Generation of solitary waves in a pycnocline by an internal wave beam

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Acronyms: IW(B) = internal wave (beam), ISW = internal solitary wave, \( \Phi \)-speed = phase speed.

Introduction

- Internal solitary waves have been detected in the Bay of Biscay far from the coast, at a distance too large for the waves to have been generated at the shelf break (cf. [1], [2], [3], [4]).
- Proposed mechanism for their presence far from the coast ([1]): internal wave beam (here the internal tide) hitting the seasonal thermocline ⇒ large interfacial displacement ⇒ trains of ISWs.
- Such a mechanism is rare (western Europe, southwestern Indian ocean) ⇒ very restrictive conditions should hold for it to occur.
- Theoretical works ([5], [6]): the value of the density jump across the pycnocline has to be of moderate strength so that the \( \Phi \)-speeds of the interfacial wave and of the forcing IW have the same order of magnitude.
- This mechanism has never been observed nor simulated numerically up to now. We present non-rotating, direct numerical simulations which confirm this mechanism. We show that various modes of ISW can be generated, whereof occurrence can be controlled.

Numerical Set-up

- Reproduction of the geometry and fluid parameters of the Coriolis turntable (Grenoble), where experiments on this subject have been performed in 2008.
- MITgcm code: incompressible nonhydrostatic Boussinesq equations, centered 2nd-order finite volume scheme.
- Continuous temperature profile (see figure, red), idealized version of the summer stratification in the Bay of Biscay.
- Forcing: temporally oscillating velocity field at the left boundary (see figure, magenta), Spongy layer on the right boundary.
- Explicit scheme, no parametrization, nothing “under the hood”.
- Parameters that will be varied: \( \Delta = \) (see fig., red), \( \Lambda = \) (see fig., magenta), \( \delta \) and resolution in 3 experiments: E1, E2 and E3, in which mode-1, -2 and -3 ISWs develop respectively.

Mode-1 solitary waves

- Taylor-Goldstein equation for frequency \( \Omega = \) IW modes of frequency \( \omega \): 
  \[
  \frac{d^2W_n}{dz^2} + \omega_n^4(W_n - \frac{c_\omega^2}{c_\Phi^2}W_n) = 0
  \]
- \( W_n \) : vertical structure of \( n \)-mode
- \( c_\Phi^2 = \) \( \Phi \)-speed of \( n \)-mode
- When \( \Omega \geq \omega_n \), trapped waves: only small variations of \( c_\Phi \) with \( \omega \) ⇒ \( n = 0 \)
- Resonance condition to generate mode-1 ISWs: \( v_\phi \sim c_n \)

Mode-n solitary waves: a modal resonance condition

- Illustration of the resonance condition for mode-2 ISWs. (Experiment E2)
  \( \Delta = 3.4\% \), \( \Lambda = 26 \text{ cm/s}, \)
  \( v_\phi = 1.8 \text{ cm/s} \)
- Illustration of the resonance condition for mode-3 ISWs. (Experiment E3)
  \( \Delta = 4\% \), \( \Lambda = 15 \text{ cm/s}, \)
  \( v_\phi = 1 \text{ cm/s} \)

Mode-n \((n=2)\) solitary waves: a Bragg-like resonance condition

- Magnification of the velocity fields at the beam impact in the pycnocline for E2 (left) and E3 (right)
- Careful observation of the IWB transmission in the pycnocline reveals refraction and modification of its vertical structure.
- Simple model, approximations:
  1. Stratification ⇒ 3 layers (see ⇒)
  2. IW = plane wave
  3. WKB-like Ansatz (rays)

Conclusions and perspectives

Summary

- First numerical evidence of the generation of internal solitary waves by an internal wave beam.
- Possibility of generating ISWs of any mode.
- Two different resonance conditions to select the mode, both show that the slower the phase speed of the beam, the higher the ISW mode.

Perspectives

- Validation against experiments (ongoing).
- Application to realistic simulations (ongoing).
- Integration of realistic effects (shear flow, background IW field...).
- Validation against in situ measurements.