Sub-surface propagation of near-inertial waves in ocean fronts

Nicolas Grisouard & Leif Thomas – Dept. Environmental Earth System Science, Stanford University – nicolas.grisouard@stanford.edu

1. Introduction

Context:

- Oceanic fronts: horizontal boundaries between water masses (e.g. Gulf Stream separating sub-polar from sub-tropical waters)
- Oceanic fronts characterized by:
  - * strong lateral density gradient, thermal wind shear,
  - * strong ageostrophic, vertical motions, enhanced turbulence,
  - * **strong internal wave activity.**
- Understanding frontal mixing: crucial to understand air-sea exchanges => climate modeling, biology…

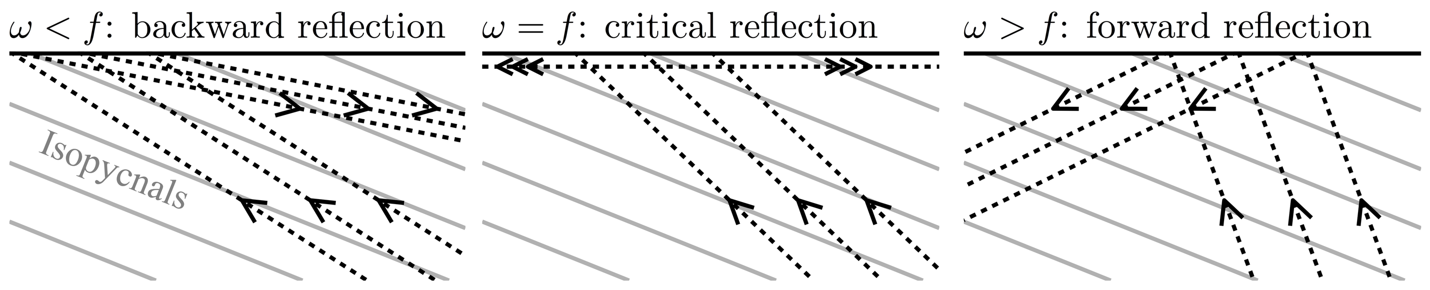
Internal waves in fronts:

- Peculiar properties due to slanted isopycnals (*Whitt & Thomas 2013*),
- Can "classical" (flat isopycnals) internal wave physics give insight about "frontal" internal wave physics?

How are the reflection properties of internal waves modified by the presence of an oceanic front?

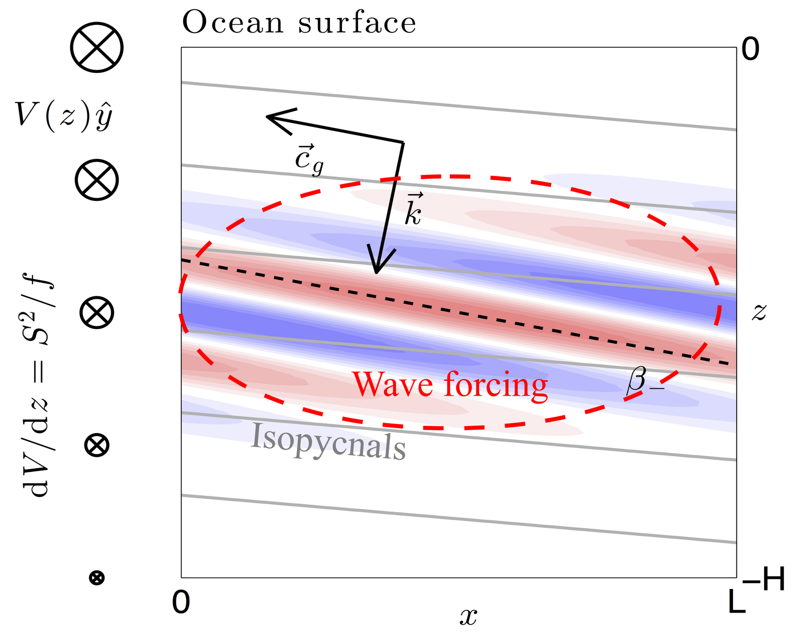
2. Critical, forward and backward reflections

