# Critical reflection of internal waves off the sea surface

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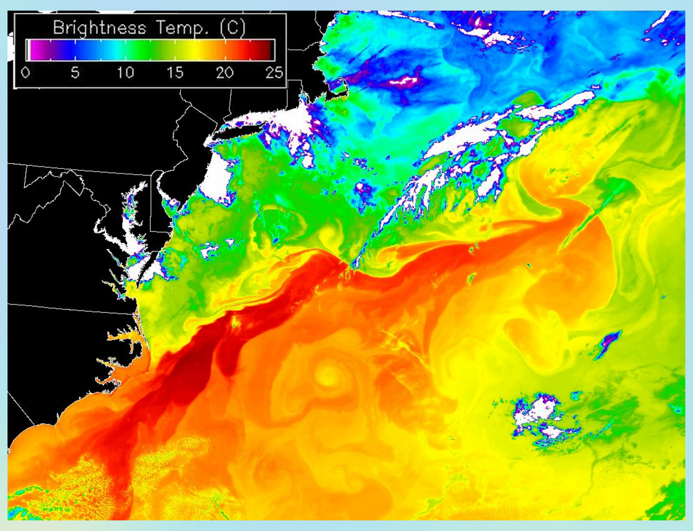
## 1. Motivation

- Mesoscale vortices (~100 km):
  - 90% of the ocean's kinetic energy
  - Geostrophic, very robust (inverse energy cascade)
- Oceanic fronts: horizontal boundaries between water masses (e.g. Gulf Stream), ~10 km wide, featuring:
  - strong lateral density gradient, thermal wind shear,
  - strong ageostrophic motions, enhanced turbulence,
  - strong internal wave activity.
- Oceanic fronts: hotspots for the dissipation of geostrophic energy?

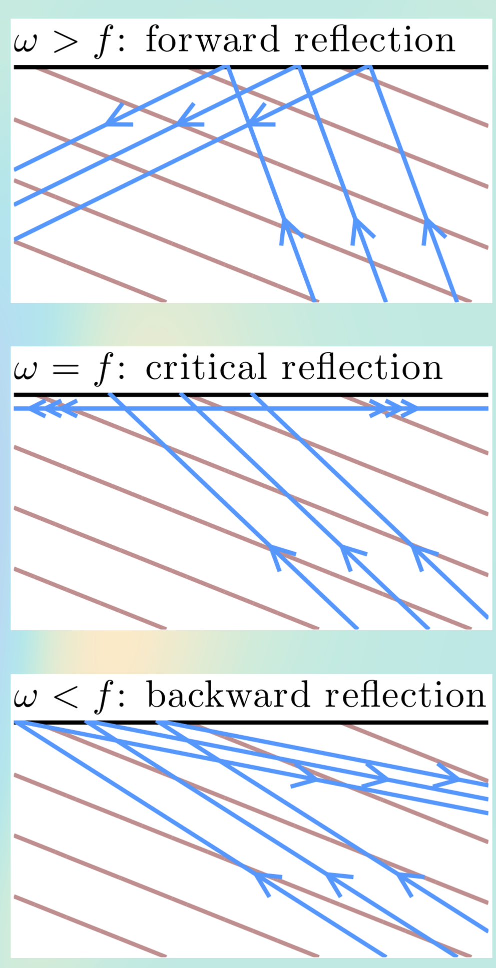
*↓ Gulf Stream: strong oceanic front (MODIS)*




Can internal waves extract geostrophic energy from fronts?

## 2. Critical, forward and backward reflections

- Lateral buoyancy gradient: $S^2 = -\frac{g}{\rho_0}\frac{\partial \bar{\rho}}{\partial x}$
- Thermal wind shear with $Ri_G = \frac{f^2 N^2}{S^4}$,
- No lateral shear ($Ro_G$=0)
- Dispersion relationship for internal waves: $m^2\omega^2 = k^2N^2 + m^2f^2 - 2kmS^2$
- Waves can oscillate at $\omega < |f|$: $\omega_{min} = f\sqrt{1-1/Ri_G}$
- Slope of characteristics: $(k/m)_\pm = S^2/N^2 \pm \sqrt{S^4/N^4 + (\omega^2 - f^2)/N^2}$
- For ω = *f*, critical reflection against the ocean surface: *slope* = 0.
- **Internal waves reflecting off the surface can experience critical reflection for ω = *f*.**
- Viscous effects?




