Weak Interactions

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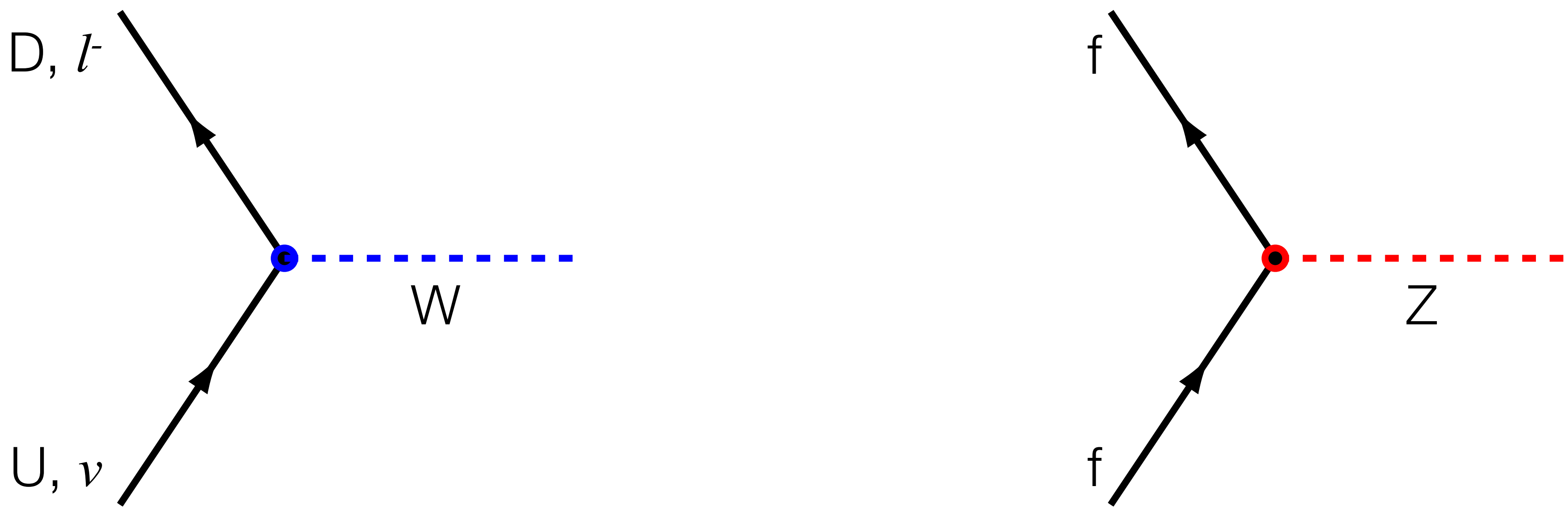
Weak interactions

- In terms of Feynman calculus:
  - good news is everything is pretty much the same as before.
    - We'll have some γ^5 floating around, but we know how to deal
  - bad news is everything is pretty much the same as before.
    - You'll deal first with expression inflation and boil down them down

Weak Interactions:

- The fundamental vertices:

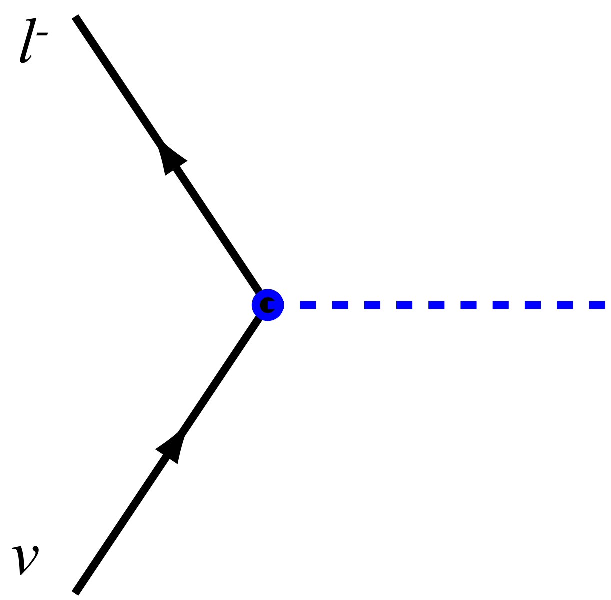


- For the weak charged current:
  - Change a charged lepton (e, μ, τ) into its corresponding neutrino (ν_e, ν_μ, ν_τ)
  - Change a down-type quark (d, s, b) to an up-type quark (u, c, t)
    - Cross-generational transitions governed by CKM matrix
  - Likewise for the corresponding antiparticles.
- For the weak neutral current, particle identity does not change
  - Same particle in and out (like the photon)

The Weak Charged Current:

- Turns out to be easier to deal with than the NC:
- Feynman Rules:

Vertex Factor for Leptons:



$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

W propagator

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2}$$

Weak Charge Current vs. Photon

QED vertex

$$ig_e\gamma^\mu$$

photon propagator

$$\frac{-ig_{\mu\nu}}{q^2}$$

Weak CC vertex

$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

W propagator

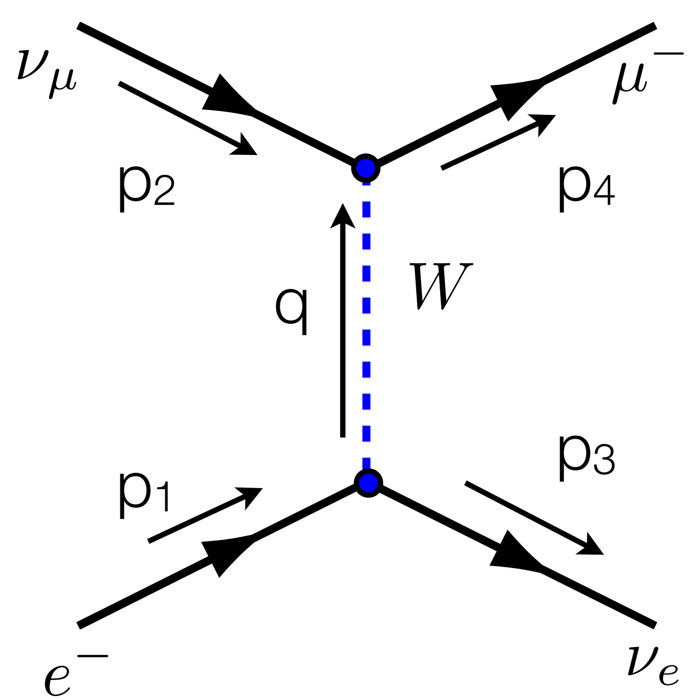
$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2}$$

- Vertex:
  - coupling constant
  - Dirac structure: γ^μ vs. γ^μ - $\gamma^\mu\gamma^5$
    - (Vector, V-A)
  - Charge: W carries one unit of charge
- Propagator:
  - Massive particle, 3 polarizations
  - Both are propagators for a vector particle
  - At low energies $q<M^2_Wc^2$

$$\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2} \Rightarrow \frac{ig_{\mu\nu}}{M_W^2c^2}$$

"Inverse Muon Decay" IMD:

$$\nu_\mu + e^- \to \mu^- + \nu_e$$



- Upper fermion leg

$$\left[\bar{u}_4 \; \frac{ig_W}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)\; u_2\right](2\pi)^4\delta^4(p_2+q-p_4)$$

- Lower Fermion leg

$$\left[\bar{u}_3 \; \frac{ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\; u_1\right]\times(2\pi)^4\delta^4(p_1-q-p_3)$$

- Propagator

$$\int \frac{1}{(2\pi)^4}d^4q \quad \frac{ig_{\mu\nu}}{M_W^2c^2}$$

$$\frac{-ig_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_4\gamma_\mu(1-\gamma^5)u_2\right]\times(2\pi)^4\delta(p_1+p_2-p_3-p_4)$$

$$\mathcal{M} = \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_4\gamma_\mu(1-\gamma^5)u_2\right]$$

Spin Summation:

- We can apply our spin summation relations directly to our amplitude

$$\mathcal{M} = \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_4\gamma_\mu(1-\gamma^5)u_2\right]$$

$$\sum|\mathcal{M}|^2 = \frac{g_W^4}{64M_W^4c^4}$$
$$\mathrm{Tr}\left[\gamma^\mu(1-\gamma^5)(\not{p}_1+m_ec)\gamma^\nu(1-\gamma^5)(\not{p}_3)\right]$$
$$\mathrm{Tr}\left[\gamma_\mu(1-\gamma^5)\ \not{p}_2\gamma_\nu(1-\gamma^5)(\not{p}_4+m_\mu c)\right]$$

- How does it come about?

$$\mathcal{M}\mathcal{M}^* = \left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_3\gamma^\nu(1-\gamma^5)u_1\right]^*\left[\bar{u}_4\gamma_\mu(1-\gamma^5)u_2\right]\left[\bar{u}_4\gamma_\nu(1-\gamma^5)u_2\right]^*$$

- Use our spin summation relation

$$\sum_{\text{a, b spins}}\left[\bar{u}(a)\Gamma_1u(b)\right]\left[\bar{u}(a)\Gamma_2u(b)\right]^* = \mathrm{Tr}\left[\Gamma_1(\not{p}_b+m_bc)\bar{\Gamma}_2(\not{p}_a+m_ac)\right]$$

  - where a=3, b=1, $\Gamma_1 = \gamma^\mu(1-\gamma^5)$ $\Gamma_2 = \gamma^\nu(1-\gamma^5)$
  - where a=4, b=2, $\Gamma_1 = \gamma_\mu(1-\gamma^5)$ $\Gamma_2 = \gamma_\nu(1-\gamma^5)$

Evaluate Traces:

$$\sum |\mathcal{M}|^2 = \frac{g_W^4}{64 M_W^4 c^4}$$
$$\mathrm{Tr}\left[\gamma^\mu (1-\gamma^5)(\not{p}_1 + m_e c)\gamma^\nu (1-\gamma^5)(\not{p}_3)\right]$$
$$\mathrm{Tr}\left[\gamma_\mu (1-\gamma^5)\ \not{p}_2 \gamma_\nu (1-\gamma^5)(\not{p}_4 + m_\mu c)\right]$$