- Oceanic fronts characterized by strong lateral density gradients: $S^2 = -(g/\rho_0)(d\bar\rho/dx)$
- Consequence on internal waves: unusual dispersion relationship: $\omega^2(\beta) = \beta^2 N^2 + f^2 - 2\beta S^2$
- The slope of wave phase lines are symmetric around the isopycnal slope (if non-hydrostatic): $\beta_\pm = (k/m)_\pm = S^2/N^2 \pm (S^4/N^4 + (\omega^2 - f^2)/N^2)^{1/2}$
- For $\omega = f$, critical reflection against the ocean surface: $\beta_- = 0$.
- **Similar to classical internal waves reflecting onto a slope, frontal internal waves reflecting onto the ocean surface can experience critical reflection for $\omega = f$.**
- If $\omega > f$: "forward" (sub-critical) reflection; if $\omega < f$: "backward" (super-critical) reflection.



3. Set-Up

- Two-dimensional (x, z) simulations,
- nx = 256, nz = 512 or 1024,
- $\Delta x = 1.56$ m, $\Delta z = 9.77$ cm,
- $N^2 = 10^{-4}$ s^{-1}, $S^2 = 9.8\ 10^{-7}$ s^{-1}, $f = 10^{-4}$ s^{-1},
- Geostrophic Richardson: $Ri_G = f^2N^2/S^4 = 1.05$
- Background PV: $f^2N^2(1-1/Ri_G) > 0$,
- Waves forced in the volume (cf. Figure), minimal generation of PV
- Forcing amplitude tuned such that incident wave has Richardson number Ri_1 when reaching the surface
- Free-slip, rigid lids on top & bottom, periodic in x



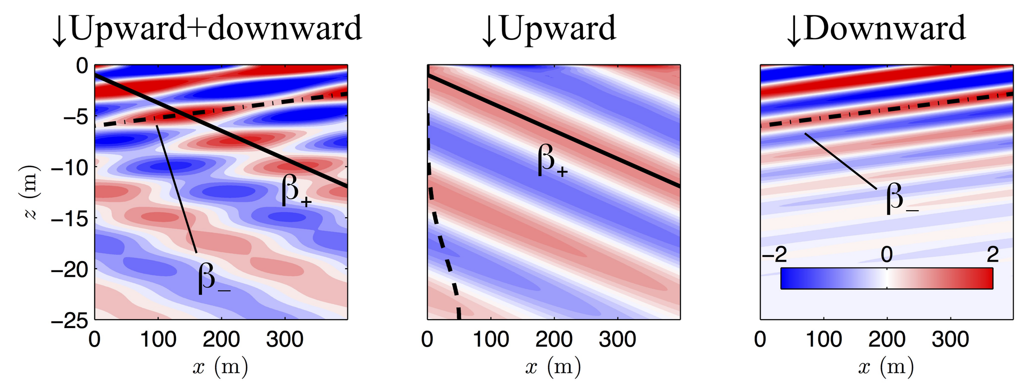
Equations solved by the code (Winters *et al.* '04):

$$\vec u_t + f\hat z\times\vec u + (S^2/f)w\hat y - b\hat z + \epsilon[(\vec\nabla\times\vec u)\times\vec u] + \vec\nabla P = D\vec u,\quad \vec u + V(z)\hat y = (u,\ v + S^2z/f,\ w)$$

$$b_t + S^2u + N^2w + \epsilon\vec u\cdot\vec\nabla b = Db,\quad u_x + w_z = 0,\quad \epsilon = 0 \text{ or } 1,\quad D = \nu^z\partial_{zz} - \nu^h\partial_{xxxx},\quad \nu^z = 2\ \text{mm}^2/\text{s}.$$