## 3. Near-critical linear reflection: theory

**In the inviscid case, simple:** $\left((f^2-\omega^2)\partial_z^2 - 2ikS^2\partial_z - k^2N^2\right)\hat{\phi} = 0$

with $\phi = \hat{\phi}(z)\exp i(kx - \omega t)$, $\phi = u, v, w, b, p, \psi$ or else…

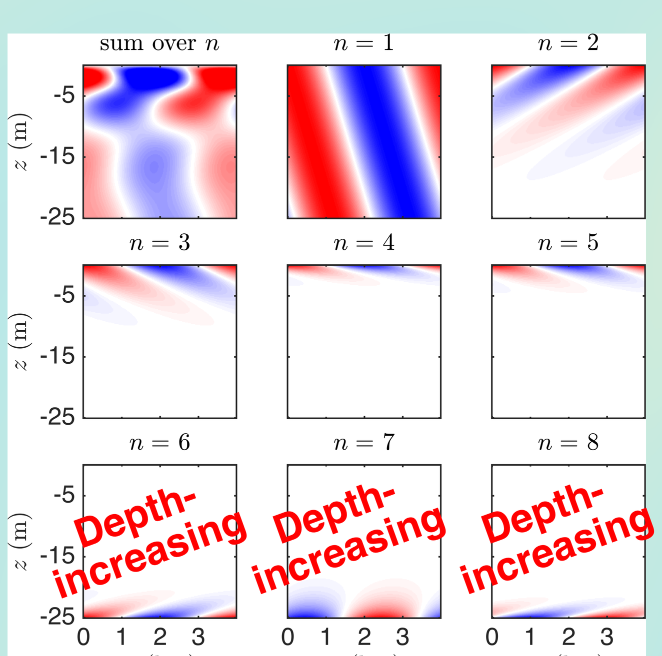
**But with viscosity, not so much:**

$$\left(i\omega + \nu\partial_z^2\right)\left[\nu^2\partial_z^6 + 2i\nu\omega\partial_z^4 + (f^2-\omega^2)\partial_z^2 - 2ikS^2\partial_z - k^2N^2\right]\hat{\phi} = 0,$$

with $\phi = \hat{\phi}(z)\exp i(kx - \omega t)$, $\phi = v, b$ or $p$ (but not $u$, $w$ or $\psi$).

$\tilde{\phi} = e^{rz}$, $r \in \mathbb{C} \Rightarrow$ **eigth possible $r$'s, four of them >0 ( ⇔ decay with depth).**

$$\phi = \sum_{n=1}^{8}\hat{\phi}_n(x,z,t), \qquad \hat{\phi}_n = \tilde{\phi}_n \exp(r_n z + ikx - i\omega t)$$




*← top left: an example of a near-critical reflection*
*Other panels: exp(ikx + $r_n$z);*
*n = 1, 6, 7 and 8 increase with depth: n=1 is the incident wave, the rest is unphysical.*

