Lecture 17: QED experiments

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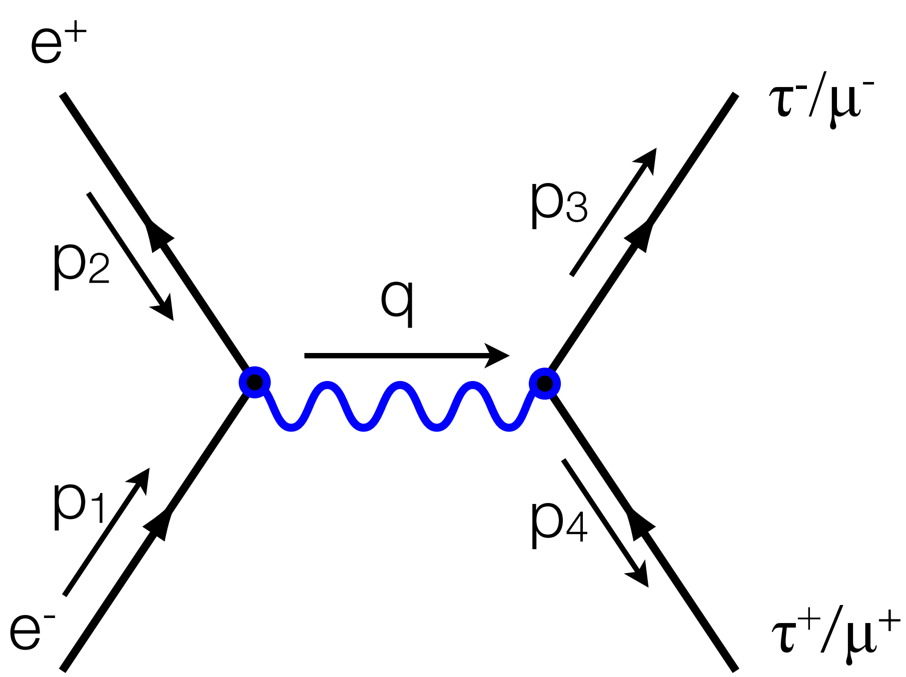
Step I/II: The Feynman Diagram and rules



$$\frac{1}{(2\pi)^4}\int d^4q \;\; \frac{-ig_{\mu\nu}}{q^2}$$

$$\bar{u}(3)\; ig_e\gamma^\mu\; v(4) \quad (2\pi)^4\delta^4(q - p_3 - p_4)$$

$$\bar{v}(2)\; ig_e\gamma^\nu\; u(1) \quad (2\pi)^4\delta^4(p_1 + p_2 - q)$$

$$[\bar{u}(3)\;\gamma^\mu\; v(4)]\;\; g_{\mu\nu}\;\; [\bar{v}(2)\;\gamma^\nu\; u(1)]$$

$$i(2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4) \times \frac{g_e^2}{(p_1+p_2)^2}$$

$$\mathcal{M} = -\frac{g_e^2}{(p_1+p_2)^2}\,[\bar{u}(3)\;\gamma^\mu\; v(4)]\,[\bar{v}(2)\;\gamma_\mu\; u(1)]$$

Step III: Summing over spins:

- To get $|M|^2$ we need to take the complex conjugate of the M:

$$\mathcal{M} = -\frac{g_e^2}{(p_1+p_2)^2}\left[\bar{u}(3)\ \gamma^\mu\ v(4)\right]\left[\bar{v}(2)\ \gamma_\mu\ u(1)\right]$$

$$\mathcal{M}^* = -\frac{g_e^2}{(p_1+p_2)^2}\left[\bar{u}(3)\gamma^\nu v(4)\right]^*\left[\bar{v}(2)\gamma_\nu u(1)\right]^*$$

$$|\mathcal{M}|^2 = \frac{g_e^4}{(p_1+p_2)^4}\left[\bar{u}(3)\gamma^\mu v(4)\right]\left[\bar{u}(3)\gamma^\nu v(4)\right]^*\left[\bar{v}(2)\gamma_\mu u(1)\right]\left[\bar{v}(2)\gamma_\nu u(1)\right]^*$$

$$\sum_{\text{spins}}\left[\bar{u}(3)\gamma^\mu v(4)\right]\left[\bar{u}(3)\gamma^\nu v(4)\right]^* = \mathrm{Tr}\left[(\gamma^\mu(\not{p}_4 - Mc)\gamma^\nu(\not{p}_3 + Mc)\right]$$

$$\sum_{\text{spins}}\left[\bar{v}(2)\gamma^\mu u(1)\right]\left[\bar{v}(2)\gamma^\nu u(1)\right]^* = \mathrm{Tr}\left[\gamma_\mu(\not{p}_1 + mc)\gamma_\nu(\not{p}_2 - mc)\right]$$

$$\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{g_e^4}{(p_1+p_2)^4}\mathrm{Tr}\left[\gamma^\mu(\not{p}_4 - Mc)\gamma^\nu(\not{p}_3 + Mc)\right]\mathrm{Tr}\left[\gamma_\mu(\not{p}_1 + mc)\gamma_\nu(\not{p}_2 - mc)\right]$$

Step IV (continued)

- Put it all together:

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{g_e^4}{(p_1+p_2)^4}\mathrm{Tr}\left[\gamma^\mu(\not{p}_4 - Mc)\gamma^\nu(\not{p}_3 + Mc)\right]\mathrm{Tr}\left[\gamma_\mu(\not{p}_1 + mc)\gamma_\nu(\not{p}_2 - mc)\right]$$

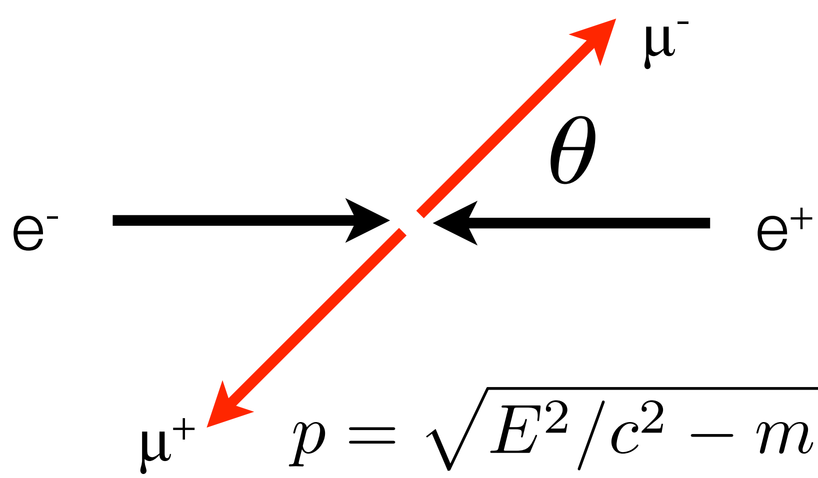
$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{g_e^4}{(p_1+p_2)^4} 32 \times [(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) + m^2c^2(p_3\cdot p_4) + M^2c^2(p_1\cdot p_2) + 2m^2c^2M^2c^2]$$

- Since we are averaging over the initial spins, we need to divide by 4:

$$\langle|\mathcal{M}|^2\rangle = \frac{g_e^4}{(p_1+p_2)^4} 8 \times [(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) + m^2c^2(p_3\cdot p_4) + M^2c^2(p_1\cdot p_2) + 2m^2c^2M^2c^2]$$

Step V: The Kinematics:

$$\langle|\mathcal{M}|^2\rangle = \frac{g_e^4}{(p_1+p_2)^4} 8 \times [(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) + m^2c^2(p_3\cdot p_4) + M^2c^2(p_1\cdot p_2) + 2m^2c^2M^2c^2]$$



$$p = \sqrt{E^2/c^2 - m^2c^2}$$

$$p_1 = (E/c, 0, p)$$
$$p_2 = (E/c, 0, -p)$$
$$p_3 = (E/c, p'\sin\theta, p'\cos\theta)$$
$$p_4 = (E/c, -p'\sin\theta, -p'\cos\theta)$$

