ON THE DEVELOPMENT OF POLAR LOW WAVETRAINS

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ABSTRACT

Our previous work on the stability of deformation-induced frontal zones has led to the discovery of a new cyclone-scale mode of baroclinic instability. A crucial factor that led to this discovery was the use of primitive equations in the formulation of the stability problem. In fact, we have shown that this mode is filtered out by both the quasi-geostrophic and geostrophic momentum approximations to the primitive equations. In this paper, we apply our methodology to the problem of identifying the dynamic processes responsible for the development of polar low wavetrains. Observational evidence has shown that these wavetrains or families develop along shallow baroclinic zones that are situated north of the primary polar front. Usually three or four coherent disturbances make up a wavetrain. Each member of the wavetrain is typically observed to develop from a small amplitude perturbation into a fully developed cyclone with a characteristic wavelength of approximately 500 km. Such a development is indicative of the existence of a dynamic instability. Indeed we propose that the new cyclone-scale mode of baroclinic instability is responsible for the development of the polar low wavetrains. To demonstrate this, we will show that the stability characteristics of a typical baroclinic zone in which a wavetrain was observed to develop are very similar to those of the model frontal zones that we have previously studied. In addition, we will demonstrate that the wavelength and structure of the most unstable wave predicted by our theory are in good agreement with observations.

1. INTRODUCTION

The conventional interpretation of all midlatitude cyclones is that they are manifestations of the baroclinic instability mechanism first described by Charney

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(1947) and Eady (1949). Charney (1975) writing in his preface to the "Selected Papers of J.A. Bjerknes" clearly recognized that this was not the case. He stated that there appeared to be some fundamental dynamic difference between the upper level long waves that quasi-geostrophic theory so successfully describes, and the shorter wavelength frontal cyclones upon which Bjerknes (1919) had focused his attention. However, until recently no clear theoretical explanation supporting the necessity of such distinction has been forthcoming.

Moore and Peltier (1987) described a detailed analysis of the stability of realistic atmospheric frontal structures against arbitrary three-dimensional perturbations. The frontal structures employed in this initial investigation were those generated by the action of a hyperbolic deformation field on a previously existing large-scale horizontal potential temperature gradient. The semi-geostrophic theory of Eliassen (1948) and Hoskins and Bretherton (1972) was employed to describe the process of frontogenesis. The frontal zones generated in this way, for which the assumption of uniform potential vorticity was also employed, were used as basic states for the purpose of the stability analysis. In this analysis the quasi-geostrophic approximation was *not* invoked and the complete nonseparable eigenvalue problem was solved *without* approximation.

The main result from this work was the demonstration of the existence of a short wave branch of unstable normal modes in the eigenspectrum. The fastest growing mode in this new branch was found to have a horizontal wavelength somewhat less than 1000 km, in close accord with the observed scale of frontal cyclones (Bjerknes and Solberg, 1922; Harrold and Browning, 1969; Reed, 1979). An analysis of the energy budget for the new cyclone scale mode demonstrated that it was also driven by the baroclinic instability mechanism.

The relationship between this new mode of baroclinic instability and the classical Charney-Eady mode has recently been the subject of a number of further investigations. The most important of these (Moore and Peltier, 1989a) has been the demonstration that the quasi-geostrophic approximation completely filters the new mode from the dynamic system while leaving the Charney-Eady mode only slightly affected. This can be understood on the basis of the fact that the short wavelength mode in the primitive equations system is boundary confined. It is therefore excluded from the instability spectrum of a constant potential vorticity basic state by the Charney-Stern theorem of quasi-geostrophic theory. When semi-geostrophic theory was employed in the stability analysis (Moore and Peltier, 1989b), it was found that it is similarly incapable of supporting the new short wavelength cyclone mode.

An important question that arises out of the discovery of this mode is that of the role it plays in the generation of observed cyclone scale disturbances, such as polar lows and comma clouds. Of particular interest are cases in which a coherent family or wavetrain of such disturbances develops (Harrold and Browning, 1969; Reed, 1979). A favoured location for the development of these wavetrains is the Norwegian Sea (Mansfield, 1974; Duncan, 1977). Recently, Reed and Duncan (1987) described one such case in which a family of four polar lows was observed to grow along a quasi-two-dimensional shallow baroclinic zone of low static stability. What makes their case so interesting is that each member in the wavetrain was observed to grow from a small perturbation into a fully developed polar low. They state that the time required for the disturbances to double in amplitude was in the range of 8 to 24 hr. More importantly, the polar lows in the wavetrain had a characteristic wavelength of approximately 500 m.