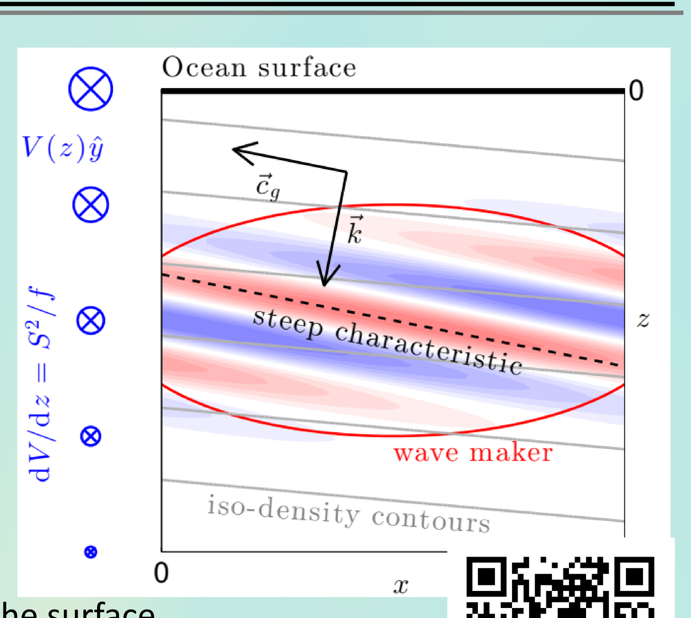
**Viscous solutions matter only for near-critical reflections.**

**Boundary conditions + Polarization relations + assume incident wave is known**
=> 4 equations X 4 unknown
=> **we can compute the full flow analytically**

$$\partial_z u|_{top} = \partial_z v|_{top} = w|_{top} = b|_{top} = 0 \quad \Rightarrow \quad \sum_{n=1}^{5}\tilde{u}_n r_n = \sum_{n=1}^{5}\tilde{v}_n r_n = \sum_{n=1}^{5}\tilde{w}_n = \sum_{n=1}^{5}\tilde{b}_n = 0$$

## 4. Numerical Set-Up

- Two-dimensional (*x*, *z*) simulations,
- nx = 256, nz = 1024,
- Geostrophic Richardson number: $Ri_G$ = 1.05
- Waves forced in the volume, minimal generation of PV
- Forcing amplitude tuned such that the incident wave has a given Richardson number when reaching the surface
- Spectral (Fourier/SinCos) code (Winters *et al.* 2004)




## 5. Energy extraction

$$\frac{\partial\langle\overline{E}\rangle}{\partial t} + \langle pw\rangle\big|_{z=z_{box}} + S^2\left\langle\overline{\frac{ub}{N^2} + \frac{vw}{f}}\right\rangle - \nu\left\langle\overline{\vec{u}_h\cdot\frac{\partial^2\vec{u}_h}{\partial z^2} + \frac{b^2}{N^2}\frac{\partial^2 b}{\partial z^2}}\right\rangle = 0$$

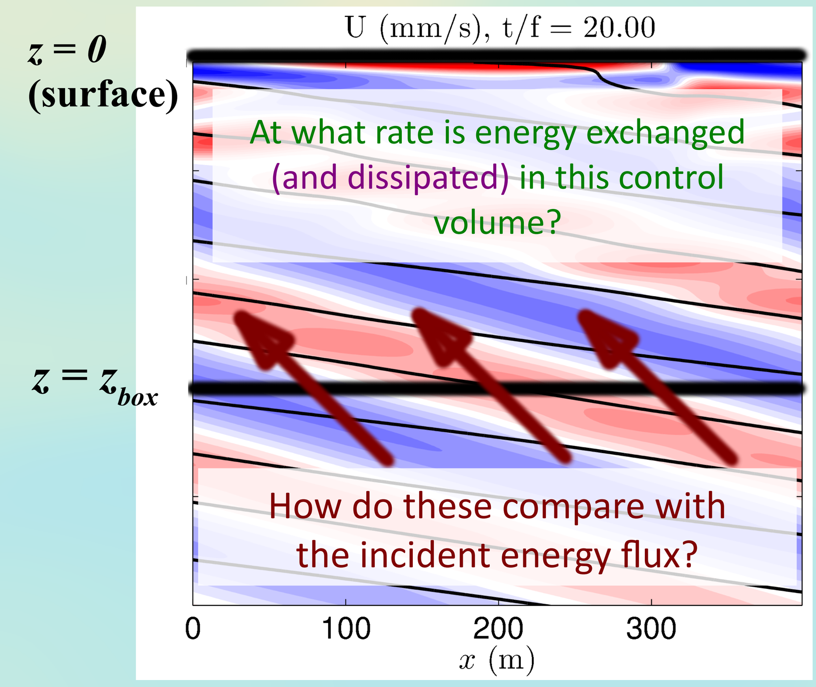
Vertical flux

Exchange with front (potential + kinetic)

