Isospin

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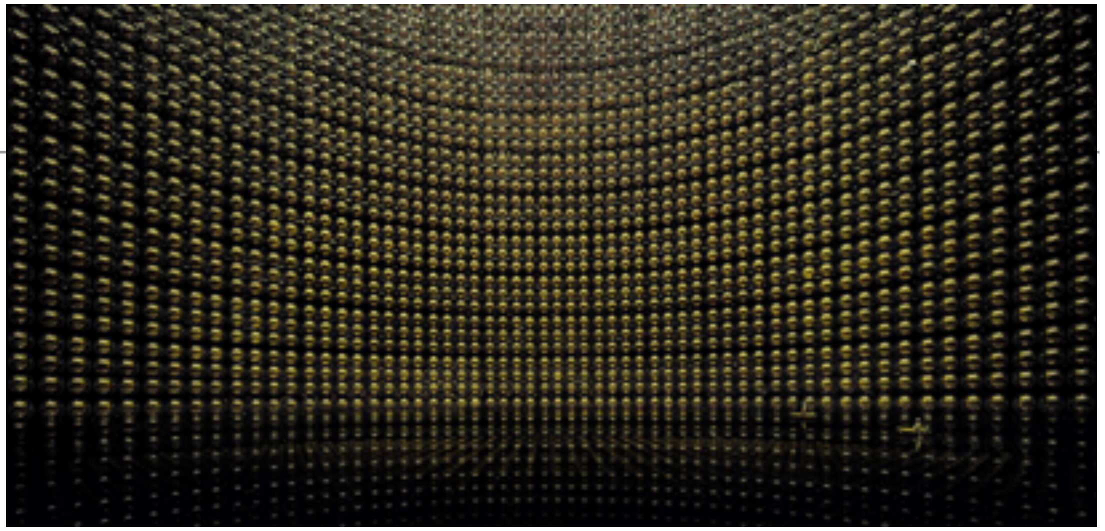
Announcements

- Problem Set 1 due today at 5 PM
    - Box #7 in basement of McLennan
- Problem set 2 will be posted today





The Nobel Prize in Physics 2015

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2015 to

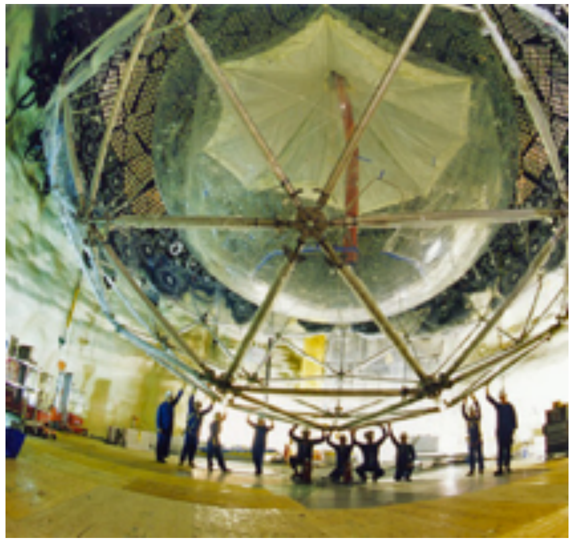
Takaaki Kajita

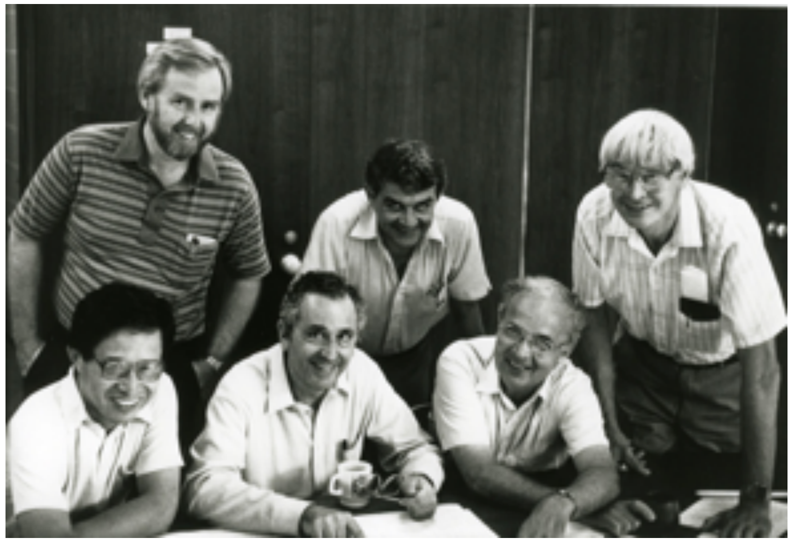
Super-Kamiokande Collaboration
University of Tokyo, Kashiwa, Japan