The growth of a family of waves from small amplitude perturbations to fully developed disturbances is indicative of the existence of a dynamic instability of the environment in which the waves appeared. In an attempt to identify this instability, Reed and Duncan (1987) performed a linear stability analysis of the underlying baroclinic zone. In this analysis, they made use of the quasi-geostrophic approximation. There are two apparent problems with their approach. First, the static stability in the environment in which the polar lows grow varies strongly in the horizontal. Second, the background Richardson number field is of order unity. Both of these factors imply that the quasi-geostrophic approximation to the primitive equations is invalid. If suitable modifications were made to the environment, i.e., the elimination of the observed horizontal variations in the static stability and along-front wind fields, then the results of their quasi-geostrophic analysis indicated that unstable perturbations with wavelengths on the order of 500 km could grow via the baroclinic instability mechanism. The doubling times for these quasi-geostrophic waves, however, were found to be long compared to the observed doubling times. This realization led Reed and Duncan (1987) to propose that some sort of convective instability was also needed to account for the rapid development of the disturbances.

We will show that when the primitive equations are employed in the stability analysis, the results are far more robust and in closer accord with the observations. Most importantly, they demonstrate that the stability characteristics of the baroclinic zones in which the polar low wavetrains develop are very similar to those of the frontal zones investigated by Moore and Peltier (1987). To accomplish this, we will consider the stability of two-dimensional nonseparable baroclinic zones to three-dimensional small amplitude perturbations. We begin by reviewing the theory of nonseparable baroclinic instability and its application to the phenomenon of frontal cyclogenesis.

2. THE THEORY OF NONSEPARABLE BAROCLINIC INSTABILITY AND FRONTAL CYCLOGENESIS

The problem that we are obliged to solve is that of the determination of the stability of a two-dimensional baroclinic zone, consisting of an along-front wind field $\bar{V}$ and corresponding potential temperature field $\bar{\theta}$, against arbitrary three-dimensional small amplitude perturbations that obey the full hydrostatic primitive equations.

There is an unfortunate inconsistency between the nomenclature used to describe the baroclinic zone and that used to describe the mechanisms by which unstable waves can grow on such a zone. The root of this inconsistency concerns the choice for the orientation of the coordinate system that is to be employed. Frontogenesis theory (Hoskins and Bretherton, 1972) assumes that the baroclinic zone varies in x and z but not y, while conventional baroclinic instability theory (Charney, 1947; Eady, 1949) assumes that the mean state is a function of y and z but not x! Prior to the work of Moore and Peltier, (1987), no one had considered the problem of determining the stability characteristics of realistic baroclinic zones. As a result, no one has been obliged to face this inconsistency. In this and our previous analyses we have chosen to retain the coordinate system that arises out of frontogenesis theory. As such, our x axis is in the cross-front direction and our y-axis is in the along-front direction. Provided that $\bar{V}$ and $\bar{\theta}$ are in thermal wind balance, viz:

$$f\frac{\partial \bar{V}}{\partial z} = \frac{g}{\theta_o}\ \frac{\partial \bar{\theta}}{\partial x}\ , \tag{1}$$

then the mean state constitutes a steady two-dimensional solution to the hydrostatic primitive equations. It should be noted that in writing (1) we have employed the pseudoheight of Hoskins and Bretherton (1972) as our vertical coordinate. The stability analysis of such a mean state leads to the formulation of a nonseparable two-dimensional boundary value problem. The atmospheric dynamics group at Toronto has solved a number of such problems (Klaassen and Peltier, 1985; Moore and Peltier, 1987, 1989a, 1989b; Laprise and Peltier, 1989) by making use of ideas from the Floquet theory (Jordan and Smith, 1977) for differential equations with periodic coefficients. The assumption made is that the stability characteristics of a given state $(\bar{V},\bar{\theta})$ are the same as those of a lattice of such mean states periodic in x. It then follows from this imposed periodicity, that the normal modes of the set of linear partial differential equations that describe the evolution of small amplitude perturbations have the following functional form:

$$
\begin{aligned}
F'(x,y,z,t) &= Re\left[F^{\dagger}(x,z)\, e^{i(ax+by)}\, e^{st}\right] \\
F^{\dagger}(x,z) &= F^{\dagger}(x+L,z)
\end{aligned}
\tag{2}
$$

where F' represents any of the five hydrodynamic fields that describe the perturbation, a is the cross-front Floquet number, b is the along-front wave-number, s is the complex growth rate, and L is the underlying periodicity of the lattice.