Dissipation

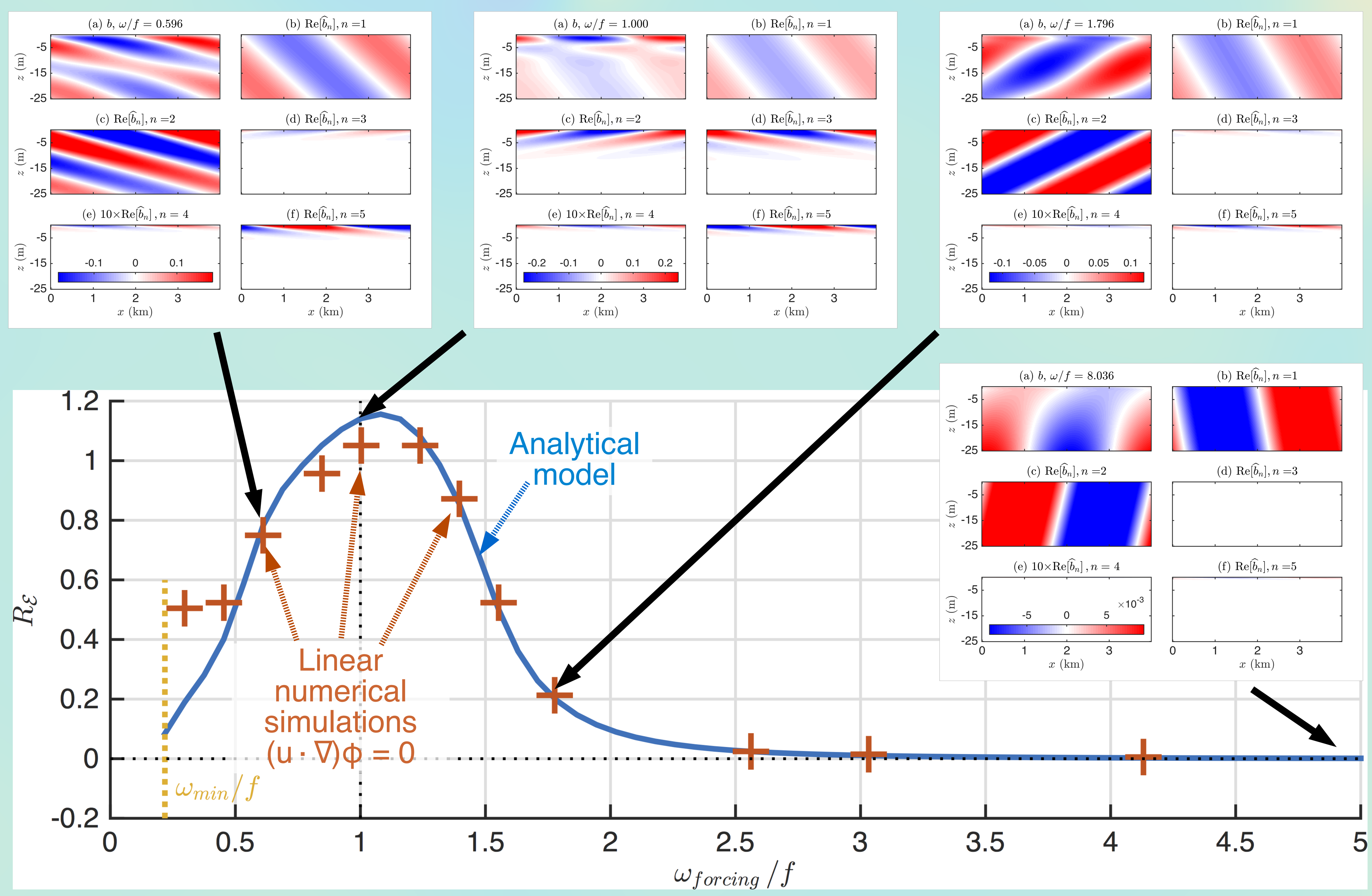
$$E = \frac{1}{2}\left(|\vec{u}^2| + b^2/N^2\right); \quad \langle\Phi\rangle = \frac{1}{L}\int_0^L \Phi\, dx; \quad \overline{\Phi}(z) = \int_0^{z_{box}}\Phi\, dz$$