Arthur B. McDonald

Sudbury Neutrino Observatory Collaboration
Queen's University, Kingston, Canada

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"





Overview

- Assign isospin values to states
- Understand basic principle of “isospin symmetry” and why it works
- Translate isospin symmetry into consequences for scattering amplitudes
  - Use Clebsch-Gordan Tables
  - Using phenomenology, infer isospin assignment when not known *a priori*

What is isospin?

- Heisenberg noticed that protons and neutrons are very close in mass

p 938.272 MeV/c^2 $\quad p \rightarrow |\frac{1}{2}, \frac{1}{2}\rangle$

n 939.565 MeV/c^2 $\quad n \rightarrow |\frac{1}{2}, -\frac{1}{2}\rangle$

- We now know that the pions also closely spaced:

π^+ 139.570 MeV/c^2 $\quad \pi^+ \rightarrow |1, 1\rangle$

π^0 134.977 MeV/c^2 $\quad \pi^0 \rightarrow |1, 0\rangle$

π^- 139.570 MeV/c^2 $\quad \pi^- \rightarrow |1, -1\rangle$

- Likewise the Delta Resonances:

$\Delta^{++/+/0/-}$ 1232 MeV/c^2

$\Delta^{++} \rightarrow |\frac{3}{2}, \frac{3}{2}\rangle \qquad \Delta^{+} \rightarrow |\frac{3}{2}, \frac{1}{2}\rangle$

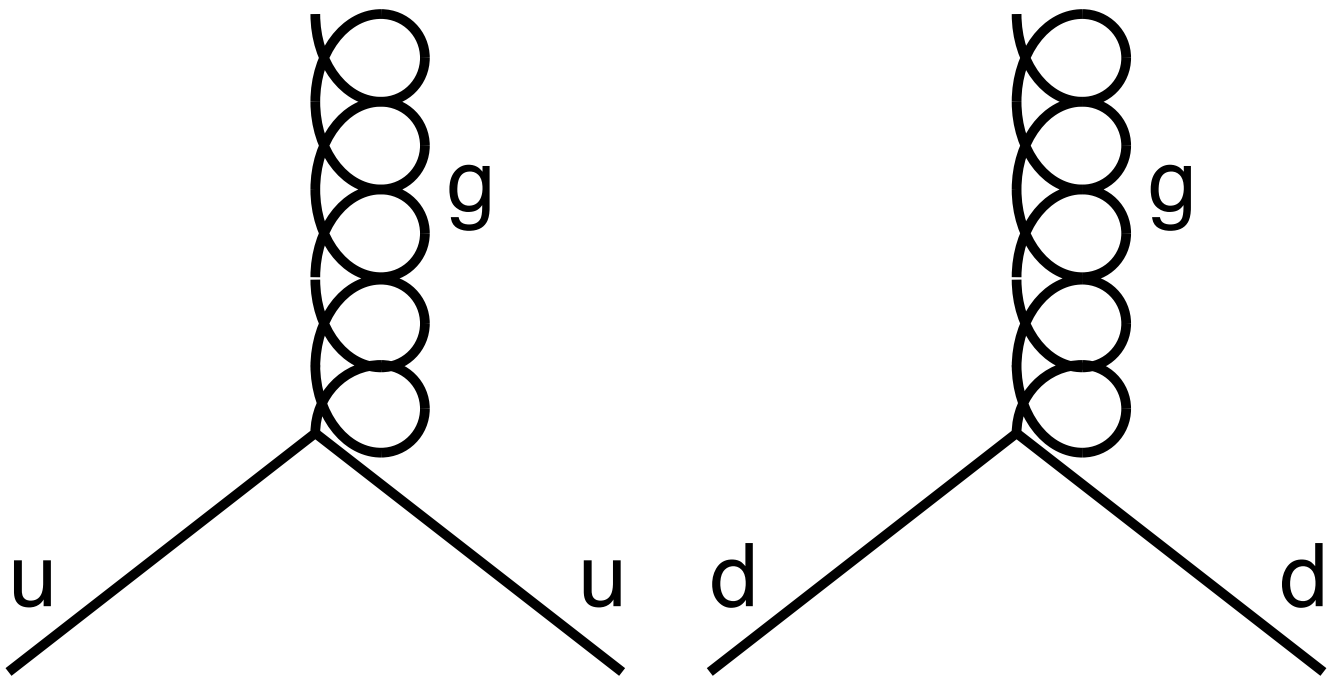
$\Delta^{0} \rightarrow |\frac{3}{2}, -\frac{1}{2}\rangle \qquad \Delta^{-} \rightarrow |\frac{3}{2}, -\frac{3}{2}\rangle$