$$p = \sqrt{E^2/c^2 - m^2c^2}$$

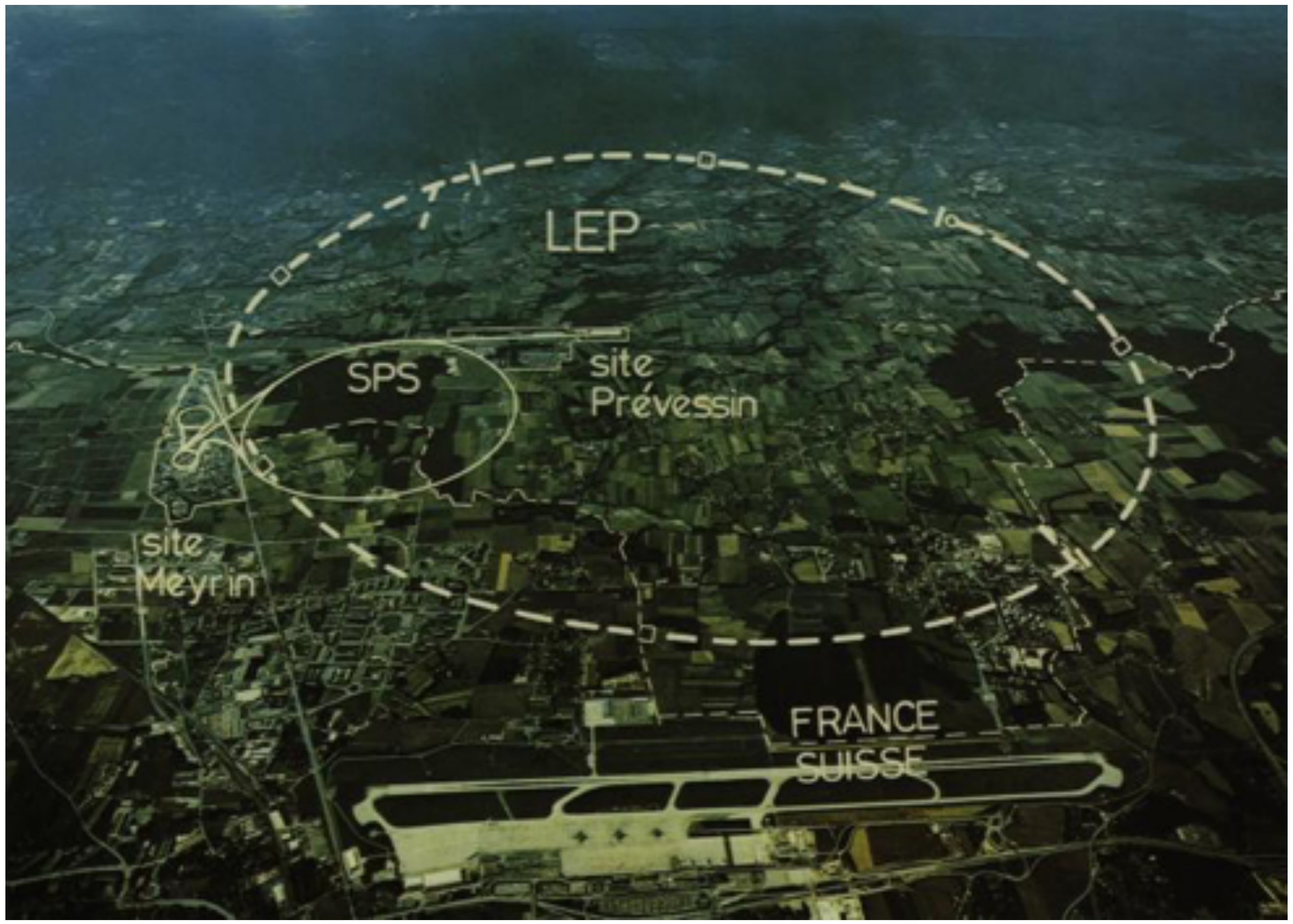
$$p' = \sqrt{E^2/c^2 - M^2c^2}$$

$$\langle|\mathcal{M}|^2\rangle = g_e^4\left[1 + \left(\frac{mc^2}{E}\right)^2 + \left(\frac{Mc^2}{E}\right)^2 + \left[1-\left(\frac{mc^2}{E}\right)^2\right]\left[1-\left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right]$$

Electron/Positron Machines around the World

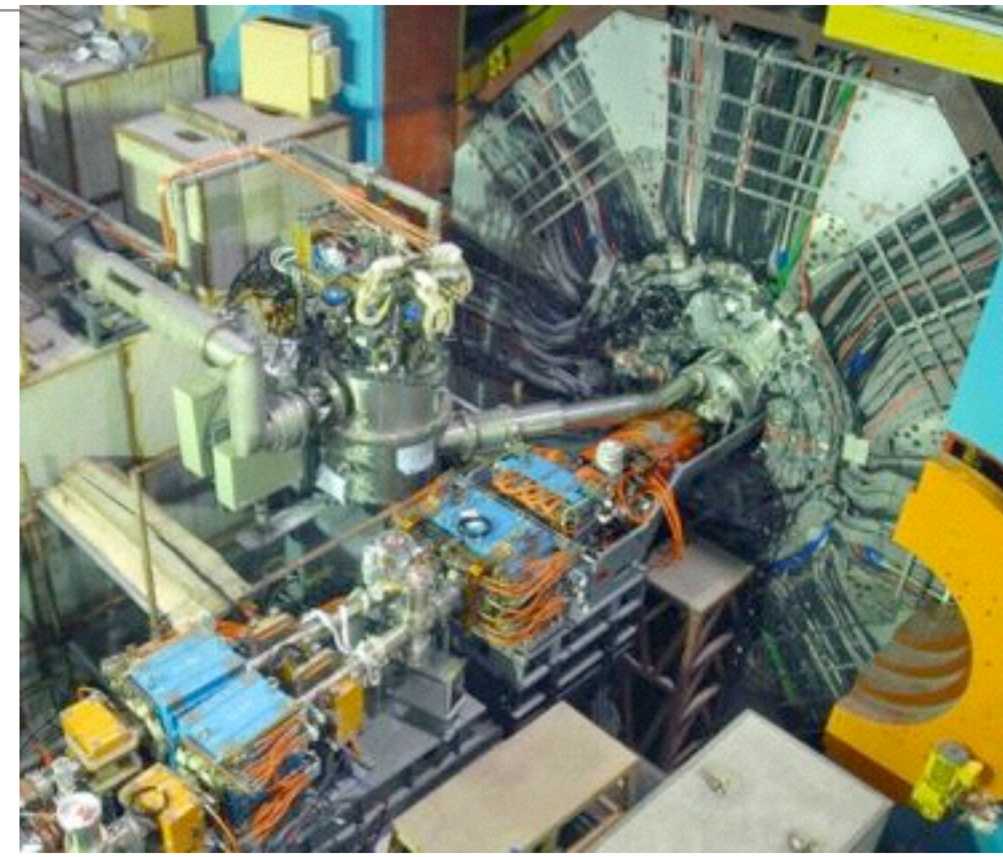
- In the US:
  - SLAC (SPEAR, PEP, PEP-II)
  - CESR
- Older machines “retire” to become synchrotron radiation sources

LEP



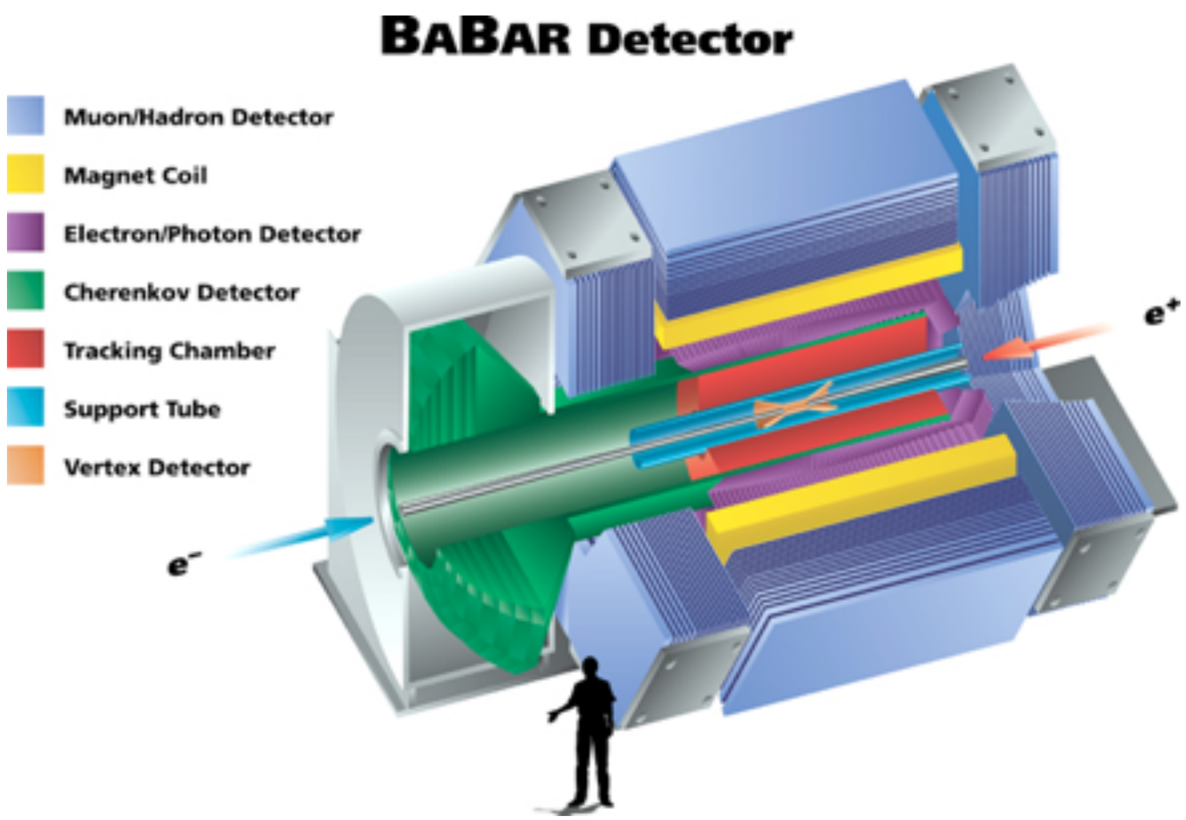
- Before there was the LHC there was LEP
  - “Large Hadron Collider”
  - “Large Electron Position” Collider