Exchange ratio: $$R_\mathcal{E} = \frac{S^2\langle\overline{ub}/N^2 + \overline{vw}/f\rangle}{\langle pw\rangle\big|_{z=z_{box}}}$$



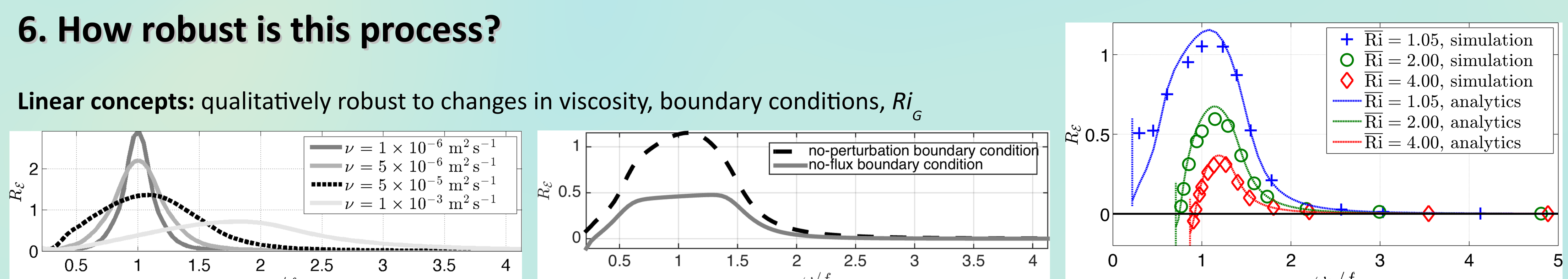

Near-inertial waves reflecting off the surface in strong fronts (RiG = O(1)) extract and dissipate a significant amount* of geostrophic energy.