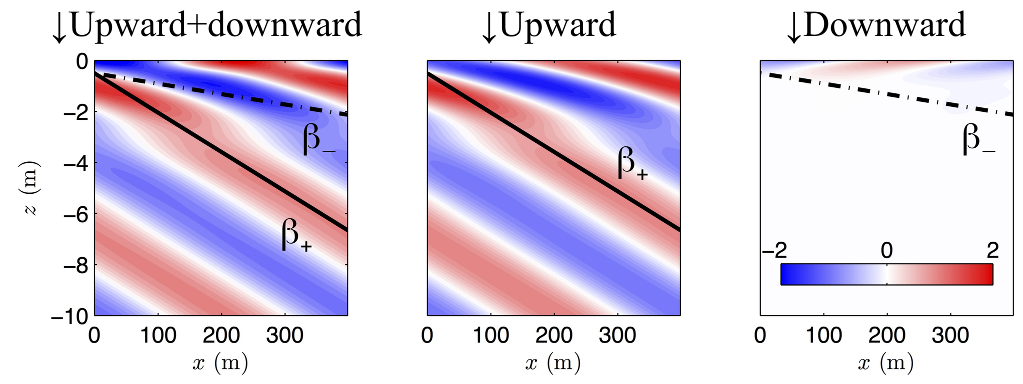
4. Linear reflections

Forward reflection ($\omega > f$)



Unsurprising result: reflection along characteristics, viscous decay.

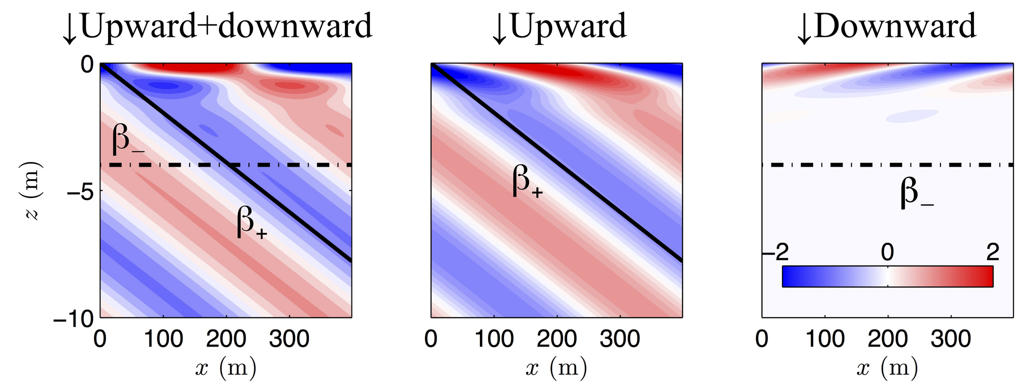
Backward reflection ($\omega < f$)



No backward reflection! Wave entirely absorbed under the surface.

Critical reflection ($\omega = f$):

Horizontal velocity field (u, mm/s):



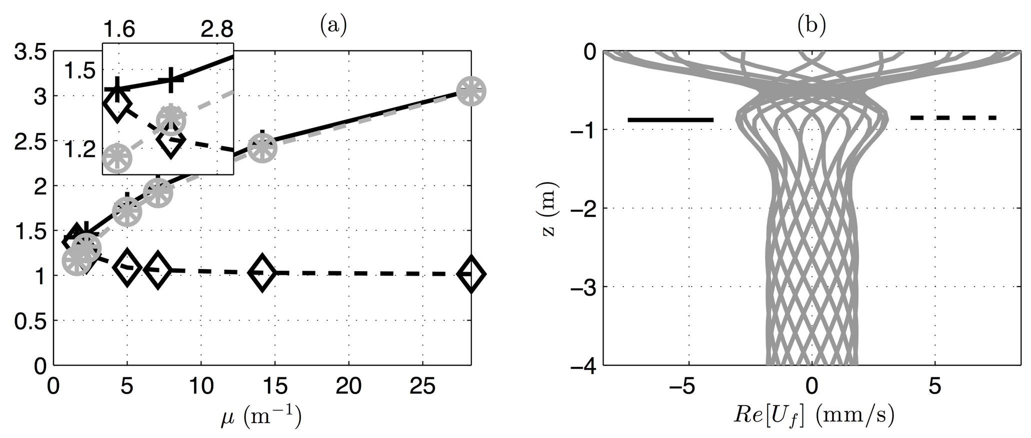
Structure of subsurface flow governed by:

$$\psi_{6z} + 2i\mu^2\psi_{4z} - 2ik_1^2\mu^2\psi_{zz} - \frac{2ik_1\mu^4S^4}{f^2}\psi_z - \frac{k_1^2\mu^4N^2}{f^2}\psi = 0,$$

with $\psi = \tilde\psi(z)\exp i(k_1x - ft)$, $(u, w) = (-\psi_z, \psi_x)$, $\mu^2 = f/\nu^z$

Made possible because $\omega \equiv f$ and $k \equiv k_{incident}$ for linear reflections.

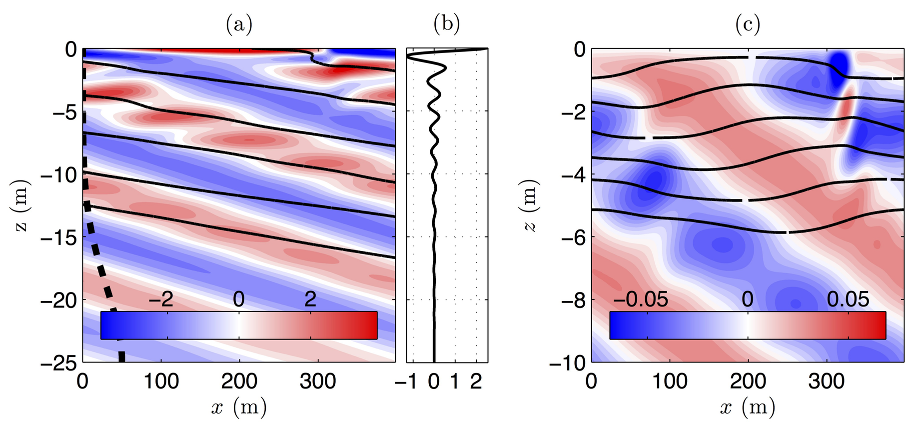
$\tilde\psi = e^{rz}$, $r \in \mathbb{C} \Rightarrow$ six possible r's, three of them >0 ($\Leftrightarrow$ decay with depth).



↑ (a) Dashed lines, diamonds, circles and stars: wavelengths of the set of three r's, whose real parts decay with depth normalized by $1/\mu$, for different μ (ν^z). Solid line, crosses: wavelength of the boundary layer flow, measured as in (b). (b): envelope of the boundary layer, visible in the horizontal velocity field. Horizontal lines: measured depth of the local maximum (solid) and predicted half-wavelength of two of the r's (dashed).

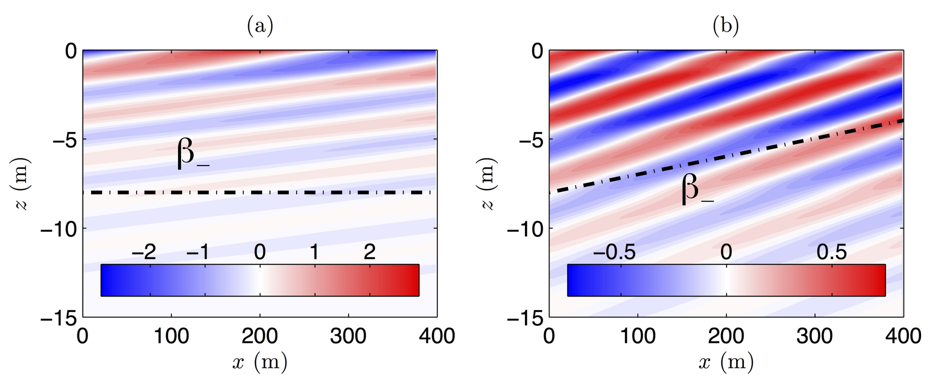
Linear critical reflection: wave absorbed under the surface, within a boundary layer well described by viscous theory.