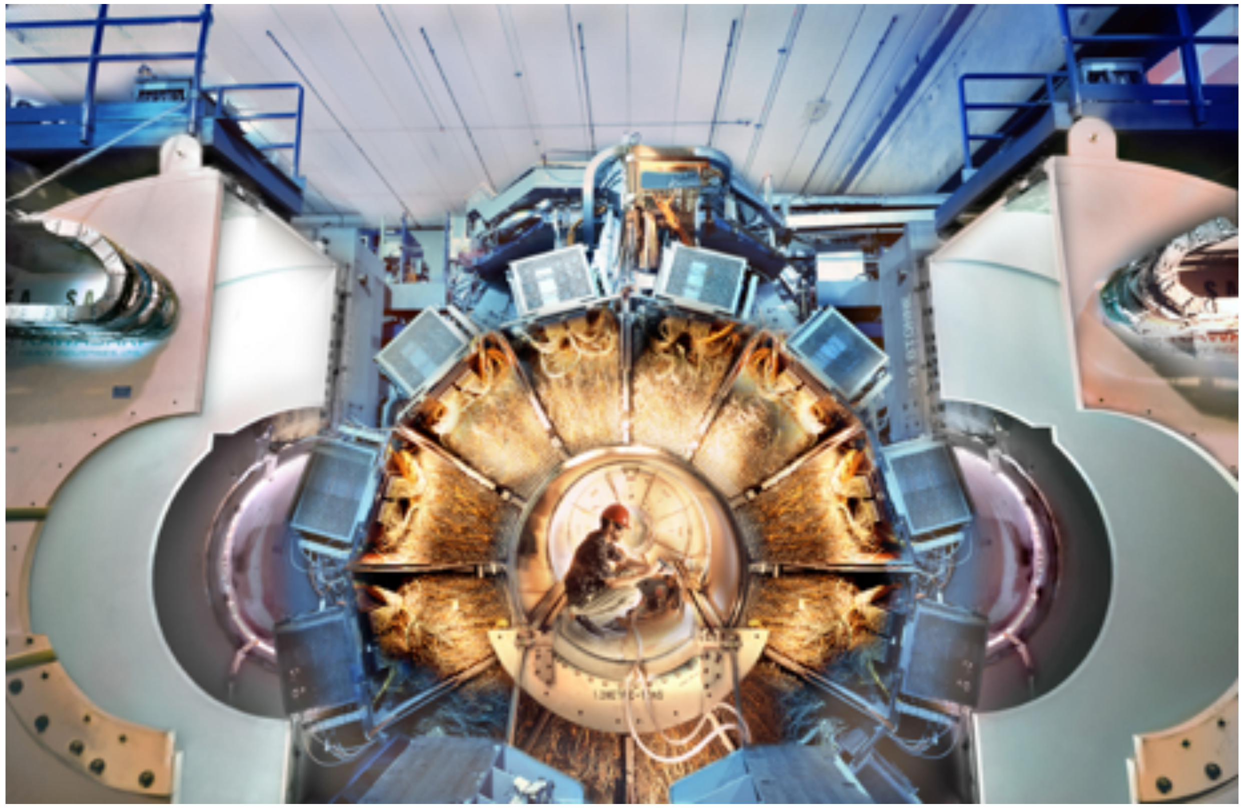
Elsewhere:



- Left: KEK-B ring at KEK (Tsukuba, Japan)
- Top: BES spectrometer (Beijing, China)
- Other machines:
  - PETRA at DESY (Hamburg, Germany)
  - VEPP at BINP (Novosibirsk, Russia)

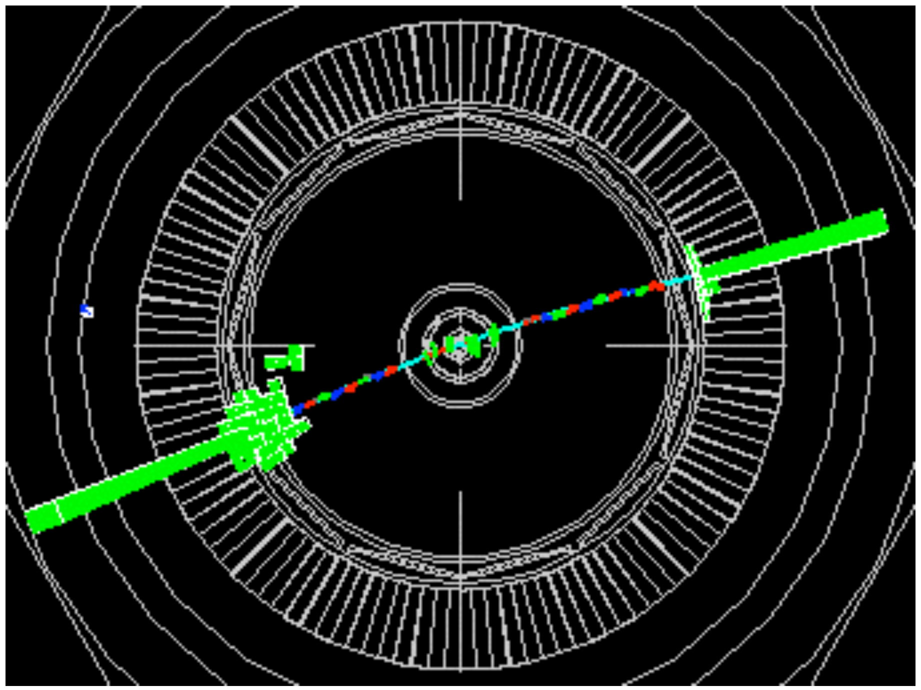
Detectors



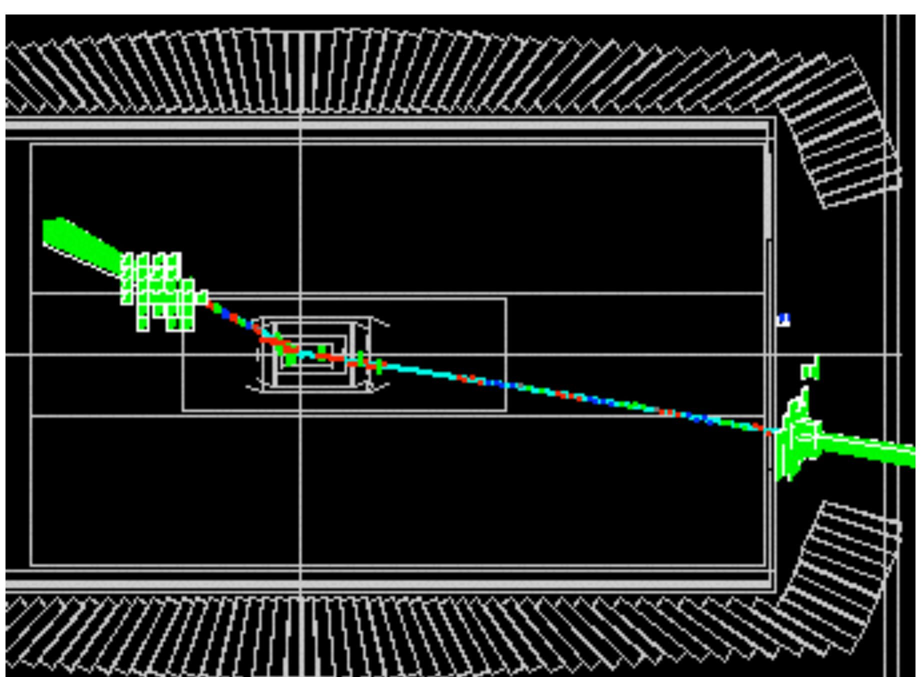


- Most detectors share a similar “cylindrical onion” design
  - Inner tracking region (silicon, drift chambers)
  - Electromagnetic calorimetry (measure and identify electron/photon energy)
  - Muon detector: identify muons by their penetration through lots of material
- Tracking and other parts of detector in magnetic field for momentum
  - Particle identification devices based on “velocity” measurements

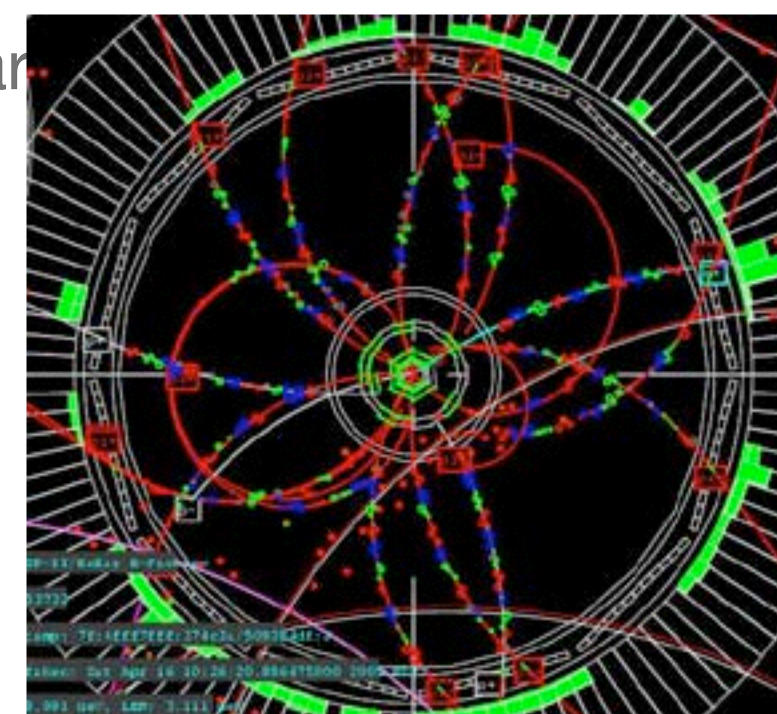
Events at BaBar



- $e^+ + e^- \rightarrow e^+ + e^-$ event at BaBar (Bhabha scattering)
- Note “straightness” of tracks:
- Large deposition in electromagnetic calorimeter
- $e^+ + e^- \rightarrow \mu^+ + \mu^-$ would look similar, but without large energy deposition in the calorimeter