* at a rate of the same order of magnitude as the wave's incident energy flux

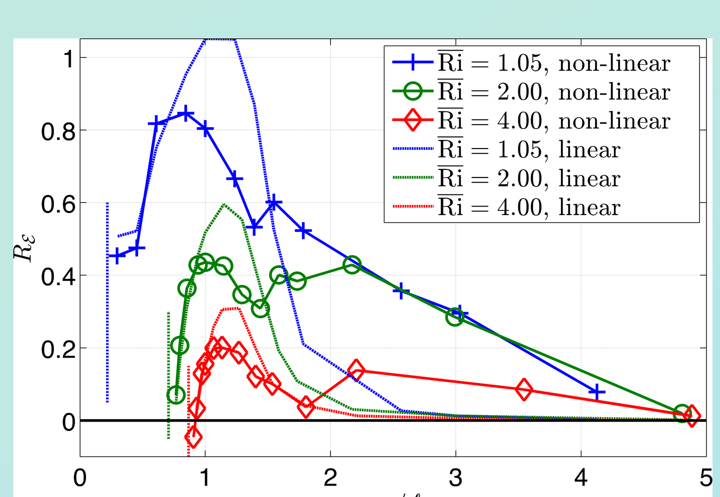



## 6. How robust is this process?

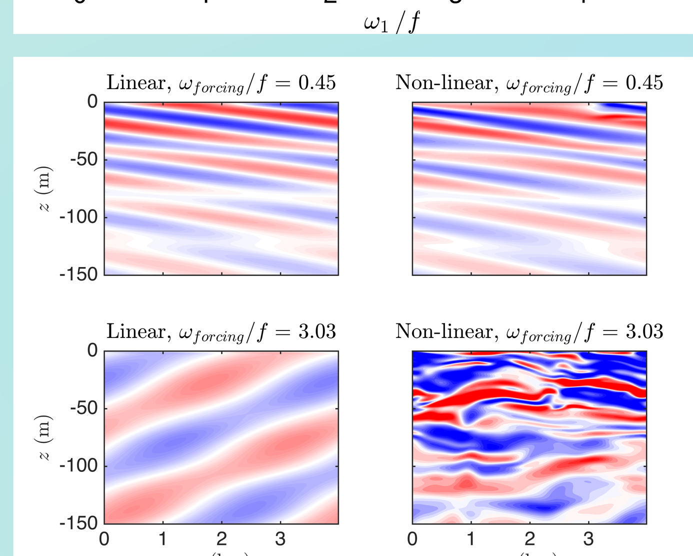
**Linear concepts:** qualitatively robust to changes in viscosity, boundary conditions, $Ri_G$

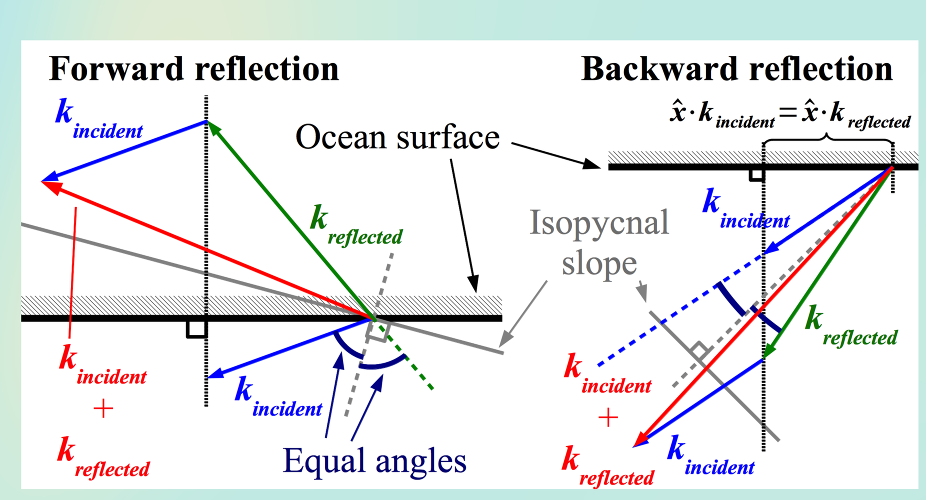



**Non-linear effects:**




- Linear and non-linear backward reflections are similar.
- Linear and non-linear forward reflections are very different.
- Backward reflections do not favour triadic resonances (cf. messy arrow sketch),
- Forward reflections do.
- Triadic resonances trigger weak, then full, turbulent-like cascades.
- Smaller scales propagate slower, pin down energy under the surface, and dissipate: increase $R_\mathcal{E}$




*← Non-linearly interacting incident and reflected waves create frequencies ω=2$\omega_{forcing}$*
- *forward reflections: shallow **k**, steep $\mathbf{c}_g$, resonances favoured*
- *backward reflections: steep **k**, shallow $\mathbf{c}_g$, resonances unlikely*

**References:**
- Winters, MacKinnon & Mills 2004. A spectral model for process studies of rotating, density-stratified flows. *J. Atmos. Ocean. Technol.* 21(1).
- Grisouard & Thomas 2015. Critical and near-critical reflections of near-inertial waves off the sea surface at ocean fronts. *J. Fluid Mech.*
- Grisouard & Thomas 2016. Energy exchanges between density fronts and near-inertial waves reflecting off the ocean surface. *J. Phy. Oceanography.*