Substitution of normal mode expansions (2) for each of the hydrodynamic fields into the full nonhydrostatic primitive equations linearized about a mean state $(\bar{V}, \bar{\theta})$ yields the following set of stability equations (Moore and Peltier, 1987):

$$(s + ib\,\bar{V})\,U^{\dagger} - fV^{\dagger} + (\partial_x + ia)\,\phi^{\dagger} = 0 \tag{3a}$$

$$(s + ib\bar{v})\,V^{\dagger} + (D^{\dagger} + f)\,\bar{V} + ib\phi^{\dagger} = 0 \tag{3b}$$

$$(s + ib\bar{V})\,W^{\dagger} - \frac{g}{\theta_o}\,\theta^{\dagger} + \partial_z\phi^{\dagger} = 0 \tag{3c}$$

$$(s + ib\bar{V})\,\theta^{\dagger} + D^{\dagger}\bar{\theta} = 0 \tag{3d}$$

$$(\partial_x + ia)\,U^{\dagger} + ibV^{\dagger} + \partial_z W^{\dagger} = 0. \tag{3e}$$

In this system the operator $D^{\dagger}$ is defined as:

$$D^{\dagger} = U^{\dagger}\partial_x + W^{\dagger}\,\partial_z. \tag{3f}$$

Subject to the hydrostatic approximation and making use of Galerkin expansions for each of the perturbation hydrodynamic fields $(U^{\dagger}$,

$V^{\dagger}, W^{\dagger}, \theta^{\dagger}, \phi^{\dagger}$), the above system may be reduced to a matrix eigenvalue problem of the form (see Moore and Peltier, 1987 for details):

$$s\vec{\chi} = M\vec{\chi} \tag{4}$$

where $\vec{\chi}$ is the vector of projections of the perturbation hydrodynamic fields onto the basis functions used in the Galerkin expansions and M is the complex stability matrix. In general M is a function of the Floquet number a, the along-front wavenumber b and the mean state $(\bar{V}, \bar{\theta})$. As described by Moore and Peltier (1987), we will restrict consideration to the case in which $a=0$.

For a given mean state, (4) is solved to yield spectra of the growth rate ($\sigma = Re\ s$) and phase speed ($Cph = -Im\ s/b$) as a function of along-front wavenumber b. A mode with wavenumber b has a wavelength $\lambda = 2\pi/b$ and is said to be unstable if its growth rate is positive. If this is the case, then the mode will double in amplitude in a time $Td = \ell n(2)/\sigma$. One of the advantages of the Galerkin method is that it simultaneously finds all the normal modes for a given wavenumber b. As a result, it allows for the identification of the harmonics of the fundamental modes of instability. Although these harmonics are not in general physically realizable (having growth rates well below those of the fundamentals), nevertheless they are indicative of the symmetries contained within the underlying basic state. The power method employed by others (Duncan, 1977; Reed and Duncan, 1987) does not have this important capability.

An examination of the energy budgets of the unstable modes provides an understanding of the physical processes responsible for their growth. From (3), one can show that:

$$2\sigma\, KE' + ib\bar{V}\, KE' = RS + VHF + \partial_x\{U^{\dagger}\phi\} + \partial_z\{W^{\dagger}\phi^{\dagger}\} \tag{5}$$

$$2\sigma\, PE' + ib\bar{V}\, PE' = HHF - VHF \tag{6}$$

where:

$$KE' \text{ (the eddy kinetic energy density)} = \frac{1}{2}\left[\{U^{\dagger 2}\} + \{V^{\dagger 2}\}\right] \tag{7}$$

$$PE' \text{ (the eddy potential energy density)} = \frac{1}{2}\{\theta^{\dagger 2}\} \tag{8}$$

$$RS \text{ (the Reynolds stress term)} = -\frac{1}{2}[\partial_z\bar{V}\, Re\{V^{\dagger}W^{\dagger *}\} + \partial_x\bar{V}\, Re\{V^{\dagger}U^{\dagger *}\}] \tag{9}$$

$$VHF \text{ (the vertical heat flux term)} = \frac{1}{2}\, Re\{W^{\dagger}\theta^{\dagger *}\} \tag{10}$$

and

$$HHF \text{ (the horizontal heat flux term)} = -\frac{1}{2}\partial_x\bar{\theta}\, Re\{U^{\dagger}\theta^{\dagger *}\} \tag{11}$$