5. Non-linear, critical reflection ($\omega_{forcing} = f$)



Snapshots:
(a) u (mm/s). Solid: isopycnals. Dashed: wave forcing position.
(b) $\int u dx/L$ (mm/s).
(c) w (mm/s). Lines: passive tracer contours.

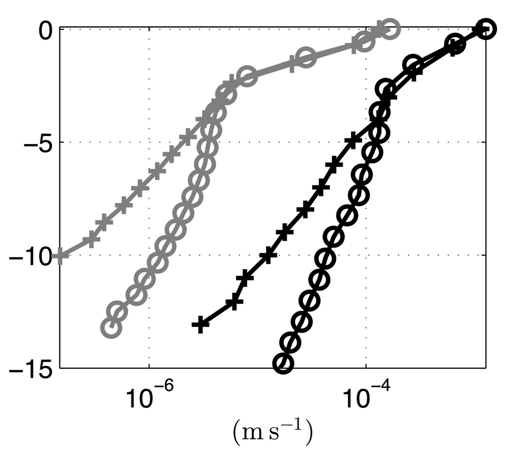
Non-linear flow active well below the surface.



Downward-propagating oscillations filtered at (a) $\omega = f$ and (b) $\omega = 2f$.

- Harmonics are present, but do not align with β_-

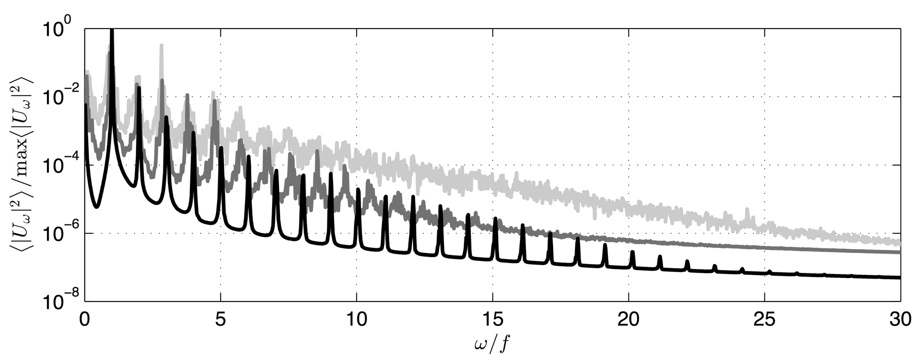
=> forced motions, not freely propagating waves (even for $\omega > f$)



Local extrema of $(\int u dx)/L$ for Ri_1=10 (black) and Ri_1 = 100 (gray), for ν^z = 2 mm²/s (circles) and ν^z = 4 mm²/s (crosses).

- Top 2.5 m: strong decay of the sub-surface flow, sensitive to amplitude, less so to viscosity.
- Below z = –2.5 m: smaller decay of the sub-surface flow, sensitive to viscosity, less so to amplitude.

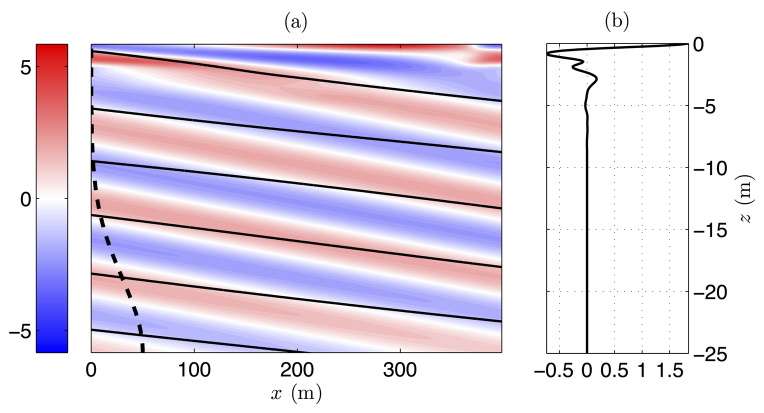
Link with the linear theory: unclear at this point…



Transition to turbulence: normalized frequency spectra of u for Ri_1=3 (black), Ri_1=1 (dark) and Ri_1=0.3 (light)

Increased amplitude => stable harmonics disappear, flow becomes turbulent. Well-known in the classical case.