- First Trace: (Tr[odd number of γ matrices] = 0)

$$\mathrm{Tr}\left[\gamma^\mu (1-\gamma^5)(\not{p}_1 + m_e c)\gamma^\nu (1-\gamma^5)(\not{p}_3)\right] = \mathrm{Tr}\left[\gamma^\mu (1-\gamma^5)\ \not{p}_1 \gamma^\nu (1-\gamma^5)\ \not{p}_3\right]$$

- Breaking out the terms:

$$\mathrm{Tr}\left[\gamma^\mu (1-\gamma^5)\ \not{p}_1 \gamma^\nu (1-\gamma^5)\ \not{p}_3\right] = \mathrm{Tr}\left[\gamma^\mu\ \not{p}_1 \gamma^\nu\ \not{p}_3\right] - \mathrm{Tr}\left[\gamma^\mu \gamma^5\ \not{p}_1 \gamma^\nu\ \not{p}_3\right]$$
$$- \mathrm{Tr}\left[\gamma^\mu\ \not{p}_1 \gamma^\nu \gamma^5\ \not{p}_3\right] + \mathrm{Tr}\left[\gamma^\mu \gamma^5\ \not{p}_1 \gamma^\nu \gamma^5\ \not{p}_3\right]$$

- One by one:
  - $\mathrm{Tr}\left[\gamma^\mu\ \not{p}_1 \gamma^\nu\ \not{p}_3\right] = 4 \times \left[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu - g^{\mu\nu}(p_1 \cdot p_3)\right]$
  - $\mathrm{Tr}\left[\gamma^\mu \gamma^5\ \not{p}_1 \gamma^\nu\ \not{p}_3\right] = \mathrm{Tr}\left[\gamma^5\ \not{p}_1 \gamma^\nu\ \not{p}_3 \gamma^\mu\right] = 4 \times i\epsilon^{\alpha\nu\beta\mu} p_{1\alpha}\ p_{3\beta}$
  - $\mathrm{Tr}\left[\gamma^\mu\ \not{p}_1 \gamma^\nu \gamma^5\ \not{p}_3\right] = \mathrm{Tr}\left[\not{p}_3 \gamma^\mu\ \not{p}_1 \gamma^\nu \gamma^5\right] = \mathrm{Tr}\left[\gamma^5\ \not{p}_3 \gamma^\mu\ \not{p}_1 \gamma^\nu\right] = 4 \times i\epsilon^{\alpha\mu\beta\nu} p_{3\alpha} p_{1\beta}$
  - $\mathrm{Tr}\left[\gamma^\mu \gamma^5\ \not{p}_1 \gamma^\nu \gamma^5\ \not{p}_3\right] = \mathrm{Tr}\left[\gamma^\mu \gamma^5 \gamma^5\ \not{p}_1 \gamma^\nu\ \not{p}_3\right] = \mathrm{Tr}\left[\gamma^\mu\ \not{p}_1 \gamma^\nu\ \not{p}_3\right]$

The antisymmetric tensor:

$$\epsilon^{\mu\nu\lambda\sigma} = \begin{array}{l} -1 \text{ if } \mu\nu\lambda\sigma \text{ is an even permutation of } 0123 \\ +1 \text{ if } \mu\nu\lambda\sigma \text{ is an odd permutation of } 0123 \\ \quad 0 \ \text{ otherwise; any of the indices are the same} \end{array}$$

- Contractions of the tensor with itself?
- Remember that each repeated index gets “summed” over from 0-3

$$\epsilon^{\mu\nu\lambda\sigma}\epsilon_{\mu\nu\lambda\tau} = -6 \times \delta^{\sigma}_{\tau}$$

$$\epsilon^{\mu\nu\lambda\sigma}\epsilon_{\mu\nu\rho\tau} = -2 \times (\delta^{\lambda}_{\rho}\delta^{\sigma}_{\tau} - \delta^{\lambda}_{\tau}\delta^{\sigma}_{\rho})$$

- Note also that the contraction with any object symmetric in two indexes is 0

$$a_{\mu}a_{\nu}\epsilon^{\mu\nu\lambda\sigma} = 0 \qquad g_{\mu\nu}\epsilon^{\mu\nu\lambda\sigma} = 0$$

Evaluating traces:

- Result:

$$\mathrm{Tr}\left[\gamma^\mu(1-\gamma^5)(\not{p}_1+m_e c)\gamma^\nu(1-\gamma^5)(\not{p}_3)\right] = 8\left[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu - g^{\mu\nu}(p_1\cdot p_3) - i\epsilon^{\alpha\nu\beta\mu}p_{1\alpha}p_{3\beta}\right]$$

- We have a very similar situation for:

$$\mathrm{Tr}\left[\gamma_\mu(1-\gamma^5)\ \not{p}_2\gamma_\nu(1-\gamma^5)(\not{p}_4+m_\mu c)\right]$$

  - $\gamma^\mu \Rightarrow \gamma_\mu \quad \gamma^\nu \Rightarrow \gamma_\nu \quad p_1 \Rightarrow p_2 \quad p_3 \Rightarrow p_4$

$$8\left[p_{2\mu}p_{4\nu} + p_{2\nu}p_{4\mu} - g_{\mu\nu}(p_2\cdot p_4) - i\epsilon_{\rho\nu\sigma\mu}p^{2\rho}p^{4\sigma}\right]$$