In the above, the along-front averaging operator { } is defined by:

$$\{\psi\} = \frac{b}{2\pi}\int_{o}^{2\pi/b} \psi\, dy \tag{12}$$

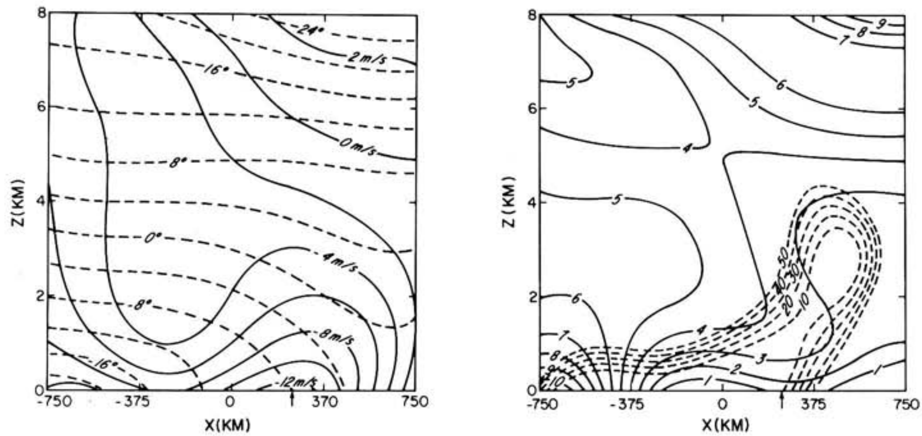
3. THE GENESIS OF POLAR LOW WAVETRAINS

We are now in a position to apply this theory to determine the stability characteristics of the baroclinic zone upon which the polar low wavetrain described by Reed and Duncan (1987) was observed to develop. Figure la shows the cross sections of the along-front velocity and potential temperature fields for this zone. The cross sections were deduced from the operational analysis done by the European Center for Medium-Range Weather Forecasts (ECMWF). As described by Reed and Duncan (1987), this analysis was not able to resolve the individual polar lows. It therefore provides a representation of the synoptic scale environment in which the wavetrain developed. The Richardson number field

$$Ri = \frac{g}{\theta_o}\ \frac{\partial_z\bar{\theta}}{(\partial_z\bar{V})^2} \tag{13}$$

and the static stability field

$$S = \partial_z\bar{\theta} \tag{14}$$



(a) *(b)*

Figure 1: (a) Cross sections of potential temperature (dashed lines, °C) and along-front velocity (solid lines, m s^{-1}) representing the basic state in which the polar low wavetrain of Reed and Duncan (1987) was observed. (b) Cross sections of static stability (solid lines, °C km^{-1}) and Richardson number (dashed lines) derived from the fields shown in Figure 1a. The arrows at the bottom of each plot indicate the location in which the polar lows were observed to develop.

associated with this baroclinic zone are displayed in Figure 1b. The arrow indicates the storm track of the polar lows in both parts of Figure 1. Examination of this figure shows that the cyclones nucleated in a region in which both the static stability and Richardson number were small. Also evident is the large horizontal variation in both the along-front wind and the static stability. The latter important characteristic of the environment in which polar lows develop has been neglected in previous studies (Mansfield, 1974; Duncan, 1977; Reed and Duncan, 1987).

Displayed in Figure 2 are the spectra predicted by our theory for the growth rate and phase speed of the unstable waves that can develop in the baroclinic zone shown in Figure 1. It should be emphasized that no modifications to the baroclinic zone have been made. Inspection of these spectra shows that there are three distinct branches of unstable waves. The branch with a growth rate maximum at $b=1.6$ (representing a wavelength of approximately 3000 km) represents the classical long wave Charney-Eady branch of baroclinic instability. That with a maximum at $b=10$ (wavelength of approximately 500 km) corresponds to the cyclone scale branch. Note that also present is the first harmonic of the cyclone scale branch. As described in Moore and Peltier (1987), the waves