6. Non-linear, non-critical reflections ($\omega_{forcing} \neq f$)



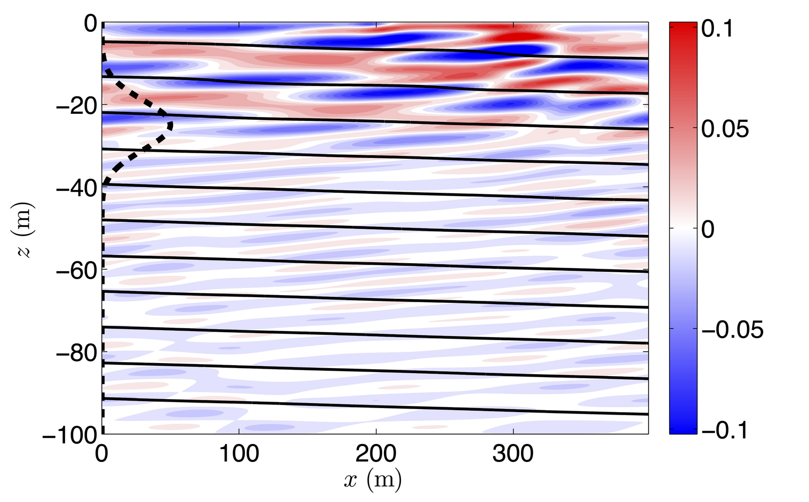
← **Background reflection ($\omega_{forcing} < f$):**
(a) snapshot of u (mm/s). Dashed line: forcing envelope. Solid lines: isopycnals.
(b) snapshot of $(\int u dx)/L$ (mm/s).

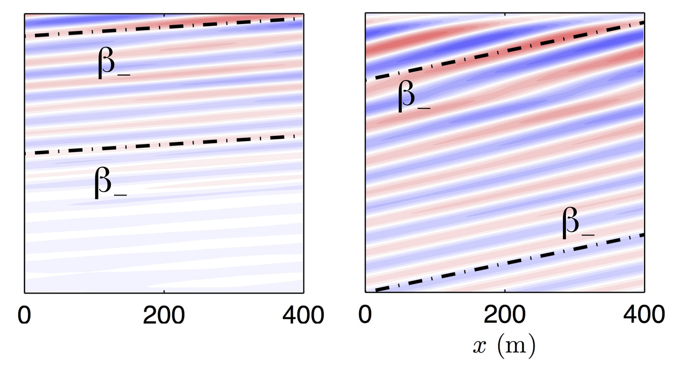
No apparent reflected wave.

Forward reflection ($\omega_{forcing} > f$):

→ Snapshot of w (mm/s). Dashed line: forcing envelope. Solid lines: isopycnals.

↓ Downward-propagating oscillations at (left) $\omega = \omega_{forcing}$ and (right) $\omega = 2\omega_{forcing}$.