The Hypothesis

- In 1932 (!), Heisenberg postulated that:
  - protons, neutrons are up/down states of isospin 1/2 system
  - pions are +1,0,-1 states of isospin 1 system
  - strong interactions are invariant under isospin rotations
    - Isospin is conserved in strong interactions

Heisenberg in 1933

Why does this work? (in hindsight)



$$u \rightarrow |\tfrac{1}{2}, \tfrac{1}{2}\rangle \qquad \bar{d} \rightarrow -|\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

$$d \rightarrow |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \qquad \bar{u} \rightarrow |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

- Strong interactions are the same for all quarks/antiquarks
- Different quarks have different properties, however
  - charge (+2/3 vs. -1/3)
  - mass (m_t=170 GeV/c^2)
- Since d and u quarks have such similar masses, interchanging them works okay
- Interchanging other quarks doesn't work as well.

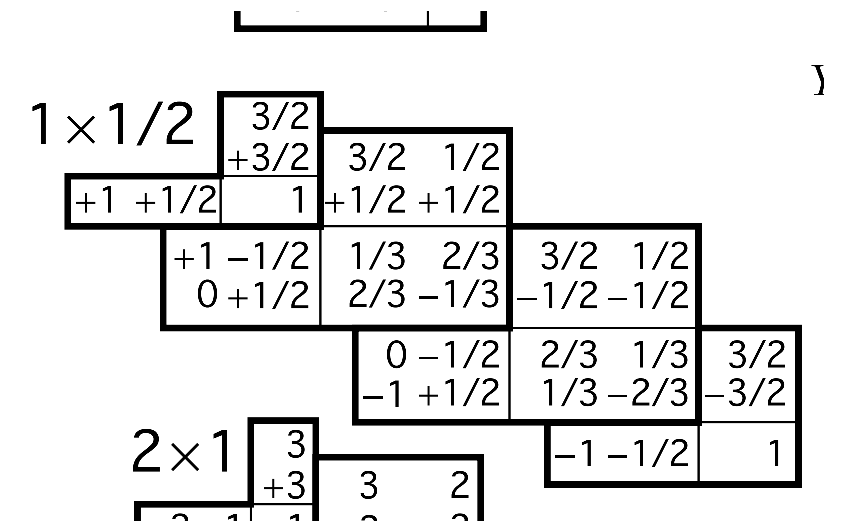
Assigning Isospin to multi-particle states:

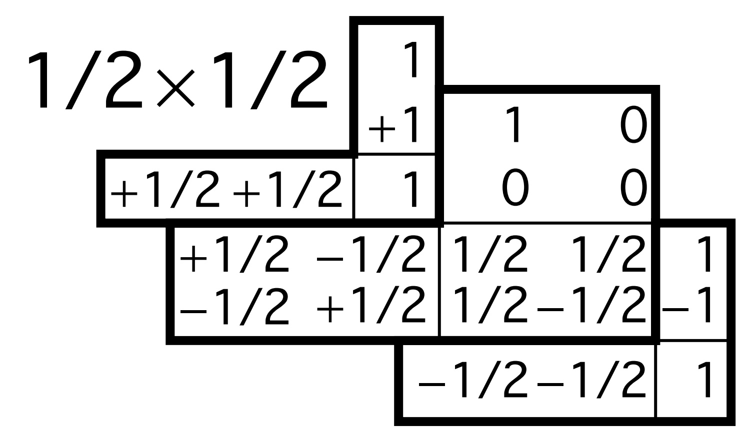
Particles	Individual States	Combined State
$p+p$	$\|\frac{1}{2},\frac{1}{2}\rangle\|\frac{1}{2},\frac{1}{2}\rangle$	$\|1,1\rangle$
$\pi^{+}+p$	$\|1,1\rangle\|\frac{1}{2},\frac{1}{2}\rangle$	$\|\frac{3}{2},\frac{3}{2}\rangle$
$\pi^{-}+n$	$\|1,-1\rangle\|\frac{1}{2},-\frac{1}{2}\rangle$	$\|\frac{3}{2},-\frac{3}{2}\rangle$
$n+n$	$\|\frac{1}{2},-\frac{1}{2}\rangle\|\frac{1}{2},-\frac{1}{2}\rangle$	$\|1,-1\rangle$