- “Hadronic” event at BaBar
- Particles like b, c quarks produced which initiate a decay chain
- “Full reconstruction” sometimes possible



Now some physics:

- We derived the amplitude for $e^+e^- \to l^+l^-$

$$\langle|\mathcal{M}|^2\rangle = g_e^4\left[1+\left(\frac{mc^2}{E}\right)^2+\left(\frac{Mc^2}{E}\right)^2+\left[1-\left(\frac{mc^2}{E}\right)^2\right]\left[1-\left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right]$$

- m = electron mass, M = lepton mass. Let's ignore the electron mass (E large enough that (mc^2/E) is very small:

$$\langle|\mathcal{M}|^2\rangle = g_e^4\left[1+\left(\frac{Mc^2}{E}\right)^2+\left[1-\left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right]$$

- Recalling our cross section formula:

$$\frac{d\sigma}{d\cos\theta d\phi} = \left(\frac{\hbar c}{8\pi}\right)^2\frac{\langle|\mathcal{M}|^2\rangle}{4E^2}\frac{|p_f|}{|p_i|}$$

- Integrate over the θ, ϕ to obtain the total cross section:

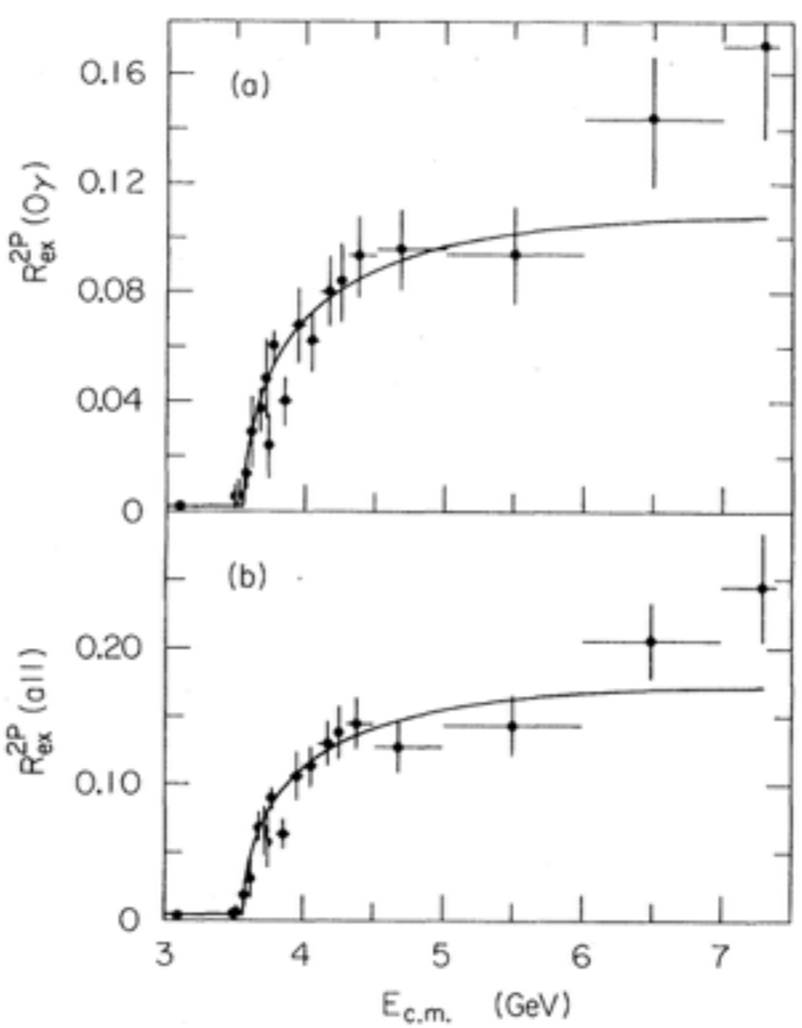
$$\sigma = \frac{\pi}{3}\left(\frac{\hbar c\alpha}{E}\right)^2\sqrt{1-(Mc^2/E)^2}\left[1+\frac{1}{2}\left(\frac{Mc^2}{E}\right)^2\right]$$

Ratio of cross sections:

- $e^+ + e^- \to \mu^+ + \mu^-$ has a very distinct signature in the detector
- "Normalize" $e^+ + e^- \to \tau^+ + \tau^-$ in the detector by taking the ratio:

$$R_{\tau\mu} = \frac{\sigma_{\tau^+\tau^-}}{\sigma_{\mu^+\mu^-}} = \frac{\sqrt{1-(M_\tau c^2/E)^2}}{\sqrt{1-(M_\mu c^2/E)^2}} \times \frac{1+\frac{1}{2}(M_\tau c^2/E)^2}{1+\frac{1}{2}(M_\mu c^2/E)^2}$$

- Note: numerator is imaginary when $E < M_\tau c^2$: this is a threshold requirement



step E, count $\tau^+ + \tau^-$ and $\mu^+ + \mu^-$ events

- Ratio is effectively $R_{\tau\mu}$

- Energy $R_{\tau\mu}(E)$ depends on the spin of the τ:
  - If the particle were a scalar or vector, it would have a different E-dependence
- Measures τ mass:

W. Bacino et al.

Angular Distribution

- From our amplitude expression:

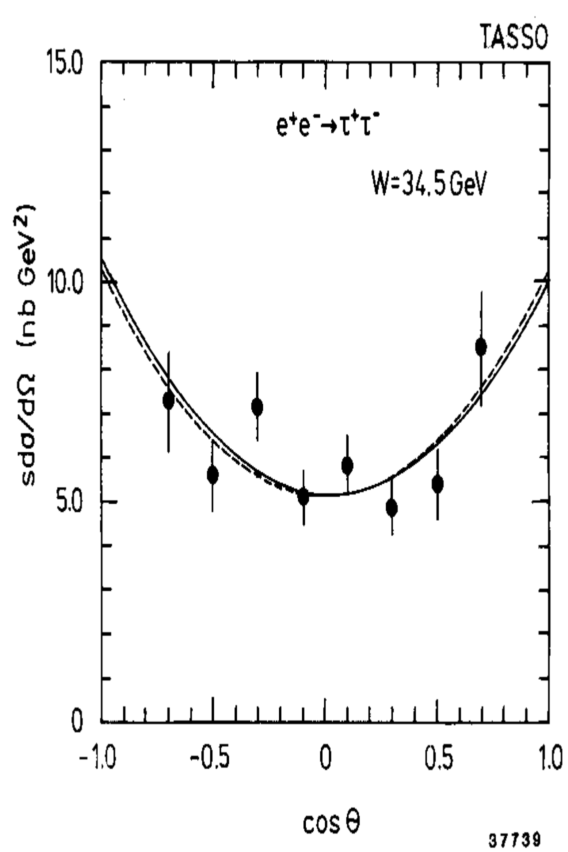
$$\langle|\mathcal{M}|^2\rangle = g_e^4\left[1+\left(\frac{Mc^2}{E}\right)^2+\left[1-\left(\frac{Mc^2}{E}\right)^2\right]\cos^2\theta\right]$$

- if we go to even higher energies E >> Mc^2, we obtain the simple form:

$$\langle|\mathcal{M}|^2\rangle = g_e^4\left[1+\cos^2\theta\right]$$

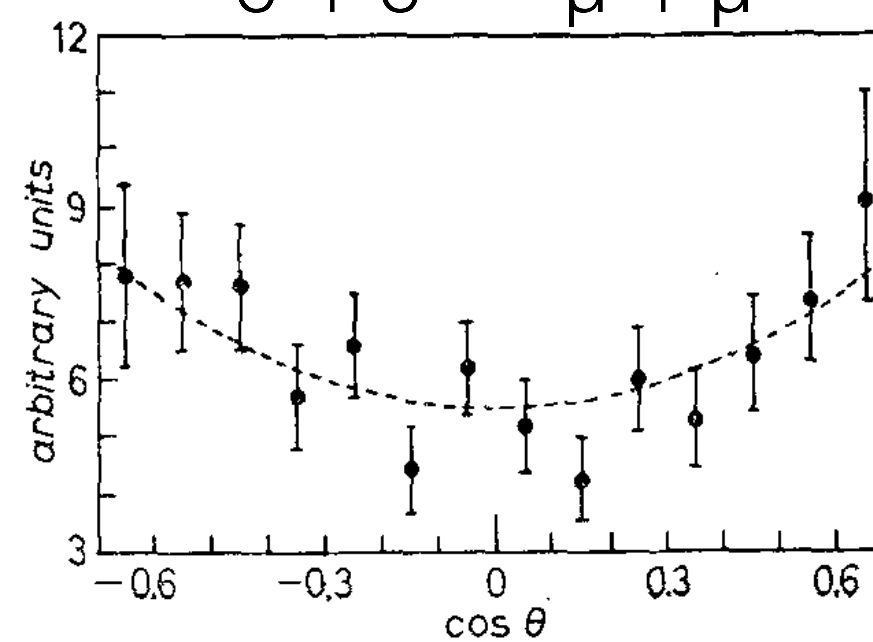
- Recalling our cross section expression $\dfrac{d\sigma}{d\cos\theta d\phi} = \left(\dfrac{\hbar c}{8\pi}\right)^2 \dfrac{\langle|\mathcal{M}|^2\rangle}{4E^2}\dfrac{|p_f|}{|p_i|}$

$$\frac{d\sigma}{d\cos\theta d\phi} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g_e^4}{4E^2}\left[1+\cos^2\theta\right] \quad |p_f|\sim|p_i|$$

