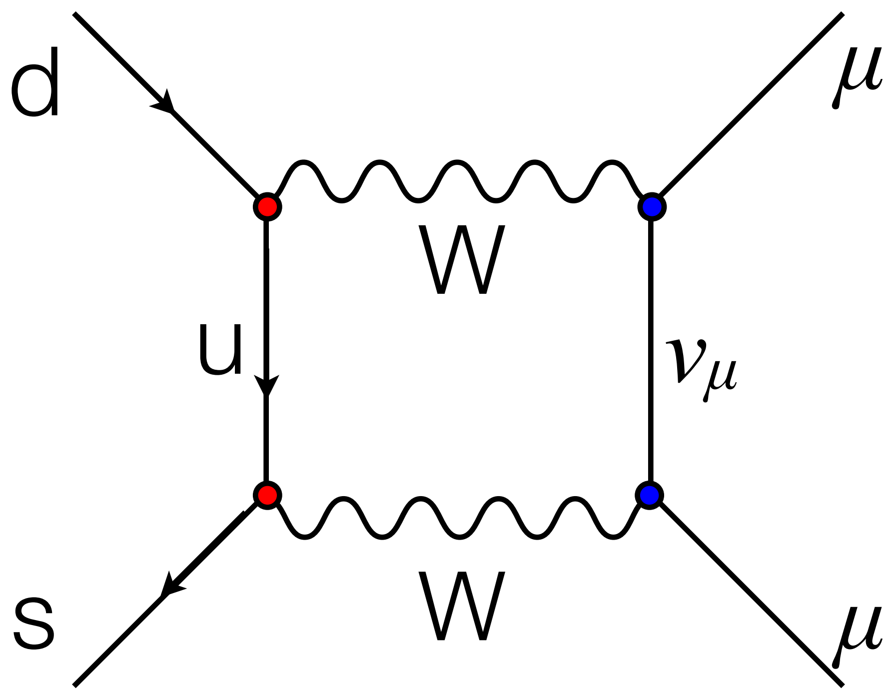
- Prior to the discovery of charm, Kobayashi and Maskawa contemplated CP violation (discovered in 1964)
- One way to introduce CP violation is by having a complex phase
  - This will switch sign from quark ↔ antiquark, changing the amplitude
  - Found no way to introduce a complex phase with 2 generations
  - Concluded that three generations are needed to have complex phase

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Cabibbo matrix incorporated as upper 2x2 part of 3x3 matrix.

$$|U_{CKM}| \sim \begin{pmatrix} 0.9738 & 0.2272 & 0.0040 \\ 0.2271 & 0.9730 & 0.0422 \\ 0.0081 & 0.0416 & 0.9991 \end{pmatrix}$$

How it works:



$$V_{ud}^* \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

$$V_{us} \frac{-ig_w}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)$$

- A factor of V_{ab} is applied for a→b transition:
  - e.g. V_{ud} refers to u → d + W^+
- A factor of V_{ab}^* is applied for b→a transition
  - e.g. V_{ud}^* for d → u + W^-
- (I think the book has it reversed)

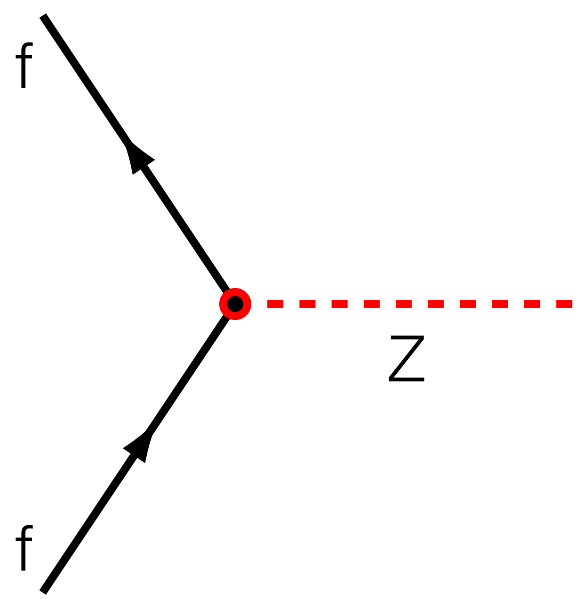
Discovery and Completion of the 3rd Generation