Easiest “highest weight” cases

Assigning Isospin to multi-particle states II:

Particles	Individual	Combined State
$p + n$	$\|\frac{1}{2}, \frac{1}{2}\rangle \|\frac{1}{2}, -\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(\|1,0\rangle + \|0,0\rangle)$
$\pi^+ + n$	$\|1,1\rangle \|\frac{1}{2}, -\frac{1}{2}\rangle$	$\frac{1}{\sqrt{3}}\|\frac{3}{2}, \frac{1}{2}\rangle + \sqrt{\frac{2}{3}}\|\frac{1}{2}, \frac{1}{2}\rangle$
$\pi^0 + p$	$\|1,0\rangle \|\frac{1}{2}, \frac{1}{2}\rangle$	$\sqrt{\frac{2}{3}}\|\frac{3}{2}, \frac{1}{2}\rangle - \frac{1}{\sqrt{3}}\|\frac{1}{2}, \frac{1}{2}\rangle$
$\pi^0 + n$	$\|1,0\rangle \|\frac{1}{2}, -\frac{1}{2}\rangle$	$\sqrt{\frac{2}{3}}\|\frac{3}{2}, -\frac{1}{2}\rangle + \frac{1}{\sqrt{3}}\|\frac{1}{2}, -\frac{1}{2}\rangle$
$\pi^- + p$	$\|1,-1\rangle \|\frac{1}{2}, \frac{1}{2}\rangle$	$\frac{1}{\sqrt{3}}\|\frac{3}{2}, -\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}\|\frac{1}{2}, -\frac{1}{2}\rangle$
Δ^+	$\|\frac{3}{2}, \frac{1}{2}\rangle$	$\|\frac{3}{2}, \frac{1}{2}\rangle$





What are the amplitudes for Δ^+ decay?

Scattering:

- Scattering is a general concept of “something in”, “something out”

$$A + B + C + D + E \ldots\ldots \xrightarrow{S} W + X + Y + Z \;\ldots$$

- What can we say about what “S” will do?
- Conservation laws:
  - Energy
  - Momentum
  - Angular Momentum
  - ?

Historically, had no idea what a “pion” is, for example.

What conservation/symmetry rules apply?

- Isospin: perhaps it is conserved in strong interactions.
  - We can then say (more about) what can happen and what can’t happen.
  - Complication: initial state can have more than one isospin value

Amplitudes and Cross Sections:

- In Quantum Mechanics, the amplitude for a transition A→B via some scattering process S is given by the product:

$$\langle A|S|B\rangle$$

- The probability for the transition is given by the absolute magnitude squared of the amplitude:

$$P = \langle A|S|B\rangle \times \langle A|S|B\rangle^* = \langle A|S|B\rangle \times \langle B|S^\dagger|A\rangle = |\langle A|S|B\rangle|^2$$

- This is related to the "cross section": in general, the cross section carries an additional "phase space" parameter associated with Fermi's Golden Rule. For now, we can "equate" cross section with probability.

- Isospin symmetry is a statement that whatever transitions are effected by S, total isospin (total and component) is conserved.

Bound state of two nucleons

- We have four possible way for two nucleons to bound
  - How does this work? $$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$ $$2 \otimes 2 = 3 \oplus 1$$
  - "two isospin 1/2 objects combine to form an isospin 1 ("isotriplet") and an isospin 0 ("isosinglet").
- What is the isotriplet?

$$|\tfrac{1}{2}, \tfrac{1}{2}\rangle|\tfrac{1}{2}, \tfrac{1}{2}\rangle \rightarrow |1, 1\rangle$$

$$\frac{1}{\sqrt{2}}\left[|\tfrac{1}{2}, -\tfrac{1}{2}\rangle|\tfrac{1}{2}, \tfrac{1}{2}\rangle + |\tfrac{1}{2}, \tfrac{1}{2}\rangle|\tfrac{1}{2}, -\tfrac{1}{2}\rangle\right] \rightarrow |1, 0\rangle$$

$$|\tfrac{1}{2}, -\tfrac{1}{2}\rangle|\tfrac{1}{2}, -\tfrac{1}{2}\rangle \rightarrow |1, -1\rangle$$

- The isosinglet?