- Now combine the terms:

$$\begin{aligned}
&64\times[\\
&\quad (p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_3)(p_2\cdot p_4) - i\epsilon_{\rho\nu\sigma\mu}p_1^\mu p_3^\nu p_2^\rho p_4^\sigma\\
&+ (p_1\cdot p_4)(p_3\cdot p_2) + (p_1\cdot p_2)(p_3\cdot p_4) - (p_1\cdot p_3)(p_2\cdot p_4) - i\epsilon_{\rho\nu\sigma\mu}p_1^\nu p_3^\mu p_2^\rho p_4^\sigma\\
&- (p_1\cdot p_3)(p_2\cdot p_4) - (p_1\cdot p_3)(p_2\cdot p_4) + 4(p_1\cdot p_3)(p_2\cdot p_4) - 0\\
&- i\epsilon^{\alpha\nu\beta\mu}(p_{2\mu}p_{4\nu} + p_{2\nu}p_{4\mu} + g_{\mu\nu}) + 2(\delta^\alpha_\rho\delta^\beta_\sigma - \delta^\alpha_\sigma\delta^\beta_\rho)p_{1\alpha}p_{3\beta}p_2^\rho p_4^\sigma]
\end{aligned}$$

End of Amplitude Calculation:

$$
\begin{aligned}
&64 \times [\\
&\quad \underline{(p_1 \cdot p_2)(p_3 \cdot p_4)} + \underline{(p_1 \cdot p_4)(p_2 \cdot p_3)} - \underline{(p_1 \cdot p_3)(p_2 \cdot p_4)} - i\epsilon_{\rho\nu\sigma\mu} p_1^\mu p_3^\nu p_2^\rho p_4^\sigma \\
&+ \underline{(p_1 \cdot p_4)(p_3 \cdot p_2)} + \underline{(p_1 \cdot p_2)(p_3 \cdot p_4)} - \underline{(p_1 \cdot p_3)(p_2 \cdot p_4)} - i\epsilon_{\rho\nu\sigma\mu} p_1^\nu p_3^\mu p_2^\rho p_4^\sigma \\
&- \underline{(p_1 \cdot p_3)(p_2 \cdot p_4)} - \underline{(p_1 \cdot p_3)(p_2 \cdot p_4)} + 4\underline{(p_1 \cdot p_3)(p_2 \cdot p_4)} - 0 \\
&- i\epsilon^{\alpha\nu\beta\mu}(p_{2\mu}p_{4\nu} + p_{2\nu}p_{4\mu} + g_{\mu\nu}) + 2(\delta_\rho^\alpha \delta_\sigma^\beta - \delta_\sigma^\alpha \delta_\rho^\beta) p_{1\alpha} p_{3\beta} p_2^\rho p_4^\sigma]
\end{aligned}
$$

$$
\begin{aligned}
64 \times [&\underline{2(p_1 \cdot p_2)(p_3 \cdot p_4)} + (-4+4)\underline{(p_1 \cdot p_3)(p_2 \cdot p_4)} + \\
&\underline{2(p_1 \cdot p_4)(p_2 \cdot p_3)} + 2(p_{1\rho}p_{3\sigma} - p_{1\sigma}p_{3\rho})p_2^\rho p_4^\sigma]
\end{aligned}
$$

$$\boxed{256 \times (p_1 \cdot p_2)(p_3 \cdot p_4)}$$

$$\langle |\mathcal{M}|^2 \rangle = \frac{g_W^4}{64 M_W^4 c^4}$$

$$\boxed{\begin{aligned}&\mathrm{Tr}\left[\gamma^\mu (1-\gamma^5)(\not{p}_1 + m_e c)\gamma^\nu (1-\gamma^5)(\not{p}_3)\right] \\ &\mathrm{Tr}\left[\gamma_\mu (1-\gamma^5)\ \not{p}_2 \gamma_\nu (1-\gamma^5)(\not{p}_4 + m_\mu c)\right]\end{aligned}}$$

$$\sum |\mathcal{M}|^2 = 4\frac{g_W^4}{M_W^4 c^4}(p_1 \cdot p_2)(p_3 \cdot p_4)$$

- Need to divide by 2 in order to average over electron (only one neutrino state)

$$\langle |\mathcal{M}|^2 \rangle = 2\frac{g_W^4}{M_W^4 c^4}(p_1 \cdot p_2)(p_3 \cdot p_4)$$

Amplitude in the CM

$$\langle|\mathcal{M}|^2\rangle = 2\frac{g_W^4}{M_W^4 c^4}(p_1 \cdot p_2)(p_3 \cdot p_4) \qquad \frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\langle|\mathcal{M}|^2\rangle}{(E_1+E_2)^2}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}$$

- If we neglect the electron mass, we have massless particles in the initial state

$$p_1 = (E/c,\ \mathbf{p}), \quad p_2 = (E/c,\ -\mathbf{p}) \qquad p_1 \cdot p_2 = (E/c)^2 + |\mathbf{p}|^2 = 2(E/c)^2$$

