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Abstract. We report preliminary results from a sequence of very high resolution simulations of the mantle convection process based upon an axisymmetric anelastically compressible spherical model with both Olivine-Spinel and Spinel-post Spinel phase transitions. Our analyses strongly suggest that at Earth-like Rayleigh numbers near 10^7 the circulation may be quite strongly layered, although a very long timescale transient exists that is characterized by brief excursions into the whole mantle state. We suggest the use of a new diagnostic tool that may be employed to determine whether or not the layered state is characteristic of the present day earth.

Introduction

The issue of the radial style of the convective circulation in the Earth's mantle that is responsible for continental drift and sea-floor spreading has yet to be settled unequivocally. In the not too distant past, arguments based upon trace element geochemistry in the Sm/Nd and Rb/Sr systems were construed to require that the lower mantle was much less differentiated than the upper mantle and therefore that the two regions must be convecting separately (O'Nions et al. 1979, Wasserburg and DePaulo 1979, Allegre 1982). Subsequent evidence, however, demonstrated these original arguments to be rather more equivocal than had earlier been supposed (e.g. Hart and Zindler 1989).

Very recently, a fairly complete review of all of the evidence for and against the layered model was assembled in Peltier (1989), in one chapter of which the issue of the radial style of convection was directly addressed (Peltier et al., 1989). There it was concluded that the mantle general circulation should be whole mantle in style unless dynamical influences associated with the phase transitions themselves were able to inhibit the penetration of one or more of these boundaries by the flow. In this letter we will demonstrate that significant layering of the convective circulation in the mantle of the present day earth due to the dynamical influence of the Spinel-post Spinel transition is to be expected.

An Axi-symmetric spherical model of the mantle general circulation

The brief sequence of analyses to be presented here is based upon extensions of the anelastic model of mantle convection recently presented in Solheim and Peltier (1990). The main modifications to this model which have been incorporated for the purpose of the analyses to be discussed here, include the influence of multiple equilibrium phase transitions, and the introduction of a depth dependence for the coefficient of thermal expansion α. The pressure (and

therefore depth) dependence of α has recently been measured by Chopelas and Boehler (1989) who have shown α to be adequately represented by the empirical expression:

$$\frac{\alpha}{\alpha_o} = \left(\frac{\rho}{\rho_o}\right)^{-\delta_T} \tag{1}$$

in which $\delta_T \approx 5$ ($4 \leq \delta_T \leq 6$) is the Anderson-Gruneissen parameter. This important result has been further discussed by Anderson et al. (1990) and Reynard and Price (1990).

The non-dimensional equations of the axi-symmetric mantle convection model that we shall employ for the purpose of the following discussion are detailed in Solheim and Peltier (1991a) and we will not reproduce them here because of space limitations. Important to our manner of handling the dynamical impact of the two phase transitions with Clapeyron slopes and latent heats of reaction β_i and ℓ_i ($i=1,2$) and density jumps $\Delta_i = \rho_{i+1} - \rho_i$, is the introduction of the transformed temperature A, defined as:

$$A = T - \frac{\rho_o}{\alpha_o T_c C_p} \left(\ell_1 \Gamma_1 + \ell_2 \Gamma_2\right), \tag{2}$$

in which the Γ_i are phase density functions defined as:

$$\Gamma_i = \frac{1}{2}\left\{\frac{1}{2} + tanh\left[\left(r_{pi}(\theta,t) - r\right)h\right]\right\}, \tag{3}$$

which represent the fraction of the $(i+1)^{st}$ phase that exists at position (r,θ) and time t. The parameters h_i determine the length scale over which the ith phase change (at radial position r_{pi}) takes place and should correspond to the actual thicknesses of the divariant phase loops of the Olivine-Spinel and Spinel-post Spinel transitions. As we will demonstrate, the values chosen for the h_i are important for the results concerning layered convection that we shall present.

The main feature of the dynamical system upon which we will focus attention here is described by the phase functions Γ_i defined above. For the Olivine-Spinel transition the loop thickness h_1 is approximately 10 km (Akaogi and Ito, 1989; Katsura and Ito, 1989) whereas for the Spinel-Post Spinel transition h_2 is closer to 5 km (Ito and Takahashi 1989). One outcome of the analyses to be reported here is the demonstration that failure to employ sufficiently small loop thicknesses for these dynamical calculations would lead to an underestimate of the ability of the Spinel-Post Spinel transition of inhibit convection across it. It is also important, of course, that the Clapeyron slopes assigned to these transitions be equal to the experimentally determined values. Based upon the laboratory experiments of Akaogi and Ito (1989) and Katsura and Ito (1989) we will therefore assign a Clapeyron slope for the exothermic Olivine-Spinel transition of either +2 MPa/°K or +3 MPa/°K (these values bracketing the range allowed by the experimental measurements), while on the basis of the work of Ito and

Takahashi (1989) we will take the Clapeyron slope for the endothermic Spinel-post Spinel transition to be -2.8 MPa/°K. Our intention will be to demonstrate that with the loop thicknesses and these slopes properly set, then convection through the 670 km phase discontinuity may be strongly inhibited.