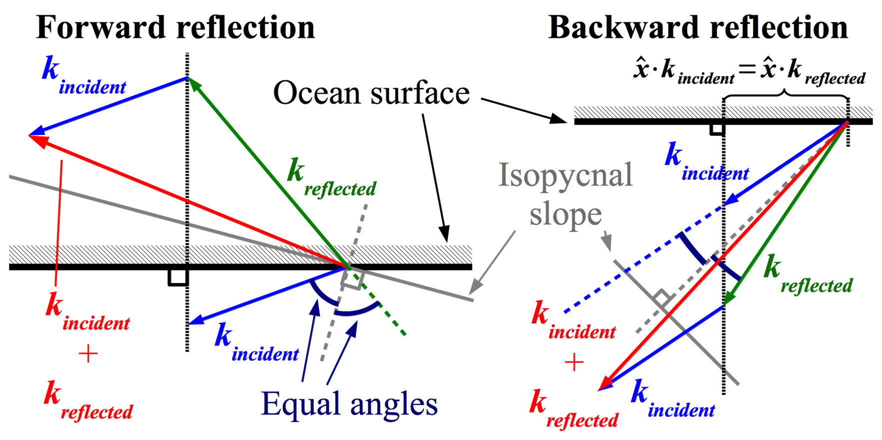


Reflections and strong generation of harmonics

A tale of two reflections: dull backward reflection, spectacular forward reflection. **Why?**

Non-linear interactions between incident and reflected waves tend to force:

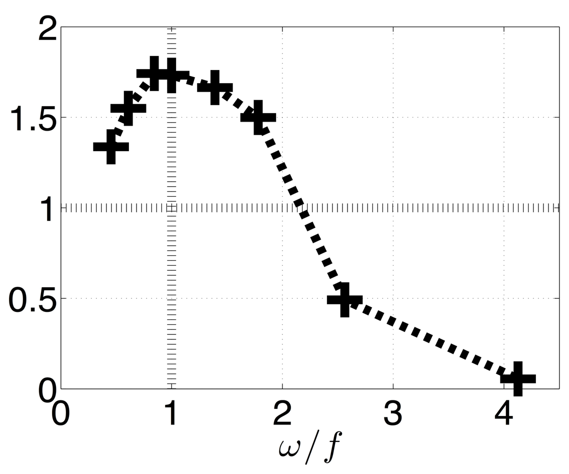
- frequencies $\omega = 2\omega_{forcing}$
- shallower $\vec k$, steeper $\vec c_g$ for forward reflections: triadic resonances favored
- steeper $\vec k$, shallower $\vec c_g$ for backward reflections: triadic resonances unlikely



7. Energetics

Ratio, averaged over x and time, of:
1. Kinetic energy dissipation in the top 15 m, over
2. Incident kinetic energy flux (Pw) at z = –15 m.

- Forward reflections → deep energy propagation,
- Around $\omega = f$: more energy is dissipated that is supplied by the incident wave!
- Geostrophic flow supplies energy to the ageostrophic flow.



Reflecting near-f waves can potentially drain energy out of fronts (in the absence of surface forcing)

Absent in classical reflections, this effect is a genuine feature of the frontal case!