- What about $p_3 \cdot p_4$? You can grind it out or use a trick:

$$p_1 + p_2 = p_3 + p_4 \qquad p_1 + p_2 = p_3 + p_4 \quad \Rightarrow \quad (p_1+p_2)^2 = (p_3+p_4)^2$$

$$(p_1+p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2$$

$$(p_3+p_4)^2 = p_3^2 + p_4^2 + 2p_3 \cdot p_4$$

- p_1, p_2, p_3 correspond to “massless” particles (electron, neutrino, neutrino)

$$p_3 \cdot p_4 = p_1 \cdot p_2 - m_\mu^2 c^2/2$$

$$\langle|\mathcal{M}|^2\rangle = 2\frac{g_W^4}{M_W^4 c^4} 2(E/c)^2 \left[2(E/c)^2 - m_\mu^2 c^2/2\right]$$

$$\langle|\mathcal{M}|^2\rangle = 8\frac{g_W^4}{M_W^4 c^4}(E/c)^4 \left[1 - \left(\frac{m_\mu c^2}{2E}\right)^2\right]$$

The Cross Sections

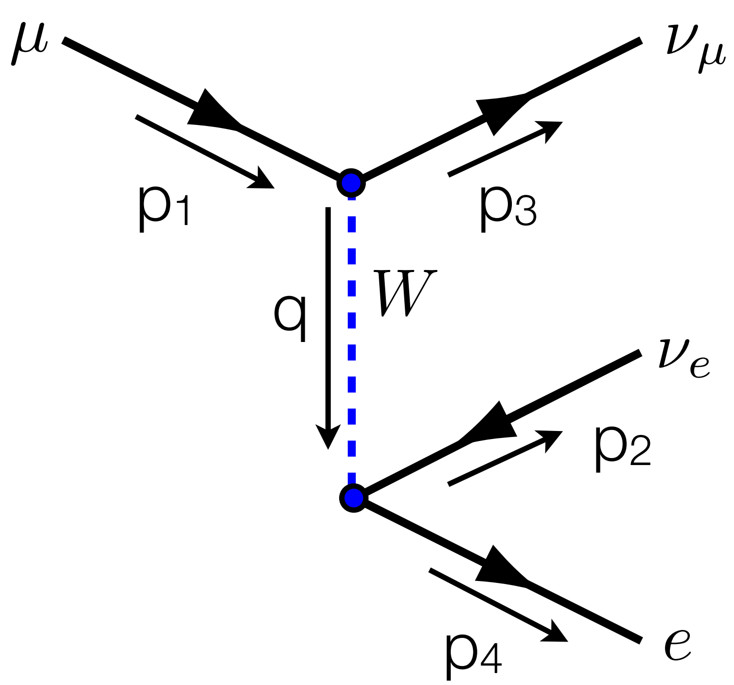
$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\langle|\mathcal{M}|^2\rangle}{(E_1+E_2)^2}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|} \qquad \langle|\mathcal{M}|^2\rangle = 8\frac{g_W^4}{M_W^4 c^4}(E/c)^4\left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left[\frac{\hbar c g_W^2 E}{4\pi(M_W c^2)^2}\right]^2\left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]^2$$

- Integration of solid angle gives just a factor of 4π

$$\sigma = \frac{1}{8\pi}\left[\frac{\hbar c g_W^2 E}{(M_W c^2)^2}\right]^2\left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]^2$$

Muon Decay:



- Upper fermion leg

$$[\bar{u}_3 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) u_1] \quad (2\pi)^4 \delta(p_1 - q - p_3)$$

- Lower Fermion leg

$$[\bar{u}_4 \frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) v_2] \quad (2\pi)^4 \delta(q - p_2 - p_4)$$

- Propagator

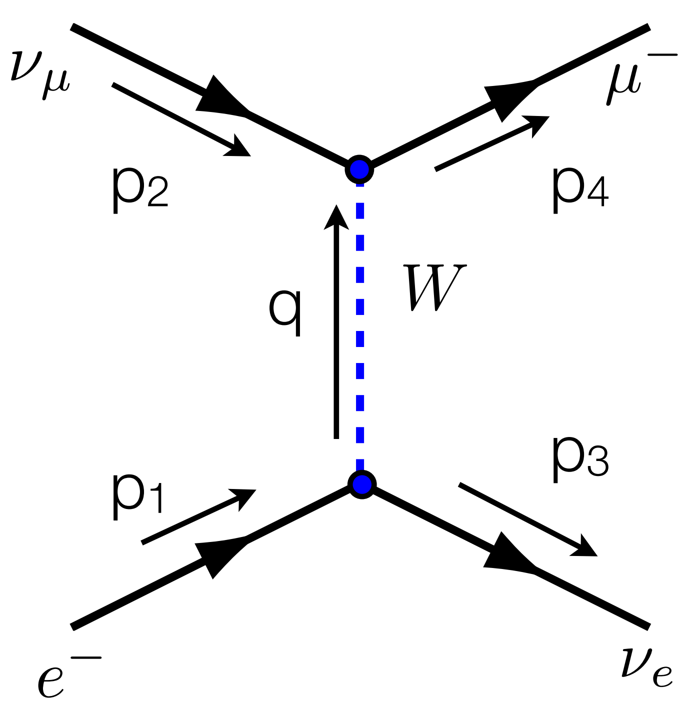
$$\int \frac{1}{(2\pi)^4} d^4 q \quad \frac{ig_{\mu\nu}}{M_W^2 c^2}$$

$$\frac{-ig_W^2}{8M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) u_1\right] \left[\bar{u}_4 \gamma_\mu (1-\gamma^5) v_2\right] \times (2\pi)^4 \delta(p_1 - p_2 - p_3 - p_4)$$