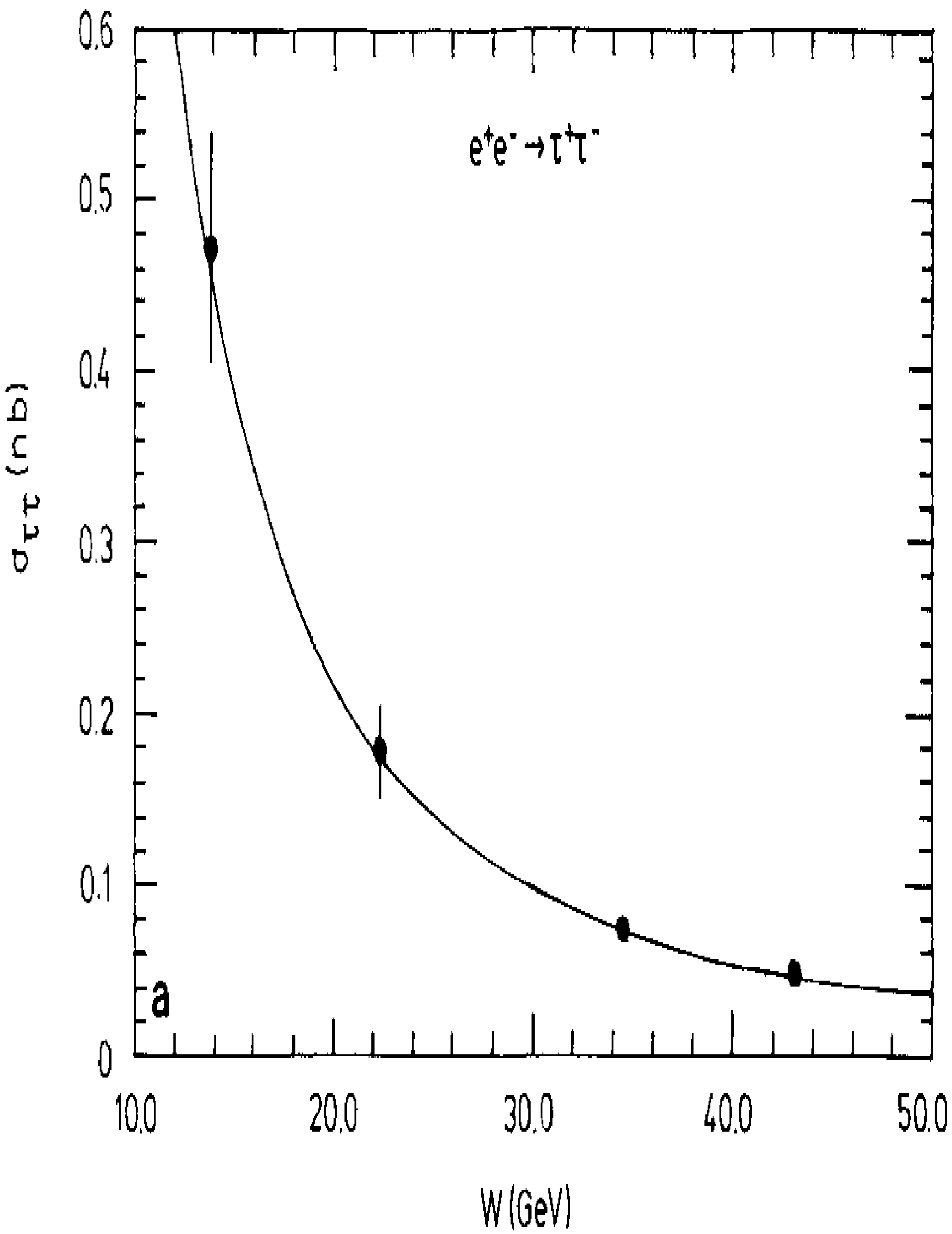


cos θ	s dσ/dΩ
0.0	5.2 nb GeV^2
1.0	10.4 nb GeV^2

$e^+ + e^- \rightarrow \mu^+ + \mu^-$



Cross Section at High Energy:



- If we use our same approximation: $Mc^2 << E$

$$\sigma = \frac{\pi}{3}\left(\frac{\hbar c \alpha}{E}\right)^2 \sqrt{1-(Mc^2/E)^2}\left[1+\frac{1}{2}\left(\frac{Mc^2}{E}\right)^2\right]$$

- becomes:

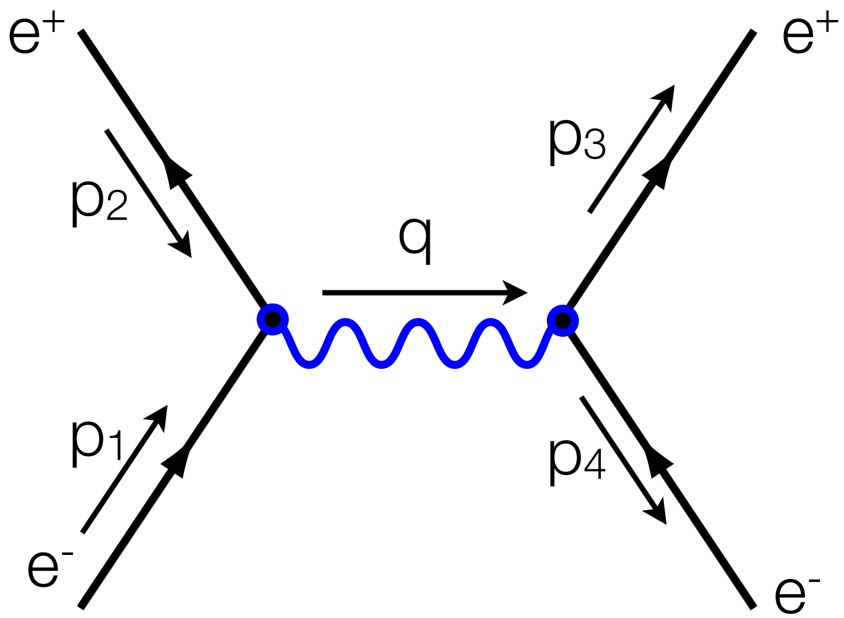
$$\sigma = \frac{\pi}{3}\left(\frac{\hbar c \alpha}{E}\right)^2$$

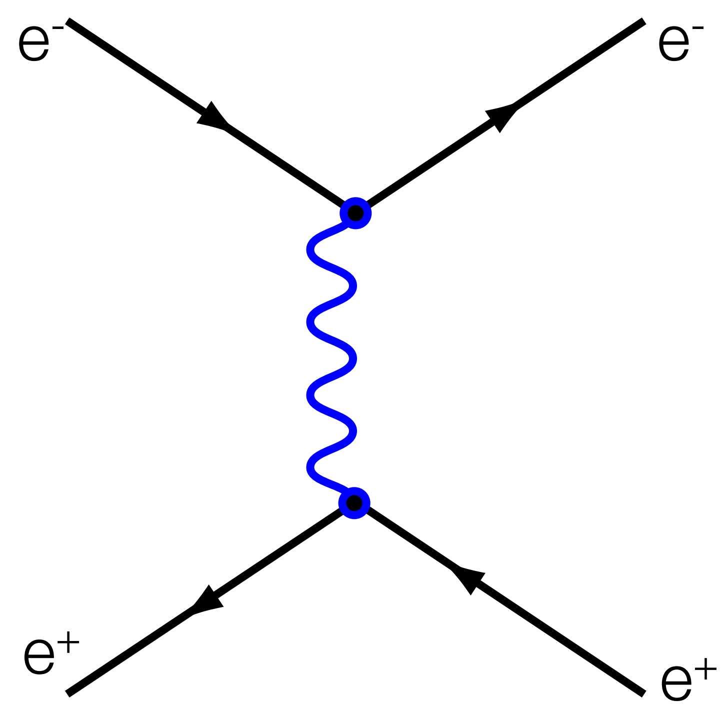
= 2.2×10^{-5} mb/E^2(GeV^2)
= 22 nb/E^2(GeV^2)