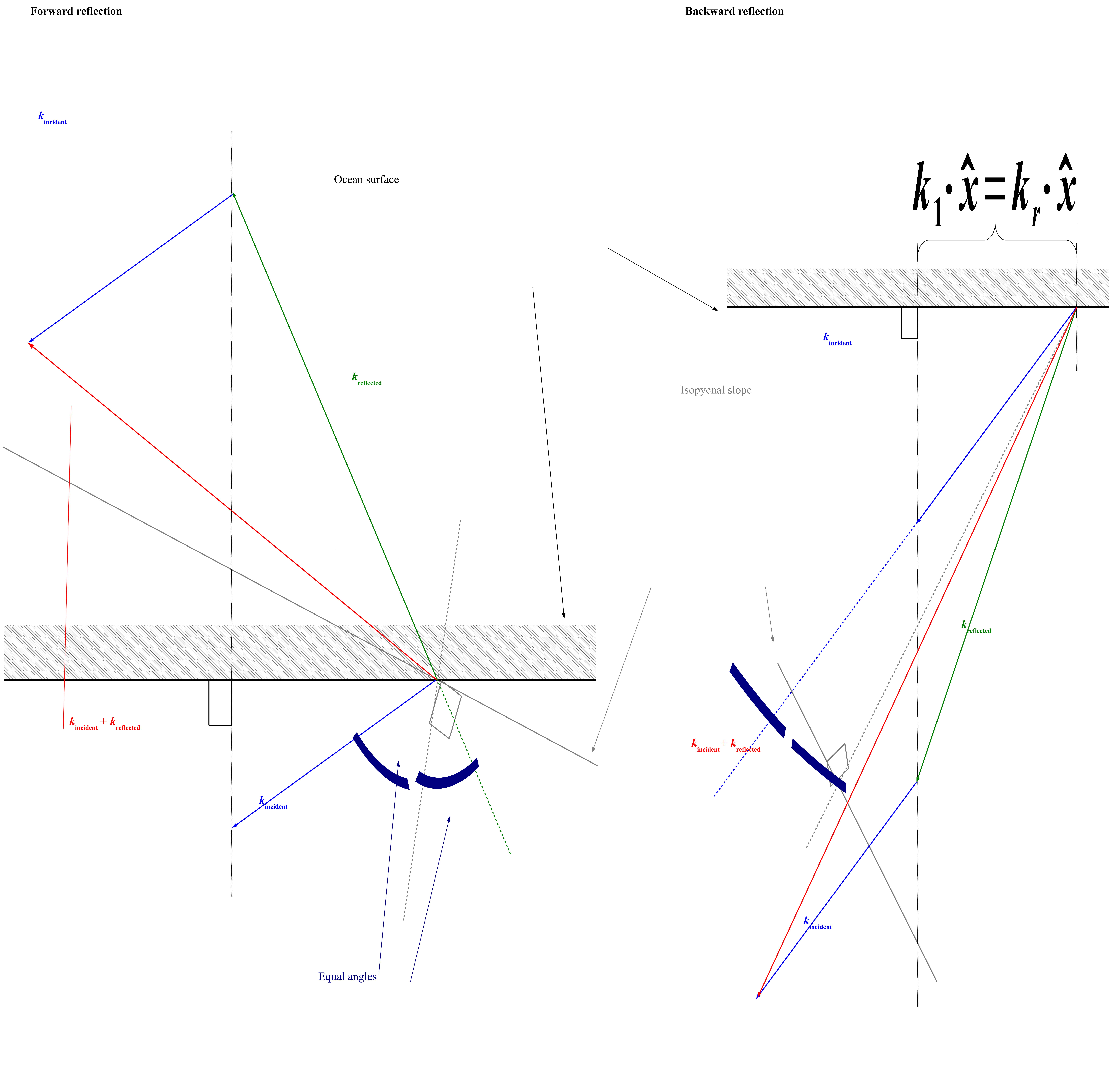
8. Conclusions

- In fronts, inertial waves experience critical reflections against the ocean surface,
- Linear reflection properties are governed by viscous theory, although more complicated than mere Ekman layer dynamics.
- Non-linear, critical reflection: ageostrophic energy present well below the surface. This flow is entirely forced, no radiation of freely-propagating waves.
- Non-linear, backward reflection: wave absorbed under the surface, no apparent reflection.
- Non-linear, forward reflection: wave reflects, non-linear interactions happen, reflections and harmonics propagate energy deep down.
- Reflecting near-f waves can potentially drain energy out of fronts, in the absence of surface forcing.

- **Multiple avenues for transfer of knowledge from classical internal wave science to frontal and sub-mesoscale dynamics: 3D effects, wave-mean flow interactions, turbulence and mixing…**

References:

- Mercier, Garnier & Dauxois 2008. Reflection and diffraction of internal waves analyzed with the Hilbert transform. *Phys. Fluids* 20(8)
- Whitt & Thomas 2013. Near-Inertial Waves in Strongly Baroclinic Currents. *J. Phys. Oceanogr.* 43(4).
- Winters, MacKinnon & Mills 2004. A spectral model for process studies of rotating, density-stratified flows. *J. Atmos. Ocean. Technol.* 21(1).


Forward reflection
Backward reflection
$k_{incident}$
Ocean surface
$k_{reflected}$
$k_{incident} + k_{reflected}$
$k_{incident}$
Equal angles
$k_1 \cdot \hat{x} = k_r \cdot \hat{x}$
$k_{incident}$
Isopycnal slope
$k_{reflected}$
$k_{incident} + k_{reflected}$
$k_{incident}$