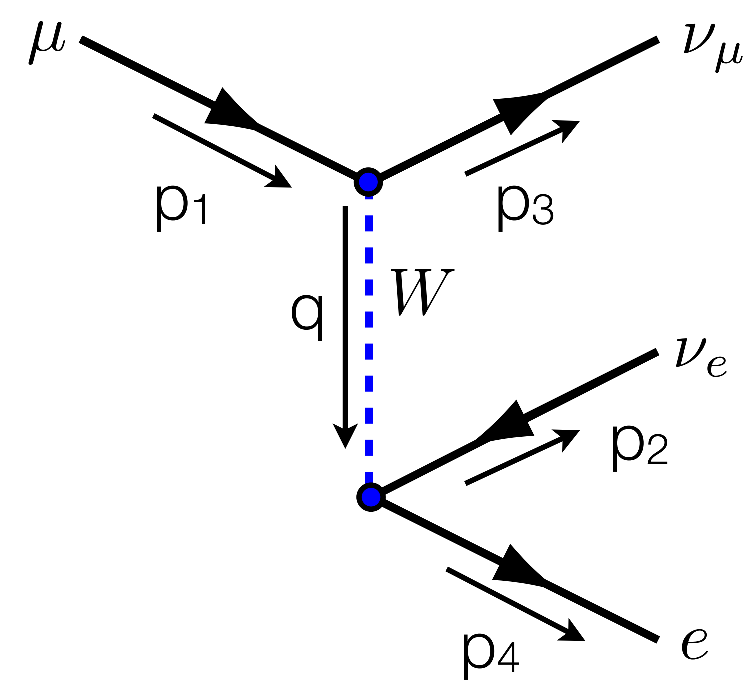
$$\frac{g_W^2}{8M_W^2 c^2} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) u_1\right] \left[\bar{u}_4 \gamma_\mu (1-\gamma^5) v_2\right]$$

Comparison

$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$



$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$



$$\mathcal{M} = \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_4\gamma_\mu(1-\gamma^5)u_2\right] \qquad \frac{g_W^2}{8M_W^2c^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_4\gamma_\mu(1-\gamma^5)v_2\right]$$

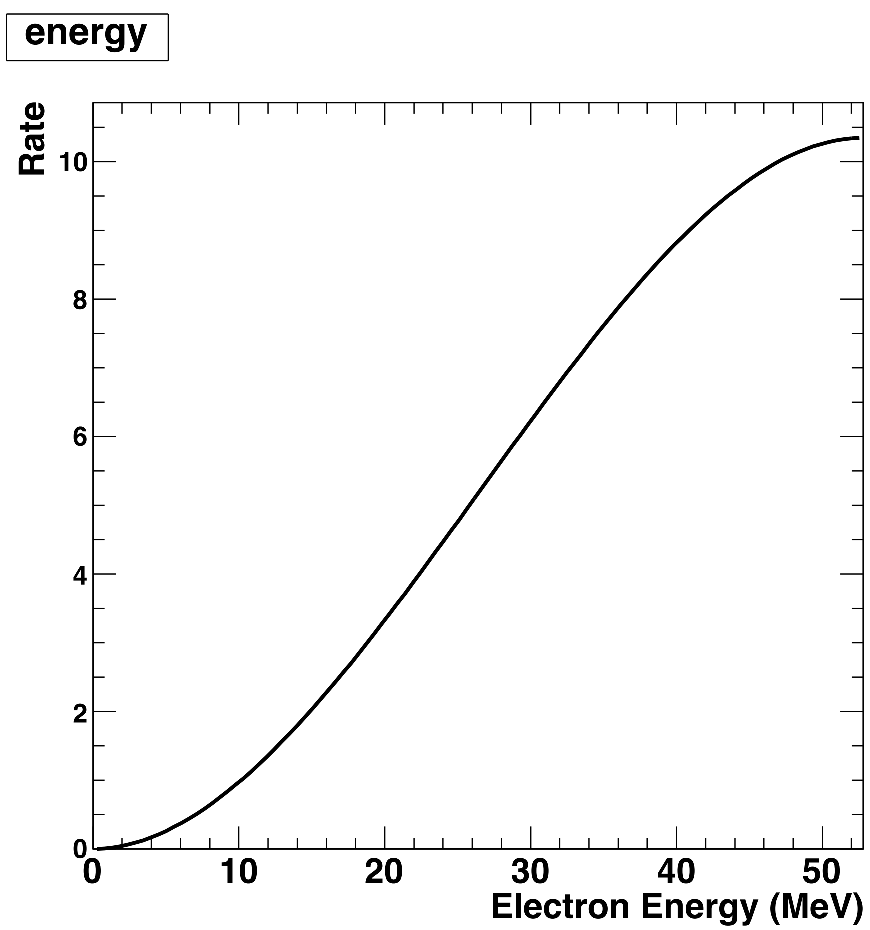
- Note that the expressions are nearly identical