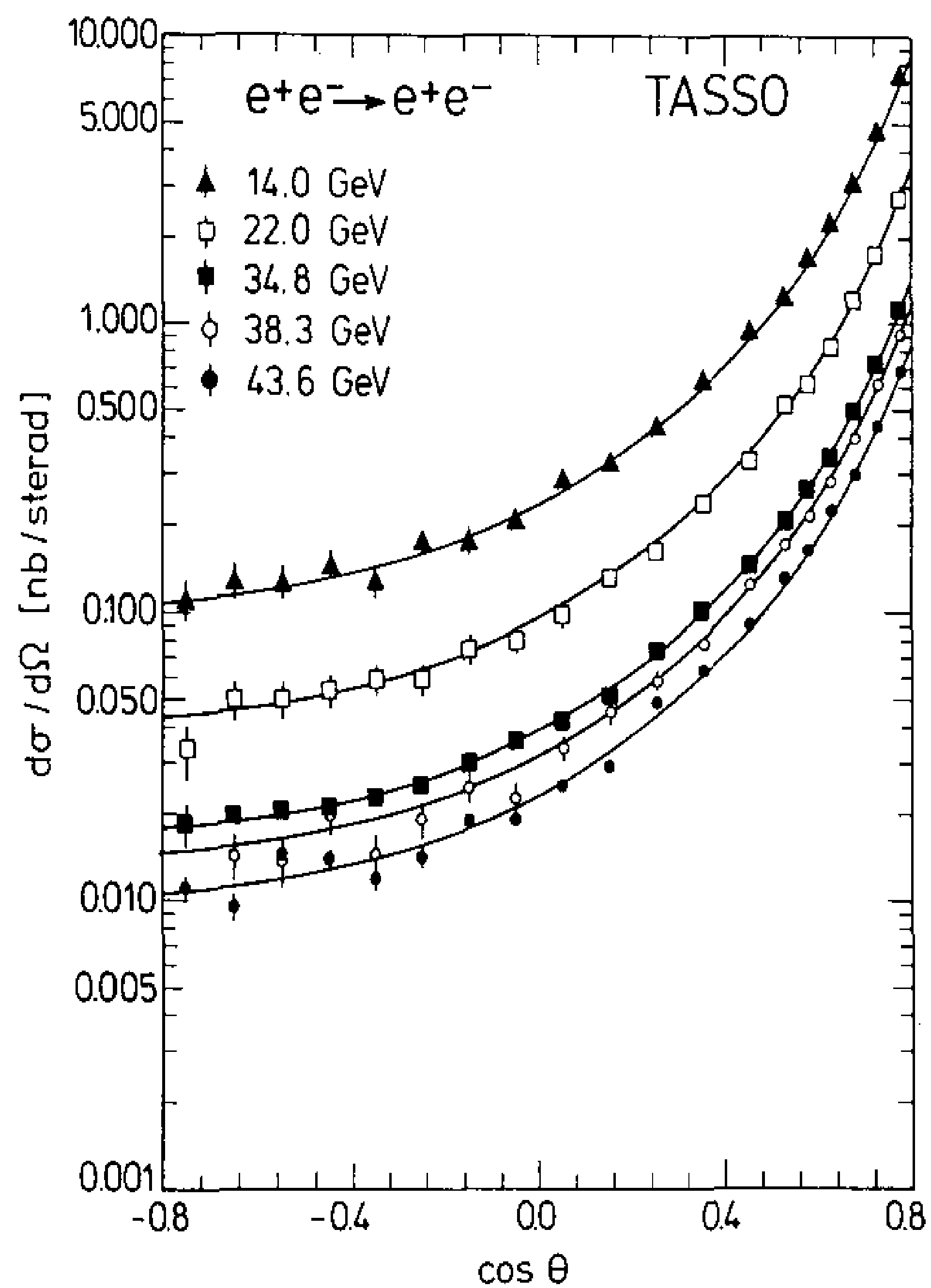
E (GeV)	Cross section (nb)
14	0.44
22	0.18
34	0.075
43	0.047

Bhabha Scattering:

- Similarly to l^+l^- production, we can also produce $e^+ e^-$ pairs





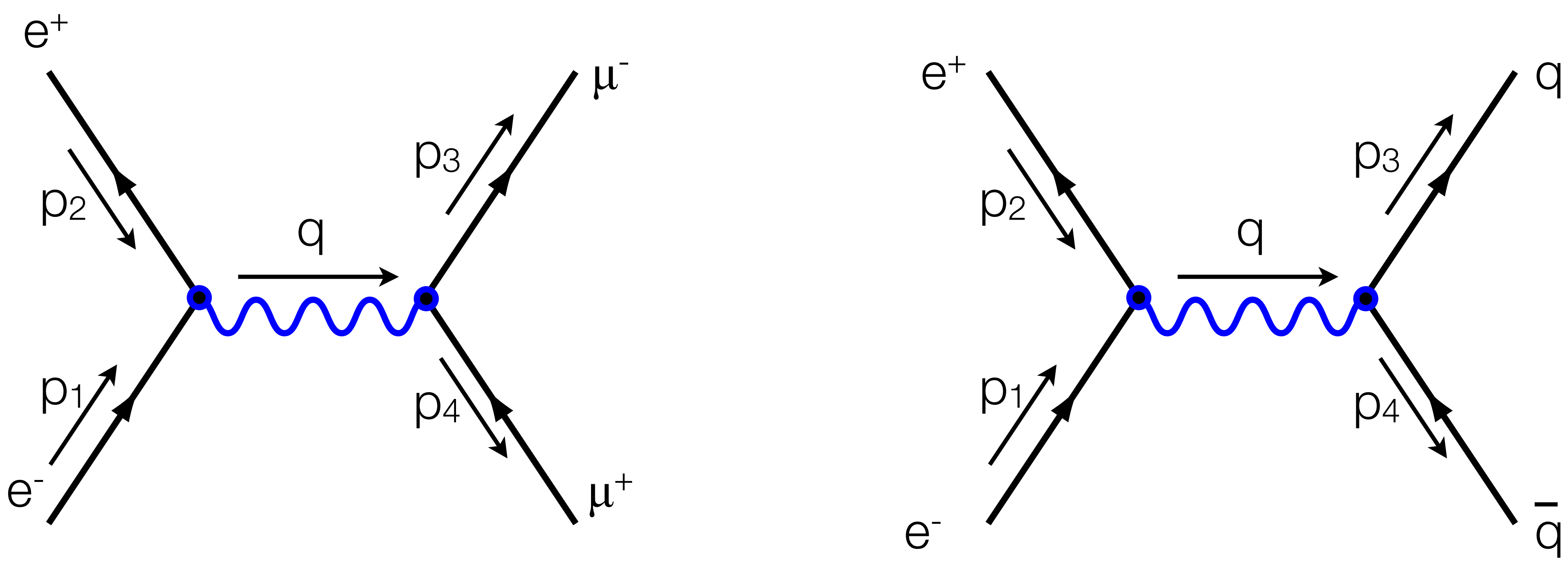


- Note extra diagram:

$$\frac{d\sigma}{d\cos\theta d\phi} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g_e^4}{4E^2}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$

Quark Production in $e^+ e^-$ annihilation

- Similarly to l^+l^- production, we can also produce quark/anti-quark pairs



- All the calculation steps are the same except:
    - Quarks do have not unit charge (1/3, 2/3) need to account for this
    - Quarks have colour: three possibility for pair production

Cross section for quark production

- Let's use our assumption that E > Mc^2, where M is now the mass of the quark

$$\sigma = \frac{\pi}{3}\left(\frac{\hbar c \alpha}{E}\right)^2 \qquad \sigma_{\mu^+\mu^-} = \frac{\pi}{3}\left(\frac{\hbar c \alpha}{E}\right)^2 \Rightarrow \sigma_{q_i\bar{q}_i} = 3Q_i^2 \times \sigma_{\mu^+\mu^-}$$

- Sum over all the quark species that are produced in the collision.

$$R = \frac{\sum_i \sigma_{q\bar{q}_i}}{\sigma_{\mu^+\mu^-}} = 3 \times \sum Q_i^2$$

- The quark species produced will depend on the energy
- Below the charm threshold (~3.8 GeV) :