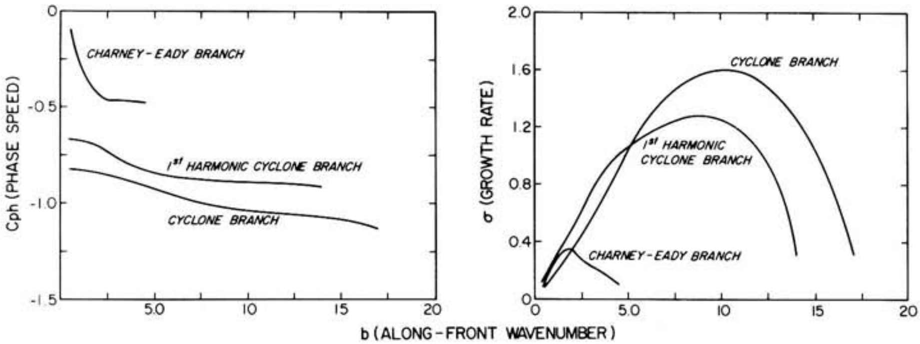


Figure 2: Nondimensional phase speed (C_{ph}) and growth rate (σ) vs. along-front wave number (b) spectra for the basic state shown in Figure 1. The phase speeds, growth rates, and wavenumbers have been scaled by $8\ m\ s^{-1}$, $10^{-5}\ s^{-1}$ and $1.25 \times 10^{-6}\ m^{-1}$, respectively.

in the Charney-Eady branch represent deep disturbances and as a result, they tend to have relatively low phase speeds (indicative of steering levels in the middle troposphere). In contrast, the waves in the cyclone scale branches are shallow and boundary confined and thus they tend to have higher phase speeds. Examination of Figure 2 shows that this is also the case for the present analysis. However, unlike the spectra described in Moore and Peltier (1987), the waves in the cyclone scale branch have much larger growth rates than those in the Charney-Eady branch. This result can be attributed to differences in the structure of the baroclinic zones analysed in Moore and Peltier (1987) as compared to the one considered here. Most importantly, the baroclinic zone under study here is relatively shallow, while those investigated by Moore and Peltier (1987) were quite deep.

From Figure 2, we see that there is a distinct maximum in the growth rate spectra. The wavelength of this most unstable normal mode, which is a member of the cyclone scale branch, is 500 km. The time required for it to double in amplitude is 9.6 hr and its phase speed is $-8\ \mathrm{m\ s^{-1}}$. The wavelength and doubling time are in good agreement with the observations made by Reed and Duncan (1987). The only serious discrepancy is in the phase speed, which is larger than the observed by a factor of approximately 2.

To identify the regions in which the normal modes develop and the mechanisms by which they develop, we present in Figures 3, 4, and 5 contour

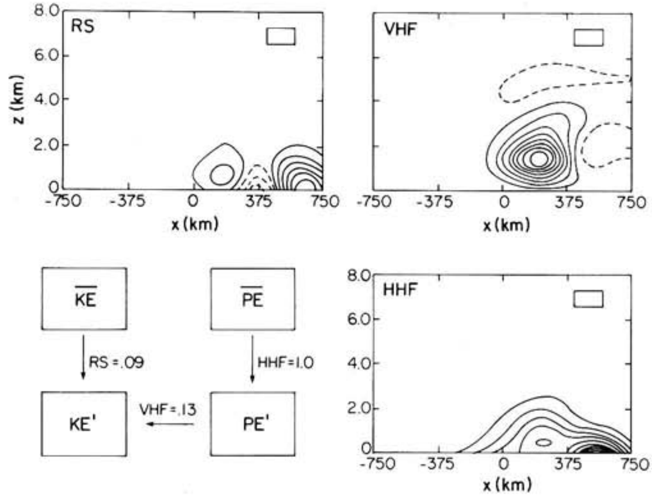


Figure 3: The energy flux terms and energy budget for the most unstable wave in the Charney-Eady branch on the basic state in Figure 1. The box in the upper right corner of the flux term plots indicates the resolution of the Galerkin expansions.

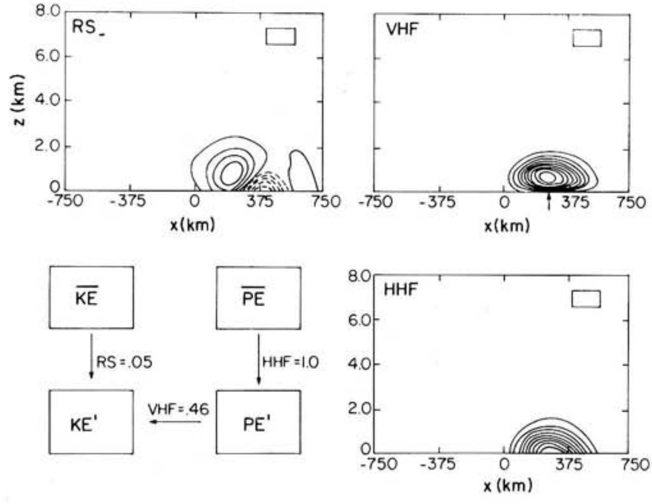


Figure 4: As in Figure 3, but for the most unstable wave in the cyclone branch. The arrow under the VHF plot shows the location in which the polar lows were observed to develop.