Previous work on the ability of mantle phase transitions to inhibit convection through them has been considerably less than definitive. The Cartesian geometry analyses of Christensen (1982) and Christensen and Yuen (1984) have revealed a tendency for some weak layering to develop at very high Rayleigh number in models with a single transition (Spinel-post Spinel), but the layering was not suggested for Earth like values of the Rayleigh number. Recently, Machetel and Weber (1991) have obtained a similar result for a single phase transition in a simple model in axially symmetric spherical geometry for a Rayleigh number that is an order of magnitude lower than that characteristic of the present day earth (which is close to 10^7). They advocate a view in which the endothermic phase change causes the flow to become layered on occasion (i.e. weakly). Yuen et al., (1990), however, have argued that when both the Olivine-Spinel and the Spinel-post Spinel transitions are included in the simulation then no layering is evident in the Earth-like regime. Numerically accurate computations are clearly required to decide the issue.

The results to be reported below were obtained on the CRAY Y-MP 2/64 computer system at the Los Alamos National Laboratory. The nonlinear system was integrated forward in time in the streamfunction - vorticity formulation for the axisymmetric fields and the in-core multigrid solver MUDPACK (from the National Centre for Atmospheric Research) was employed to invert the elliptic equations prior to each advection step of the temperature field. Using these numerical methods we have been able to integrate the convection model on grids with 769 mesh points in the radial direction ($\Delta r = 3.75$ km) and 1025 mesh points in the azimuthal direction. We believe these calculations to be the highest homogeneous resolution simulations of the mantle convection process ever performed.

Results

In Figure 1 we show two cross-sectional views of the temperature field from this high resolution model sufficiently late in the simulation that the chaotic time dependent flow (Solheim and Peltier, 1990) has fully developed. Both calculations are for heated from below circulations for which the Rayleigh number Ra = 10^7, so that both are operating in the "Earth-like" regime. Inspection of this Figure demonstrates that in the no phase transitions case, both the hot upwelling and cold downwelling plumes traverse the radial extent of the mantle unimpeded. In the case with two phase transitions and the Olivine-Spinel slope set to +2 MPa/°K, however, no hot plume is observed (at this time) to be rising through the 670 km boundary, nor is any cold plume observed to be sinking through the boundary. It is important to note, as stated on the Figure, that in the two phase boundary case the thickness of both phase loops has been set at 50 km, that is approximately an order of magnitude thicker than implied by the laboratory data. We might reasonably enquire as to whether the layering apparent

in the Figure for the case of two phase boundaries depends upon this specified thickness.

This influence has been investigated quantitatively by use of a radial mass flux diagnostic F(r) that we first introduced in Solheim and Peltier (1991). This is defined as

$$F(r) = \frac{<\rho_r |w|>}{\frac{1}{(r_s - r_{cmb})} \int_{r_{cmb}}^{r_s} <\rho_r |w|> dr} \qquad (4)$$

in which w is the radial velocity, $\rho_r(r)$ is the basic state density field in the anelastic model, and the angle brackets $<>$ denote an azimuthal average. In Figure 2a we show a sequence of time averaged diagnostics F'(r) for a sequence of models incorporating both phase transitions that differ from one another only in the widths that are assigned to the phase loops at 400 km and 670 km depth, which are taken equal for purposes of illustration. The mass-flux diagnostic is shown for models in which the phase loop thicknesses are set at 100 km, 50 km, and 25 km. For comparison purposes we have also superimposed on this Figure the time averaged mass flux diagnostic for the flow at Ra = 10^7 with no phase
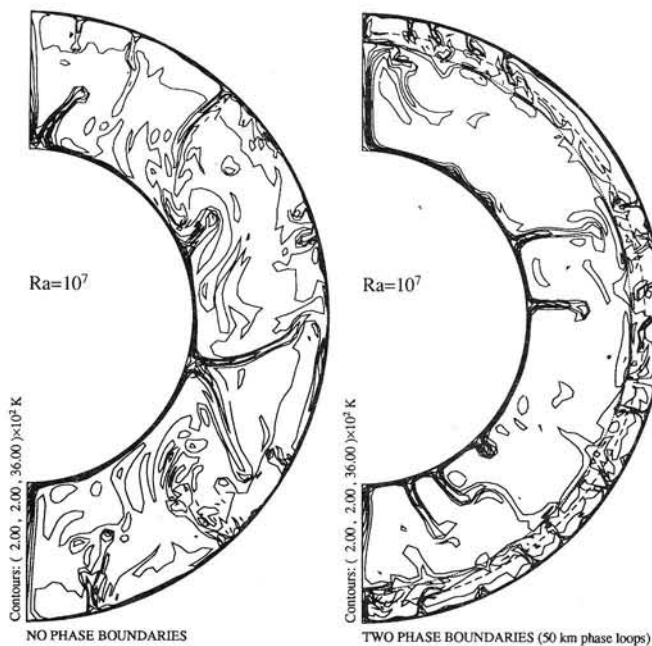