$$R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2) = 2$$

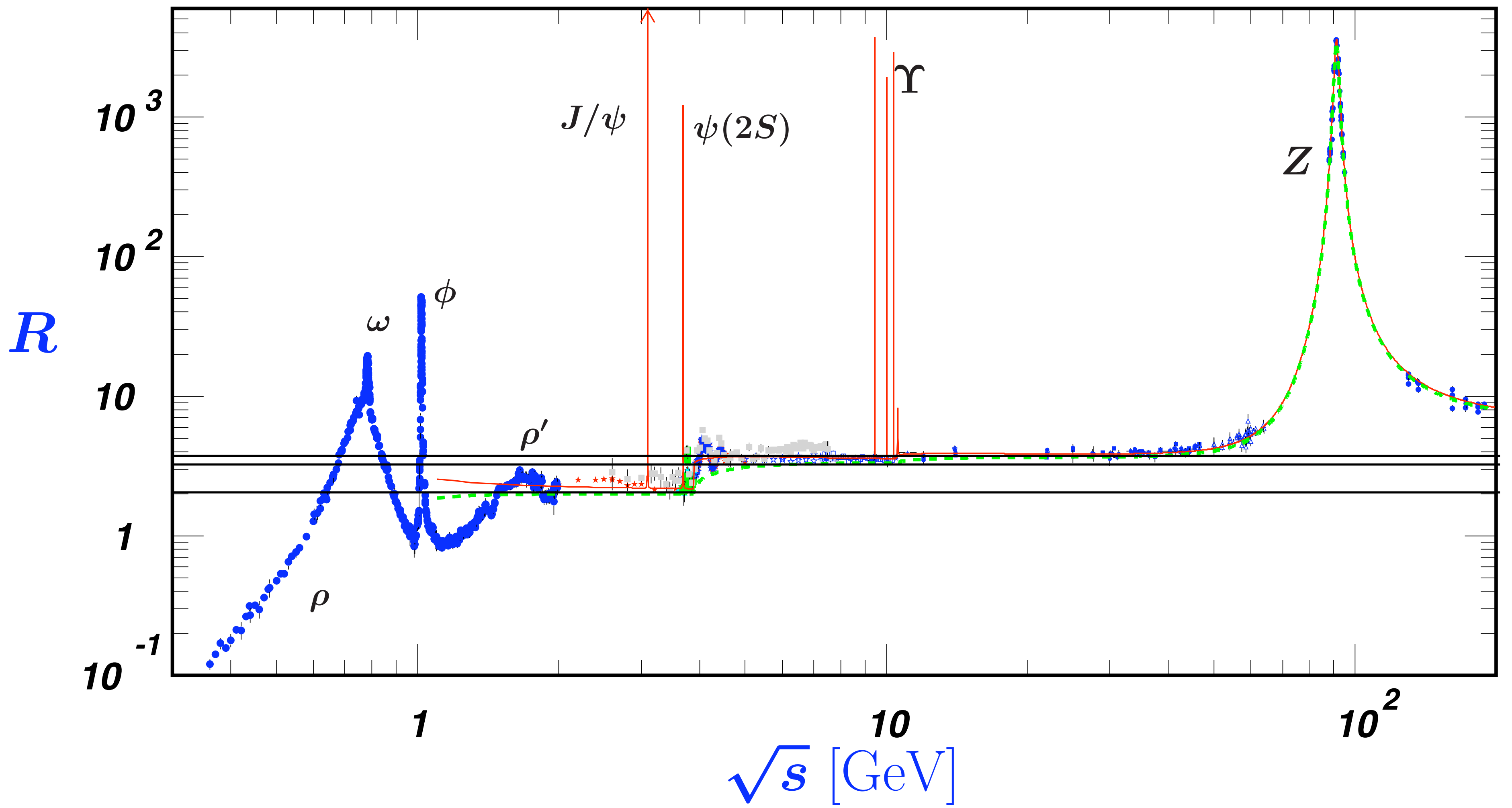
- Between charm/bottom thresholds:

$$R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2 + (2/3)^2) = 10/3$$

- Above bottom threshold: (~10 GeV)

$$R = 3 \times ((2/3)^2 + (1/3)^2 + (1/3)^2 + (2/3)^2 + (1/3)^2) = 11/3$$

Ratio of Quark/Muon Production



- Situation is much more complicated than our naive picture
- However, the need for the factor of 3 from color is unambiguous.

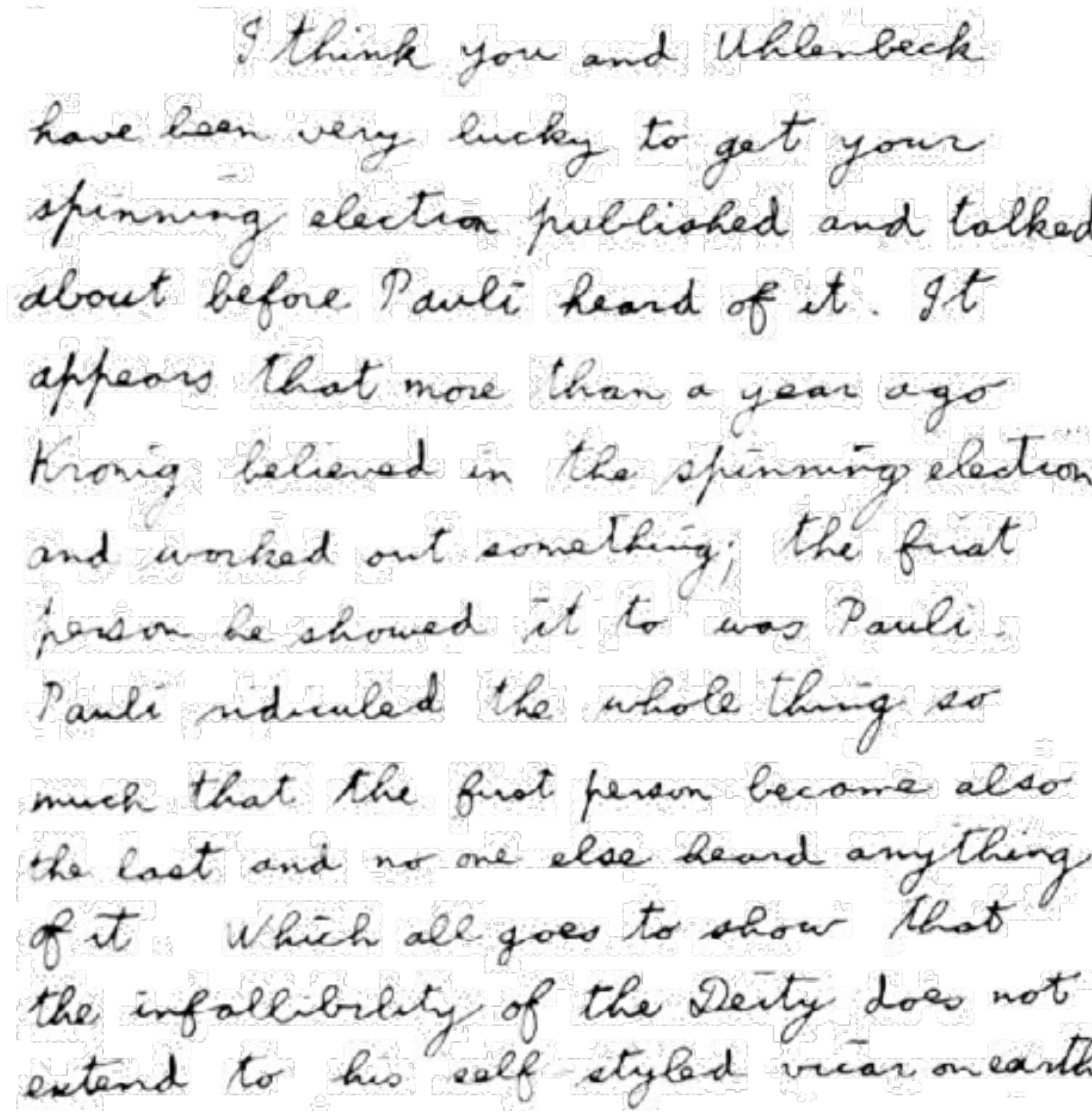
The spinning electron

- On considering the splittings in the spectra of hydrogen atom, Goudsmit and Uhlenbeck introduced the idea of a “spinning electron”
- The spinning of a charged object produces a magnetic moment that results in spin-orbit coupling
- They submitted it to Ehrenfest and met with him
- Ehrenfest: a charge that rotates like that impossible
  - Too bad, already submitted it!

“Well, it’s a nice idea, though it may be wrong. But you don’t yet have a reputation so you have nothing to lose.”

Other thoughts:

- Others had already considered this?
- Heisenberg:
    - Congratulations “on your courageous note.”
    - “What did you do with the factor of two?”
- Goudsmit and Uhlenbeck:
    - Why is it courageous?
    - What factor of two?
- As it turns out, the splittings are larger by x2 then one would naively expect from non-relativistic considerations.



I think you and Uhlenbeck have been very lucky to get your spinning electron published and talked about before Pauli heard of it. It appears that more than a year ago Kronig believed in the spinning electron and worked out something; the first person he showed it to was Pauli. Pauli ridiculed the whole thing so much that the first person became also the last and no one else heard anything of it. Which all goes to show that the infallibility of the Deity does not extend to his self styled vicar on earth.

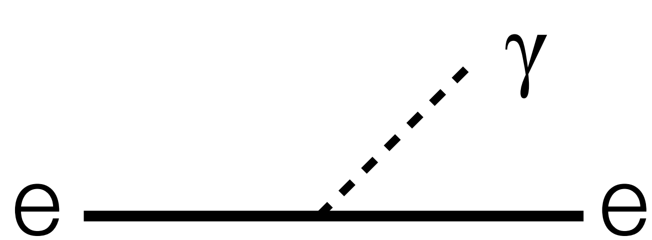
Pauli: “I was so stupid when I was young!”