ν_e	ν_μ	ν_τ
e	μ	τ

u	c	t
d	s	b

- First indications of the third generation came from the discovery of the τ in 1975 (Nobel Prize 1995)
- The bottom quark (third generation quark) 1977
- Top quark in 1994
- ν_τ in 2000
- Experiments (BaBar/BELLE) confirm Kobayashi and Maskawa's theory of CP violation
  - Nobel Prize 2008 with Nambu

The Weak Neutral Current



$$\frac{-ig_Z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)$$

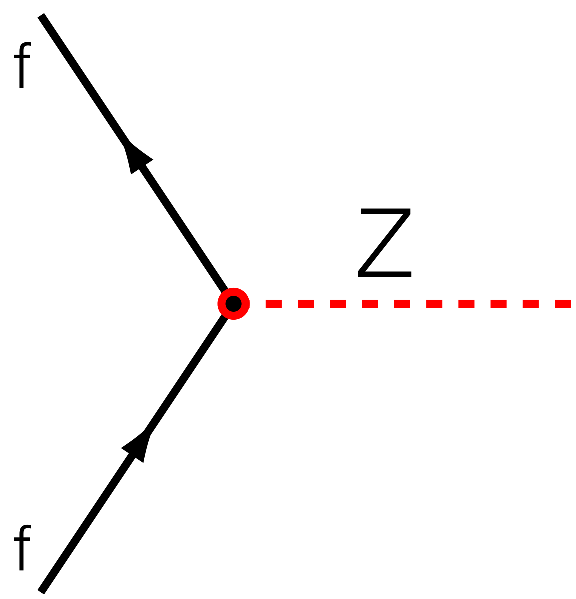
	c_V	c_A
$\nu_e\ \nu_\mu\ \nu_\tau$	1/2	1/2
$e\ \mu\ \tau$	-1/2 + 2 $\sin^2\theta_W$	-1/2
$u\ c\ t$	1/2 - 4/3 $\sin^2\theta_W$	1/2
$d\ s\ b$	-1/2 +2/3 $\sin^2\theta_W$	-1/2

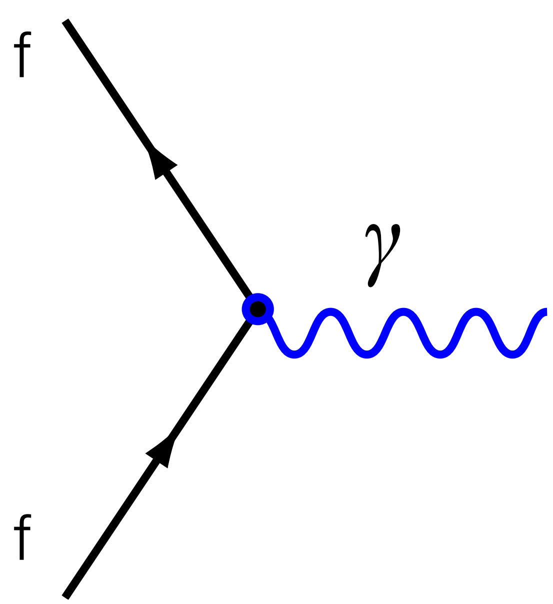
- The weak neutral current is mediated by the Z boson (M_Z=91 GeV/c^2)
- It also shows the parity-violating structure of having both vector and axial-vector couplings
- However, it is a bit more complicated than the case of the W (weak charged current)
  - The vector and axial vector components depend on the type of particle
  - $\sin^2\theta_W = 0.23126 \pm 0.00005$

ν_e	ν_μ	ν_τ
e	μ	τ

u	c	t
d	s	b

Z vs. γ

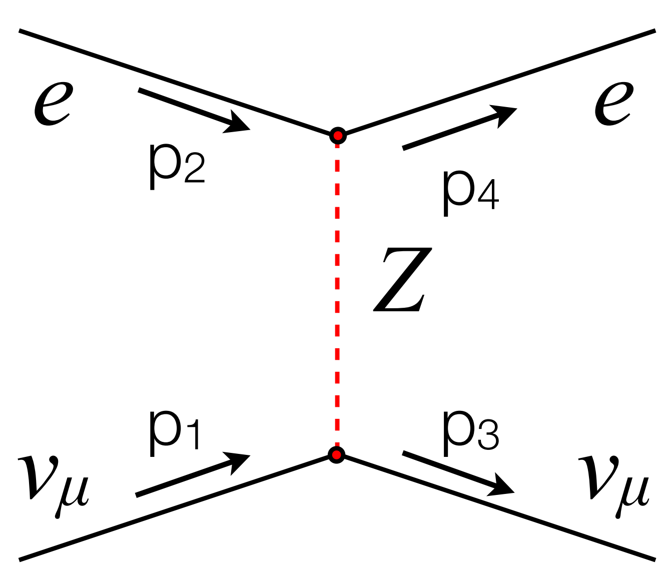




- Note that (almost) every interaction that can occur via the Z can happen via the photon
- At low energies (E << M_Zc^2)