$$\frac{1}{\sqrt{2}}\left[|\tfrac{1}{2}, -\tfrac{1}{2}\rangle|\tfrac{1}{2}, \tfrac{1}{2}\rangle - |\tfrac{1}{2}, \tfrac{1}{2}\rangle|\tfrac{1}{2}, -\tfrac{1}{2}\rangle\right] \rightarrow |0, 0\rangle$$

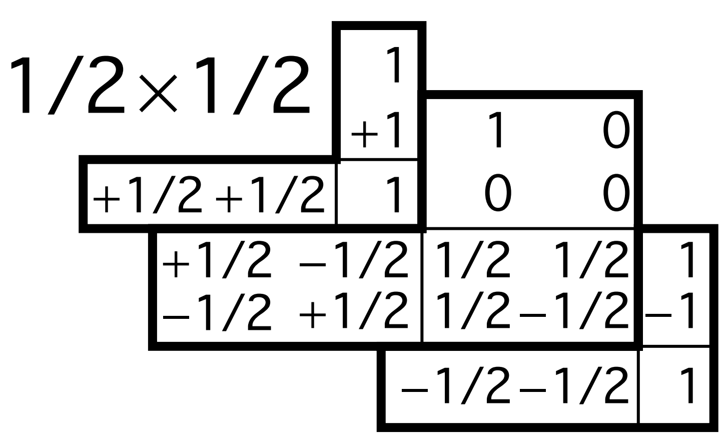
nucleon-nucleon scattering

$$\begin{aligned} p + p &\rightarrow d + \pi^+ \\ p + n &\rightarrow d + \pi^0 \\ n + n &\rightarrow d + \pi^- \end{aligned}$$

isospin

$$\begin{aligned} |\tfrac{1}{2},\tfrac{1}{2}\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle &\rightarrow |0,0\rangle|1,1\rangle \\ |\tfrac{1}{2},\tfrac{1}{2}\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle &\rightarrow |0,0\rangle|1,0\rangle \\ |\tfrac{1}{2},-\tfrac{1}{2}\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle &\rightarrow |0,0\rangle|1,-1\rangle \end{aligned}$$

$$\begin{aligned} |1,1\rangle &\rightarrow |1,1\rangle & \langle 1,1|S|1,1\rangle &\rightarrow A \\ \frac{1}{\sqrt{2}}\left[|1,0\rangle + |0,0\rangle\right] &\rightarrow |1,0\rangle & \frac{1}{\sqrt{2}}\left[\langle 1,0| + \langle 0,0|\right] S|1,0\rangle &\rightarrow \frac{1}{\sqrt{2}}A \\ |1,-1\rangle &\rightarrow |1,-1\rangle & \langle 1,-1|S|1,-1\rangle &\rightarrow A \end{aligned}$$



- Amplitudes: 1:1/√2:1
- Probability/Cross section: 1:1/2:1

Pion-Nucleon Scattering

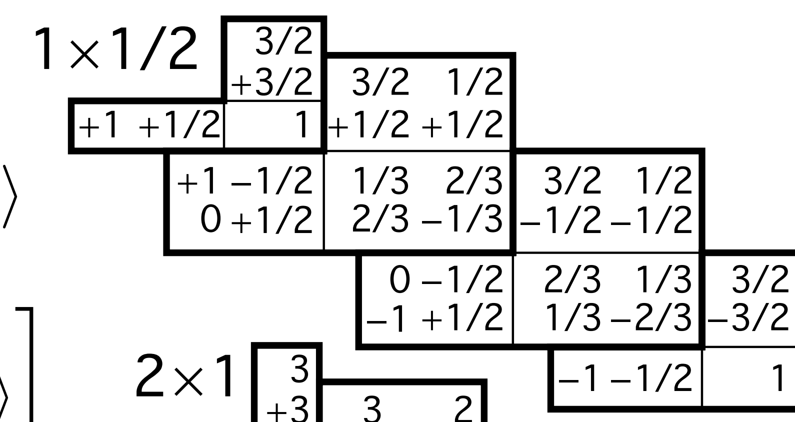
$$\pi^+ + n \quad \longrightarrow \quad \pi^0 + p \qquad\qquad |1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle \;\rightarrow\; |1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$$\pi^0 + p \quad \longrightarrow \quad \pi^+ + n \qquad\qquad |1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle \;\rightarrow\; |1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\pi^0 + n \quad \longrightarrow \quad \pi^- + p \qquad\qquad |1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle \;\rightarrow\; |1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$$\pi^- + p \quad \longrightarrow \quad \pi^0 + n \qquad\qquad |1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle \;\rightarrow\; |1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

$$\pi^+ + n \quad \longrightarrow \quad \pi^0 + p$$



$$\sqrt{\tfrac{1}{3}}|3/2,1/2\rangle + \sqrt{\tfrac{2}{3}}|1/2,1/2\rangle \quad \rightarrow \quad \sqrt{\tfrac{2}{3}}|3/2,1/2\rangle - \sqrt{\tfrac{1}{3}}|1/2,1/2\rangle$$