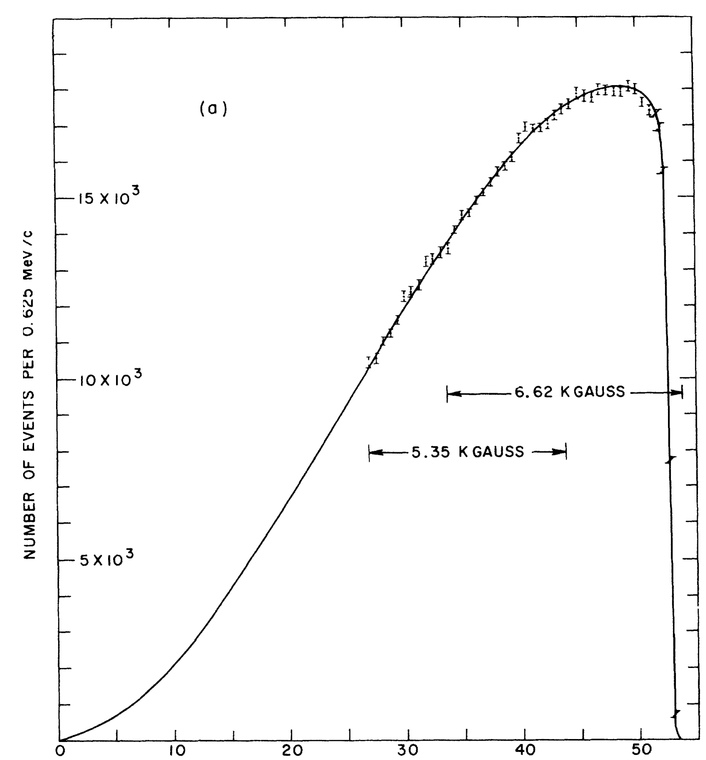
$$\langle|\mathcal{M}|^2\rangle = 2\frac{g_W^4}{M_W^4c^4}(p_1\cdot p_2)(p_3\cdot p_4) \qquad \langle|\mathcal{M}|^2\rangle = 2\frac{g_W^4}{M_W^4c^4}(p_1\cdot p_2)(p_3\cdot p_4)$$

Energy spectrum in decay

$$d\Gamma = \frac{\langle|\mathcal{M}|^2\rangle}{2\hbar m_\mu}\;\frac{1}{(2\pi)^3}\frac{d^3\mathbf{p}_2}{2|\mathbf{p}_2|}\;\frac{1}{(2\pi)^3}\frac{d^3\mathbf{p}_3}{2|\mathbf{p}_3|}\;\frac{1}{(2\pi)^3}\frac{d^3\mathbf{p}_4}{2|\mathbf{p}_4|}(2\pi)^4\delta^4(p_1-p_2-p_3-p_4)$$

$$\frac{d\Gamma}{dE} = \left(\frac{g_W}{M_W c}\right)^4 \frac{m_\mu^2 E^2}{2\hbar(4\pi)^3}\left(1-\frac{4E}{3m_\mu c^2}\right)$$





$$\tau = \Gamma^{-1} = \left(\frac{M_W}{g_W m_\mu}\right)^4 \frac{12\hbar(8\pi)^3}{m_\mu c^2}$$

$$\tau_\mu = 2.197\times 10^{-6}\mathrm{sec}$$

Measuring IMD

- Intense neutrino beam produced by FNAL proton accelerator complex (800 GeV Protons)
- p + A $\rightarrow \pi^+ + X$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

The ratio ($\mathscr{S}$) of the measured to predicted cross section for IMD is $\mathscr{S}=\sigma_{meas}/\sigma_{pred}=0.988\pm0.071$(stat) ±0.023(syst). Since the scalar coupling of the weak-