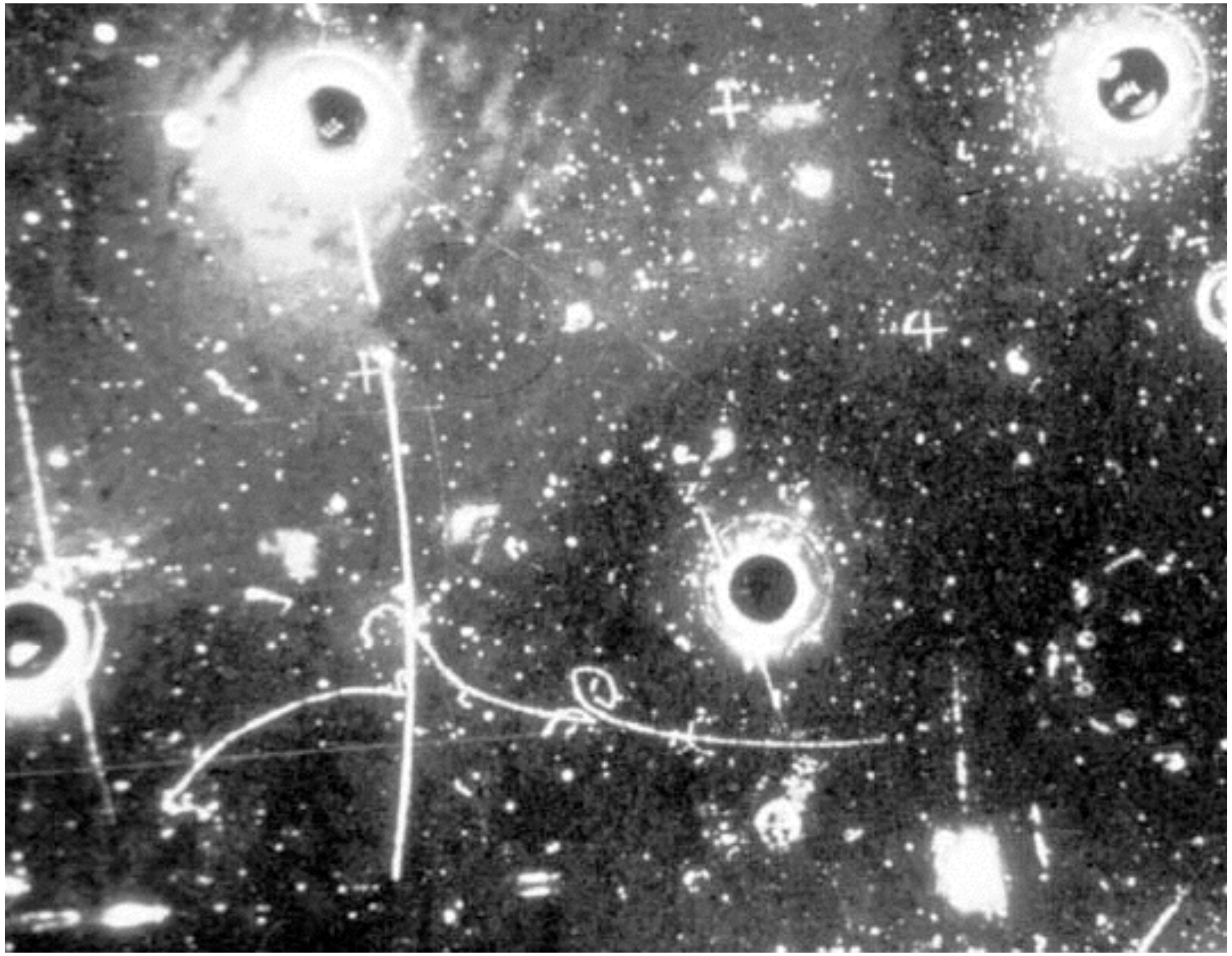
$$\frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_Z^2 c^2)}{q^2 - M_Z^2 c^2} \Rightarrow \frac{i g_{\mu\nu}}{M_Z^2 c^2}$$