Fig. 2. (a) Radial mass flux diagnostic from equation (5) in the text for axi-symmetric simulations of the mantle convection process at a Rayleigh number of 10^7. The solid line (see Figure caption) is for the model with no phase transitions. The remaining curves are for models with two phase transitions and the simulations differ from one another only in the thicknesses that are assigned to the phase loops. Note that as the thickness of the phase loops decreases the flow goes more and more strongly layered. (b) Same as (a) but comparing mass flux diagnostics for models that differ from one another only in the Clapeyron slope assumed for the Olivine-Spinel transition (β_1). (c) Time series of the mass flux diagnostic at 400 km depth (solid) and 670 km (dashed) for the model with Clapeyron slopes $\beta_1 = 3$MPa/°K and $\beta_2 = -2.8$ MPa/°K and phase loop thicknesses of 50 km.
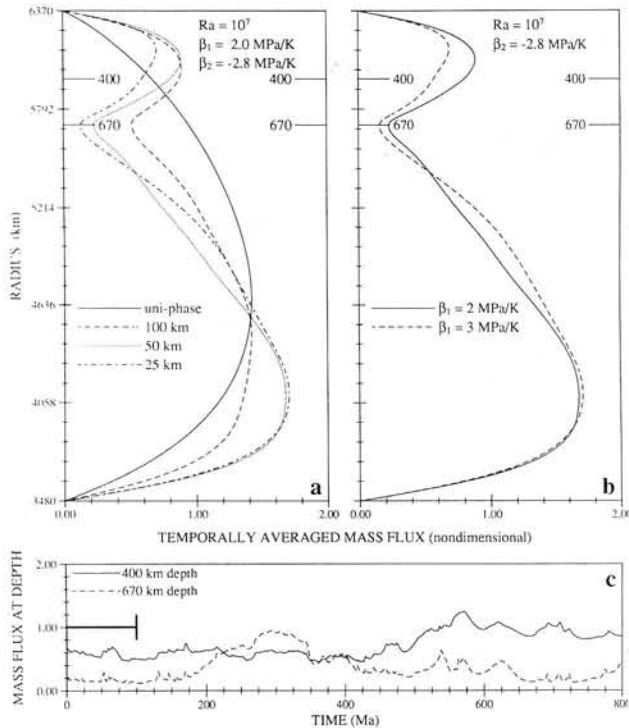
Fig. 1. Instantaneous temperature fields from two axi-symmetric simulations of the mantle convection process at a Rayleigh number of 10^7 Both flows are heated entirely from below. One contains no phase transitions whereas the second contains both a model Olivine-Spinel transition and a model Spinel-post Spinel transition. The thicknesses of the phase loops are both set at 50 km. Note that neither hot ascending plumes nor cold descending plumes appear to penetrate the 670 km discontinuity in the latter case.

transitions. Inspection of this Figure very clearly shows that the 670 km phase boundary has a strongly inhibiting influence upon the mixing of material through it over the range of times for which the averaging has been performed, and that this inhibition increases as the prescribed thickness of the phase loop decreases. This result clearly suggests that even in the absence of chemical layering across the 670 km phase transition, the mantle convective circulation in the modern earth could be substantially layered. In Figure 2(b) we compare mass flux diagnostics for two flows which differ only in the Clapeyron slope β_1 of the Olivine-Spinel transition, a comparison which demonstrates that this has relatively little influence and therefore that the expectation of Yuen et al. (1990) is not realized in our simulations - the Olivine-Spinel transition is unable to induce whole mantle flow if the deeper transition has caused layering.

In Figure 2(c) we provide time series of the massflux diagnostic $F(r,t)$ at both $r=400$ km (solid) and $r=670$ km (dashed). On this diagram the 100 Ma interval over which the averages in plates (a) and (b) were calculated is shown as the heavy solid line. During intervals of time over which the dashed line is below the solid line the circulation is strongly layered, whereas when the opposite is the case the circulation is whole mantle in style. Inspection of the 800 Ma time series in plate (c) demonstrates that, except for a relatively brief period of about 100 Ma duration, the circulation

remains in the strongly layered state. When the integration is continued (results not shown) the system once more (briefly) enters the whole mantle state following 800 Ma, suggesting the presence of a very long time scale transient in the system with a dominant period near 600 Ma. Of the many interesting issues that the results of these calculations raise, one that is especially intriguing is whether or not there might be some diagnostic tool suggested by these results that could be employed to seismically discriminate between the whole mantle and the layered states.