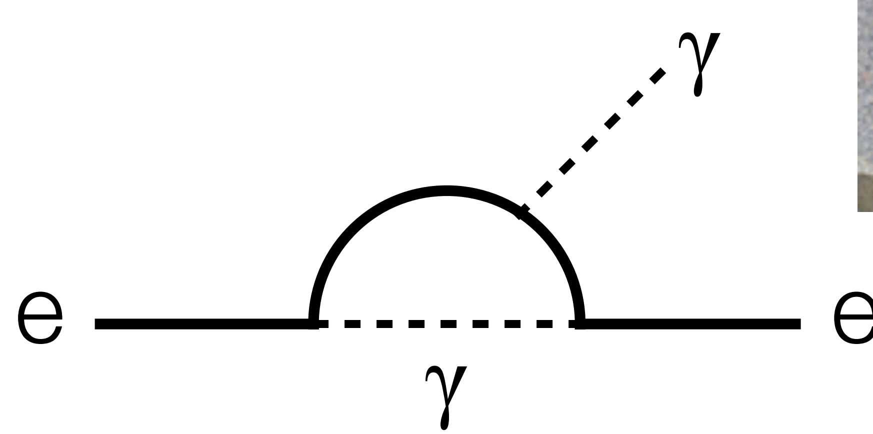
Thomas: special relativity introduces a factor of two

The “gyromagnetic ratio”

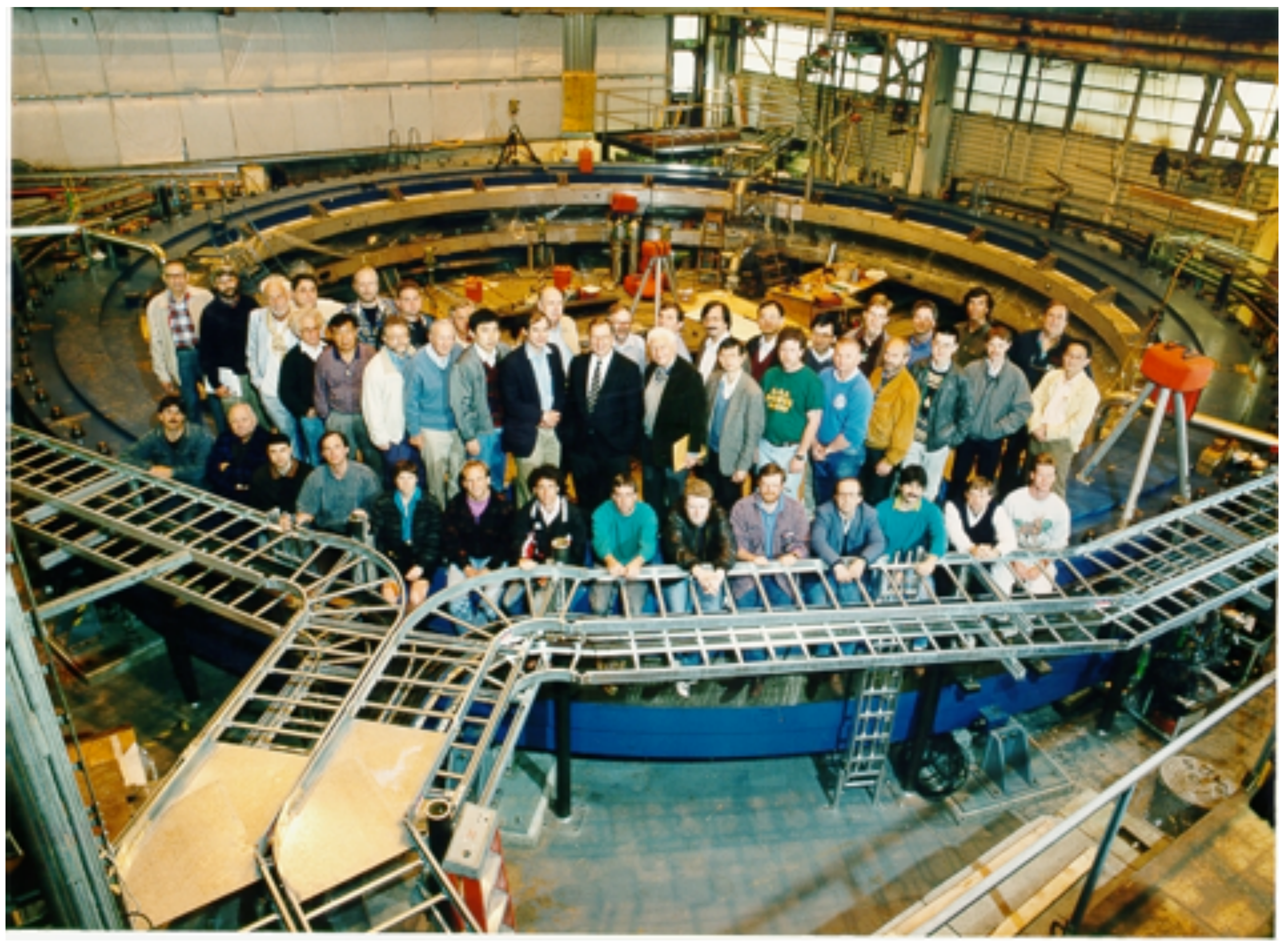
$$\mu = g\mu_B s/\hbar \qquad \mu_B = \frac{e\hbar}{2m}$$

- Ratio of the magnetic moment to spin times the Bohr magneton
- As it turns out, this is not exactly 2 for an electron
  - a = (g-2/)2 ~ 0.00115965218073(28) (current measurement)
- The departure from “2” is called the “anomalous moment”
  - results from higher order corrections
  - first calculated by Julian Schwinger in 1948
  - a ~ α/2π = 0.0011614





The muon g-2 experiment



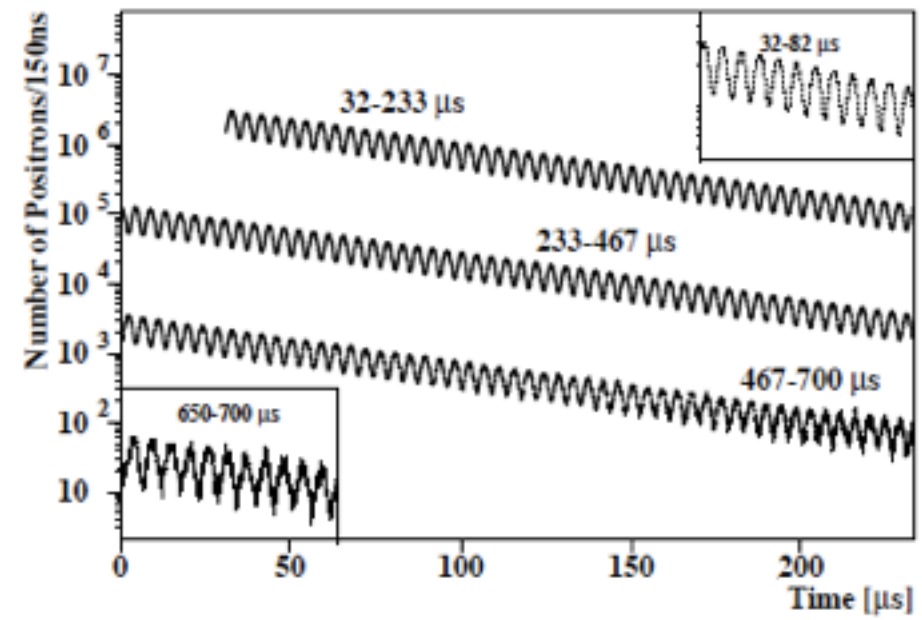
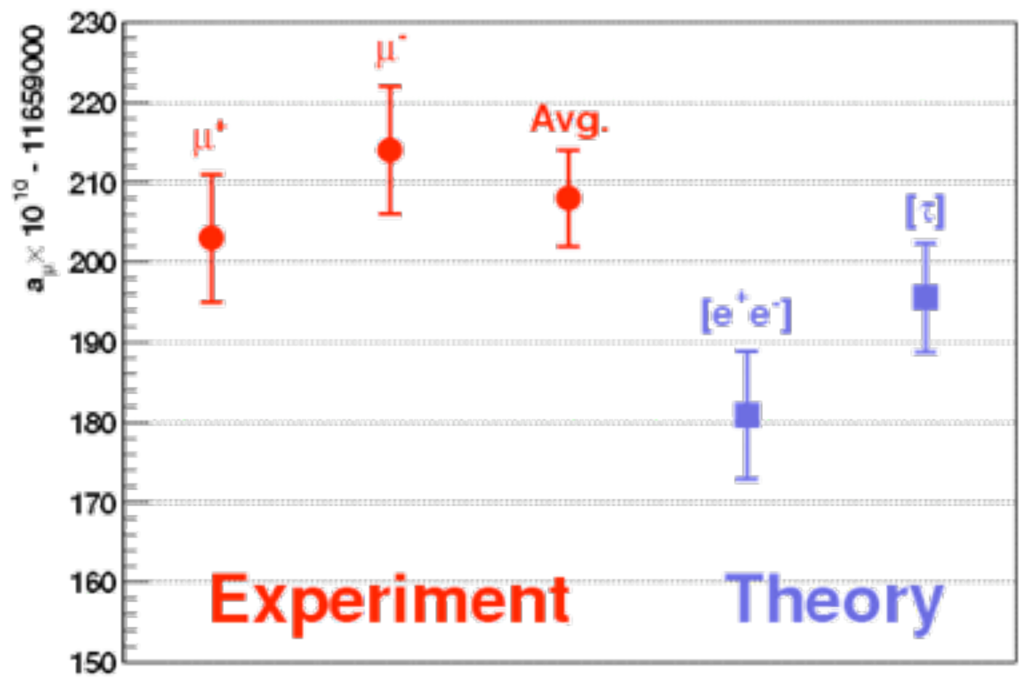


FIG. 3. Positron time spectrum overlaid with the fitted 10 parameter function (χ^2/dof= 3818/3799). The total event sample of 0.95×10^9 e^+ with $E \geq 2.0$ GeV is shown.



- Predicted: (g-2)/2 = (1165918.81±0.38)x10^{-9}
- Measured: (g-2)/2 = (1165920.80±0.63)x10^{-9}