  - Z propagator suppressed by Z mass
  - EM interaction masks weak interaction
- The exception is the neutrino
  - No electric charge, no EM interaction

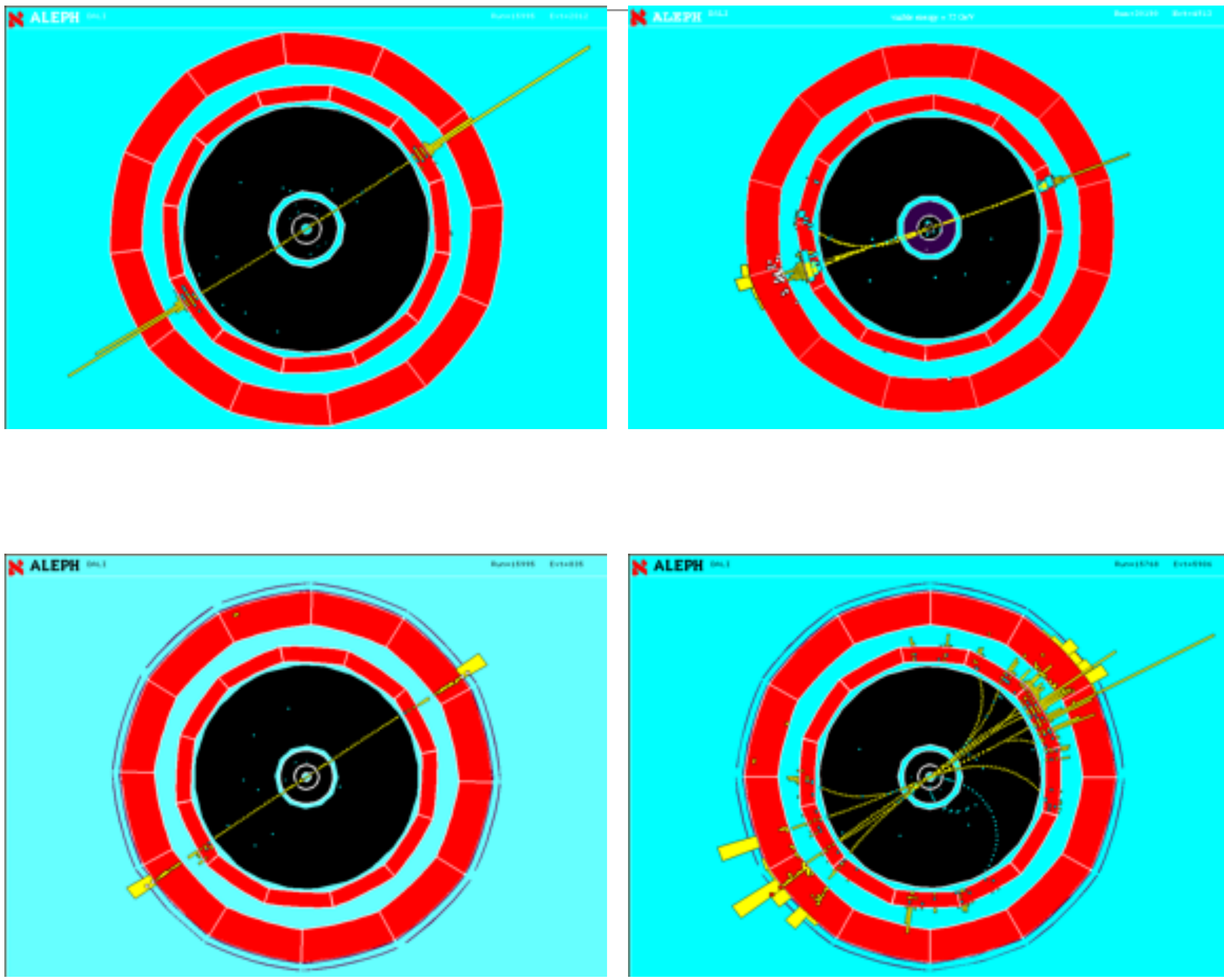


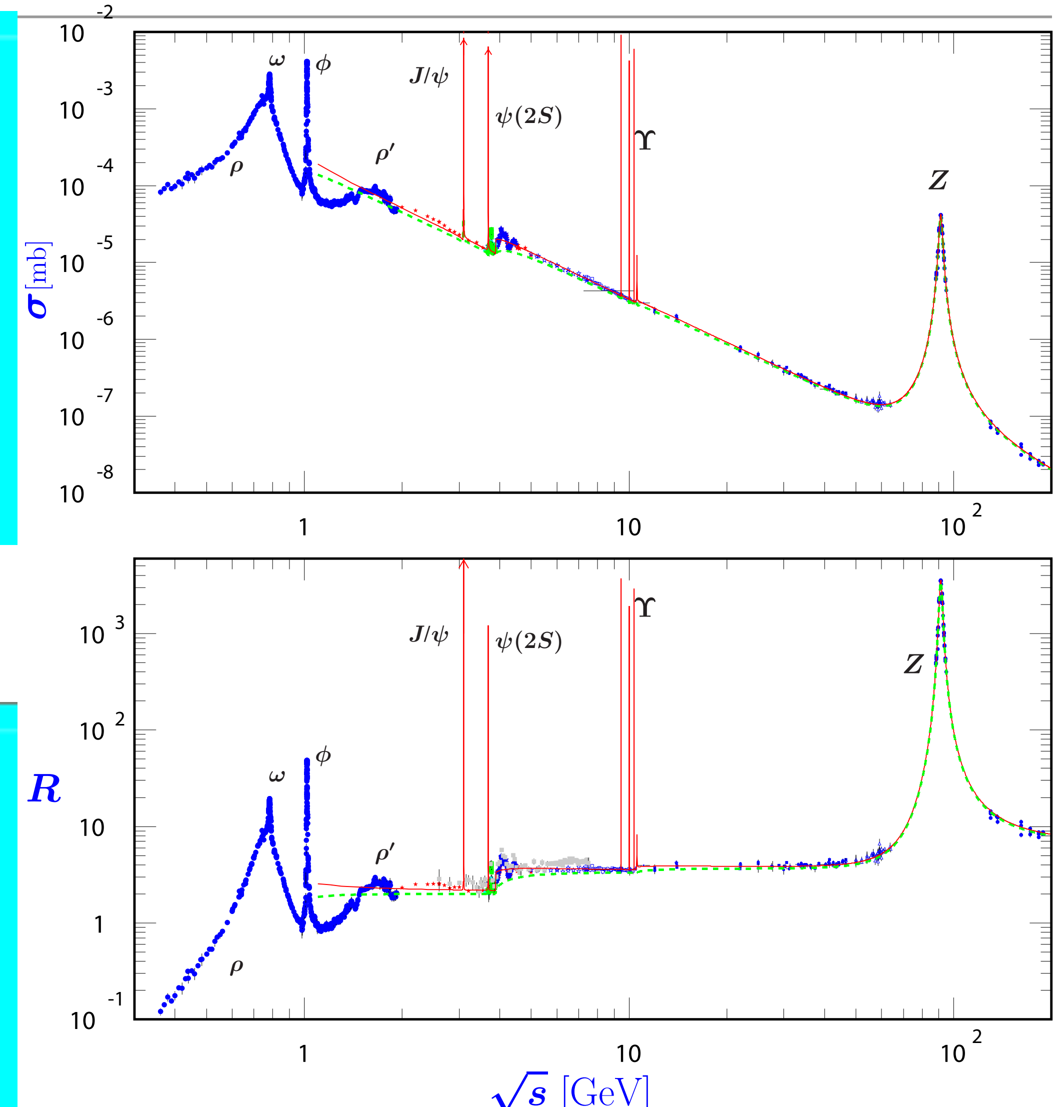
Observation:

- Intense anti neutrino beam produced
  - Scattering of atomic electron out of nowhere observed

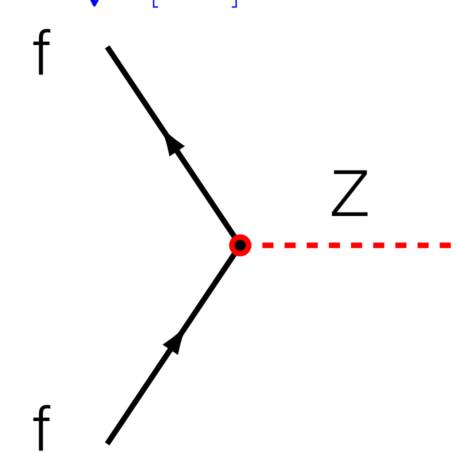


Z production in $e^+ + e^-$ collisions





- f = e, μ, τ, q



Next Time

- Today:
  - “helicity” suppression for weak decays
  - “mixing” for quarks:
    - Cabibbo angle in 2x2 quark model
    - CKM matrix for 3x3
  - weak neutral curent
- Please have a look at 9.7 on electroweak unification