In attempting to establish whether convective layering of the phase transition induced kind discussed above is actually present in the real earth, we wish to suggest here that seismologists involved in the application of tomographic imaging techniques should focus upon attempting to resolve the depth dependent power spectrum of the lateral heterogeneity resolved by their models. To illustrate the possible utility of this diagnostic we show on Figure 3 the depth dependent lateral heterogeneity (of temperature) spectrum from the simulation at $Ra = 10^7$ with two phase transitions used to construct Figure 1. On this Figure we provide three different representations of the expansion $T(r,\theta) = \sum_{\ell=0}^{\infty} C_\ell(r) P_\ell(\cos\theta)$. In the top frame we show $|C\ell|$ vs. ℓ as a function of radius r using a black dot rendering to indicate $|C\ell|$. Superimposed upon this we show the integrated spectral power as a function of depth (solid line). Below the top frame we show power spectral histograms in the top thermal boundary layer (50 km depth), at 400 km depth (the Olivine-Spinel boundary), at 670 km depth (the Spinel-post Spinel boundary), and in the bottom thermal boundary layer (D") at the base of the mantle.

Inspection of Figure 3 demonstrates that strong temperature variance is characteristic of both the top and bottom (cold and hot) thermal boundary layers. One should also note the strong systematic shift of power to lower spherical harmonic degree that is evident as one moves from the upper cold boundary layer to the lower hot boundary layer. In this two phase transition case it is also clear that there is a strong (tomographically observable?) concentration of spectral variance at the depth of the 670 km discontinuity across which mass flux may be strongly inhibited by the dynamical influence of the endothermic Spinel-post Spinel phase transformation.

Conclusions

The above described analyses demonstrate, we believe, a number of important points. Firstly it now appears quite clear that the phase transition at 670 km depth in the mantle may strongly inhibit convective mixing through this horizon and that this inhibiting effect on the circulation is not significantly decreased by the convection enhancing effect of the Olivine-Spinel transition. The mantle convective circulation may therefore be quite significantly (although imperfectly) layered on occasion, not because of an increase of mean atomic weight across the 670 km discontinuity, but by the exothermic phase transition itself.

Clearly the above described calculations must be extended to investigate a number of further impacts upon the scenario for layering presented here. Among these we consider the influence of internal heating to be important, principally because tomographic models (e.g. Forte and Peltier 1987) do
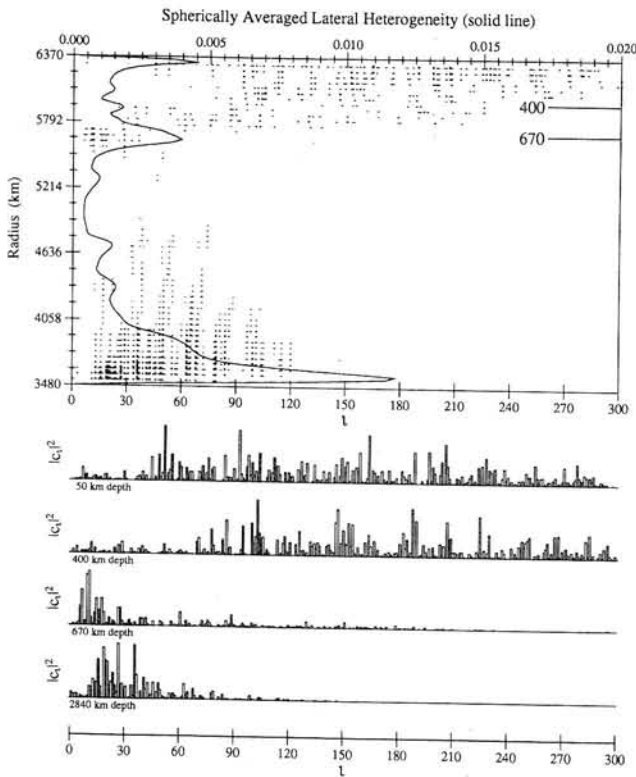
Fig. 3. Depth dependent lateral heterogeneity spectra for the axi-symmetric convection model with Ra = 10^7 and two phase transitions. The black line on the upper plate shows the integrated power in the depth dependent lateral heterogeneity spectrum.

not show the extreme concentration of variance in D" that would be expected on the basis of the heated from below convection results presented in Figures 3 and 4. Also important, we believe, is the influence of variable viscosity upon the mass flux inhibition phenomenon. Whether, and to what extent, this phenomenon is influenced by the three dimensional nature of the actual circulation is of course also an issue that will be addressed in the context of a new 3-D model that is now in the design stage (Karpik and Peltier 1991).

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