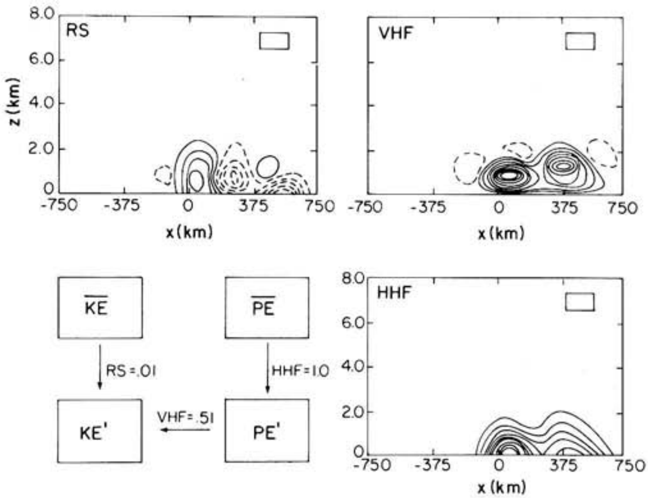


Figure 5: As in Figure 3, but for the most unstable wave in the 1st harmonic cyclone branch.

plots of the spatial distribution of the horizontal (*HHF*) and vertical (*VHF*) heat flux terms and the Reynolds stress term (*RS*) for the most unstable waves in each of the Charney-Eady, cyclone, and first harmonic cyclone branches. The sign convention used in Eqs. (5)–(11) has been adopted. Regions in which the terms are positive (negative) are contoured with solid (dashed) lines. The relative magnitudes of the various terms (normalized by *HHF*) are indicated in the corresponding energy box diagrams.

Inspection of these figures indicates that all three waves are growing by converting potential energy stored in the baroclinic zone into eddy kinetic energy. This conversion is accomplished by means of the baroclinic instability mechanism. As discussed above the Charney-Eady wave is a rather deep disturbance, while both cyclone waves are shallow and boundary confined. The waves in the fundamental branch have a singlet structure with only one maximum in the heat flux terms. By contrast, the waves in the first harmonic branch have a doublet structure with two maxima in the heat flux terms. Also illustrated in Figure 4 is the location in which the polar lows were observed to nucleate. This is exactly the region in which the cyclone wave has its maximum amplitude. It is important to note that in the quasi-geostrophic stability analyses of Reed and Duncan (1987), the region in which the unstable waves developed was very sensitive to the modifications made to the background baroclinic zone. As we have made no such modifications, our theory does not suffer from this unphysical sensitivity.

In summary, our results demonstrate that the stability characteristics of the baroclinic zone shown in Figure 1 are very similar to those of the frontal zones investigated by Moore and Peltier (1987). Of greatest importance is the fact that the baroclinic zone in Figure 1 is indeed unstable to waves in the cyclone scale branch of baroclinic instability. The structure and organization of the most unstable wave in this branch is very similar to that of the polar lows observed by Reed and Duncan (1987). This large measure of similarity leads us to propose that the cyclone scale branch of baroclinic instability discovered by Moore and Peltier (1987) is responsible for the initial development of coherent families of polar lows.

4. CONCLUSIONS

In this paper, we have demonstrated that the environment in which polar low wavetrains are observed to develop is unstable to the cyclone scale branch of baroclinic instability discovered by Moore and Peltier (1987). The predicated doubling time and wavelength of the most unstable wave in the branch are in good agreement with the observations made by Reed and Duncan (1987). Furthermore, the wave was observed to have its maximum amplitude in the region of the zone in which the static stability and Richardson number fields had their minimum. This is precisely the region in which the polar lows in the wavetrain were observed to nucleate. Our ability with a dry adiabatic theory to account for the rapid initial growth of such disturbances argues that their genesis need not involve a strong feedback with moist diabatic processes. Our prediction as to the phase speed of this wave is larger than was observed. However, it should be emphasized that our theory is valid only for the initial stages of the development of the cyclones. The observed phase speeds were those appropriate for finite amplitude disturbances. The finite amplitude behaviour of these disturbances are now being investigated with a nonlinear model that is initialized with the most unstable normal modes predicted by linear theory.

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