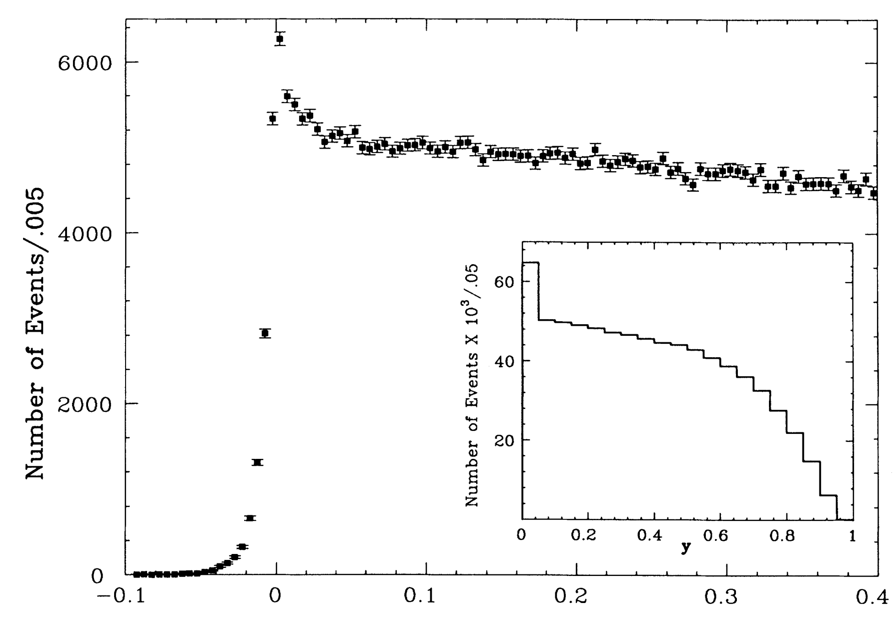
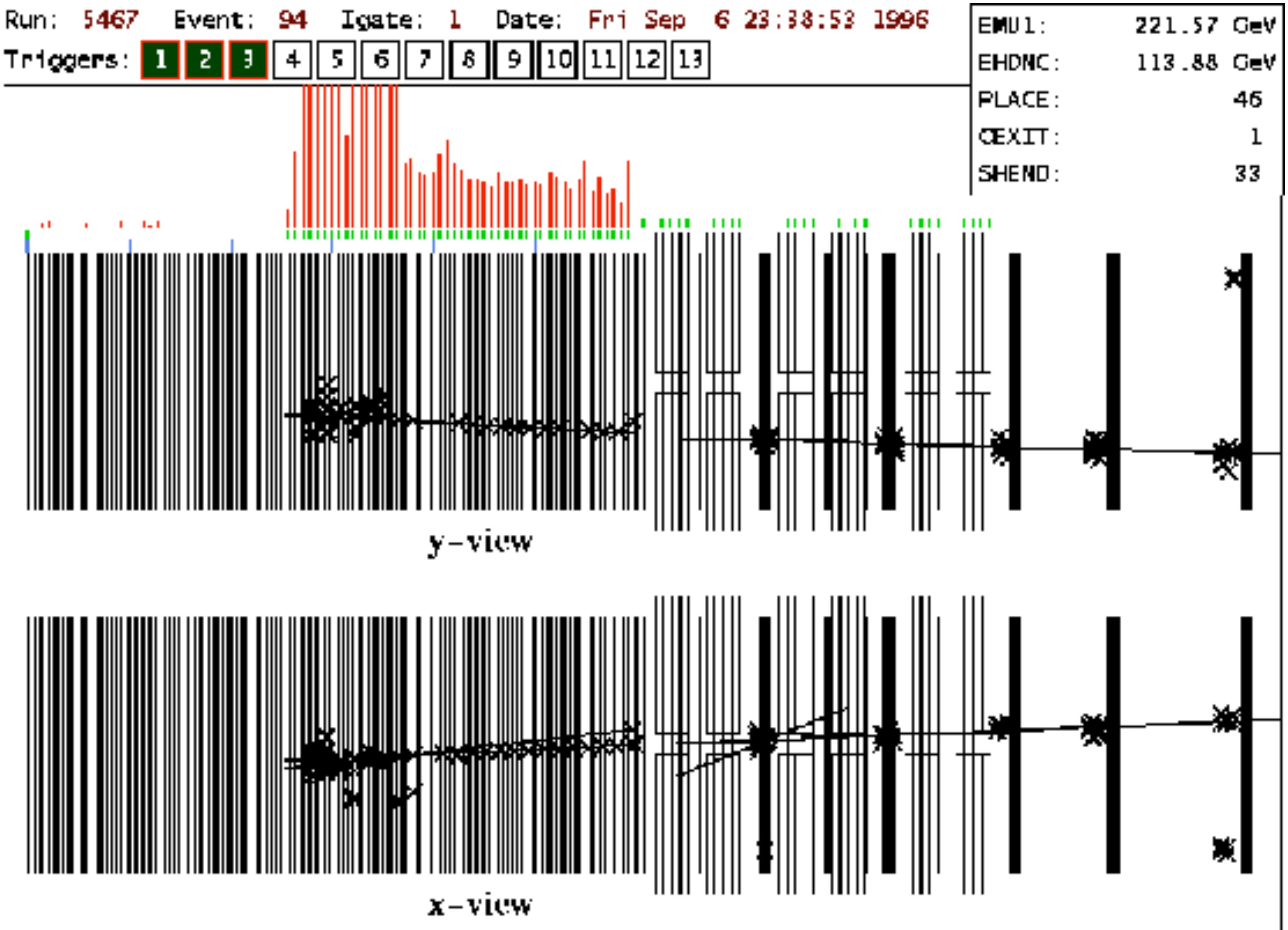


FIG. 1. Distribution of the variable $y=E_{HAD}/E_\nu$, $-0.1\le y\le 0.05$, for ν_μ-induced CC events. The calorimetric resolution smearing due to “muon-tail subtraction” in the shower region (Ref. 6) causes events with $E_{HAD}<0$ (and hence $y<0$). Inset: y distribution in the entire kinematical range $0\le y\le 1$, where events with $y<0$ are presented in the $y=0$ bin.



Run: 5467 Event: 94 Igate: 1 Date: Fri Sep 6 23:38:53 1996
Triggers: 1 2 3 4 5 6 7 8 9 10 11 12 13
EMU1: 221.57 GeV
EHDNC: 113.88 GeV
PLACE: 46
CEXIT: 1
SHEND: 33
y-view
x-view