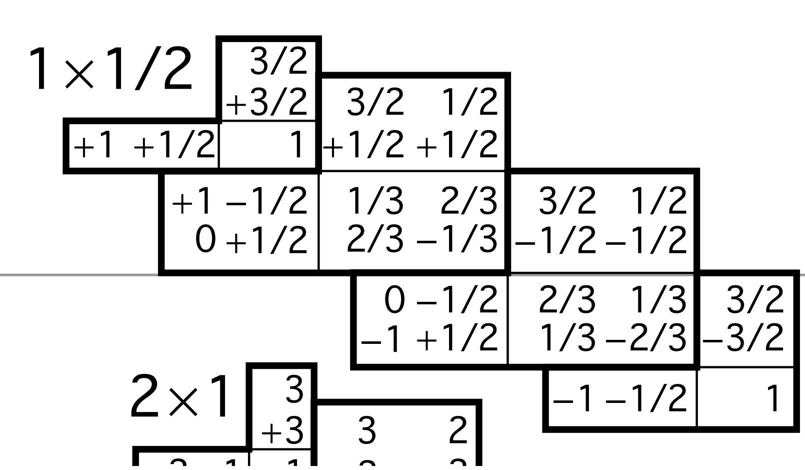
$$A = \left[\sqrt{\frac{1}{3}}\langle 3/2,1/2| + \sqrt{\frac{2}{3}}\langle 1/2,1/2|\right] S \left[\sqrt{\frac{2}{3}}|3/2,1/2\rangle - \sqrt{\frac{1}{3}}|1/2,1/2\rangle\right]$$

$$M_{3/2} = \langle 3/2,1/2|S|3/2,1/2\rangle \qquad M_{1/2} = \langle 1/2,1/2|S|1/2,1/2\rangle$$

$$A = \frac{\sqrt{2}}{3}M_{3/2} - \frac{\sqrt{2}}{3}M_{1/2}$$

- Amplitude expressed in terms of two “underlying” transitions.

One More Example:



$$\pi^0 + n \quad \rightarrow \quad \pi^- + p$$

$$|1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle \quad \rightarrow \quad |1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$$\sqrt{\tfrac{2}{3}}|3/2,-1/2\rangle + \sqrt{\tfrac{1}{3}}|1/2,-1/2\rangle \quad \rightarrow \quad \sqrt{\tfrac{1}{3}}|3/2,-1/2\rangle - \sqrt{\tfrac{2}{3}}|1/2,-1/2\rangle$$

$$A = \left[\sqrt{\frac{2}{3}}\langle 3/2,-1/2| + \sqrt{\frac{1}{3}}\langle 1/2,-1/2|\right] S \left[\sqrt{\frac{1}{3}}|3/2,-1/2\rangle - \sqrt{\frac{2}{3}}|1/2,-1/2\rangle\right]$$

$$A = \frac{\sqrt{2}}{3}M_{3/2} - \frac{\sqrt{2}}{3}M_{1/2}$$

- What about:

$$\pi^0 + p \quad \rightarrow \quad \pi^+ + n$$
$$\pi^0 + n \quad \rightarrow \quad \pi^- + p$$

Summary:

- The very close masses of the p and n is hard to accept as a coincidence
  - Other particle systems (p, K, D, etc.) have nearly degenerate masses
  - Heisenberg postulated:
    - these are multiplets of SU(2) “isospin” analogous to angular momentum
    - strong interactions are invariant under rotations of isospin
  - Today, we understand this due to the near degeneracy of the u,d masses
- Same “algebra” to determine relations between decay and scattering rates
  - add component isospin to determine total isospin of the state
  - match total isospin components before and after to determine which channels are “allowed” by conservation of isospin.
- Isospin is somewhat of an “accidental” property
  - extension to the strange quark with SU(3) doesn’t work as well
  - we will see SU(2) and “isospin” again in a more fundamental context