Generation of solitary waves in a pycnocline by an internal wave beam

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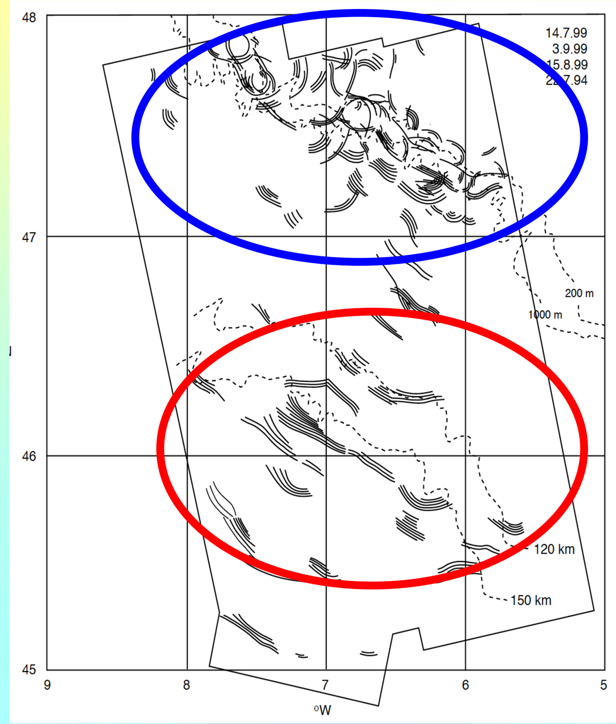
Acronyms: IW(B) = internal wave (beam), ISW = internal solitary wave, Φ-speed = phase speed.

Introduction

- Internal solitary waves have been detected in the Bay of Biscay far from the coast, at a **distance too large for the waves to have been generated at the shelf break** (cf. [1], [2], [3], [4]).
- Proposed mechanism for their presence far from the coast ([1]): **internal wave beam** (here the internal tide) **hitting the seasonal thermocline** ⇒ large interfacial displacement ⇒ trains of ISWs.
- Such a mechanism is rare (western Europe, southwestern Indian ocean) ⇒ **very restrictive conditions should hold for it to occur**.
- Theoretical works ([5], [6]): **the value of the density jump across the pycnocline has to be of moderate strength** so that the Φ-speeds of the interfacial wave and of the forcing IWB have the same order of magnitude.

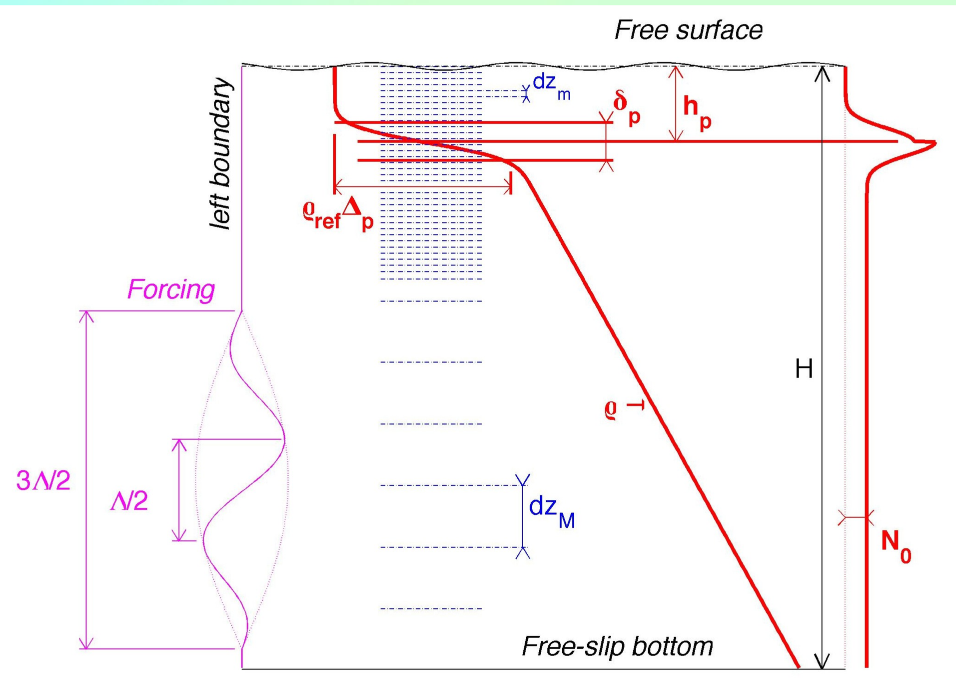


↟ *ISW packets in the Bay of Biscay (SAR images), traveling away from the shelf break.* ***Packets close to the shelf break (blue)*** *are to be distinguished from* ***those emerging far from it (red)****. From [4].*

- This mechanism has never been observed nor simulated numerically up to now. We present **non-rotating, direct numerical simulations** which **confirm this mechanism**. We show that various modes of ISW can be generated, whereof occurrence can be controlled.

Numerical Set-up

- Reproduction of the geometry and fluid parameters of the Coriolis turntable (Grenoble), where experiments on this subject have been performed in 2008.
- **MITgcm** code: incompressible nonhydrostatic Boussinesq equations, centered 2nd-order finite volume scheme.
- Continuous temperature profile (see figure, red), **idealized** version of the summer stratification in the Bay of Biscay.
- Forcing: temporally oscillating velocity field at the left boundary (see figure, magenta). Sponge layer on the right boundary.
- Explicit scheme, no parametrization, nothing "under the hood".
- Parameters that will be varied: Δ_p (see fig., red), Λ (see fig., magenta), L and resolution in **3 experiments**: E1, E2 and E3, in which mode-1, -2 and -3 ISWs develop respectively.



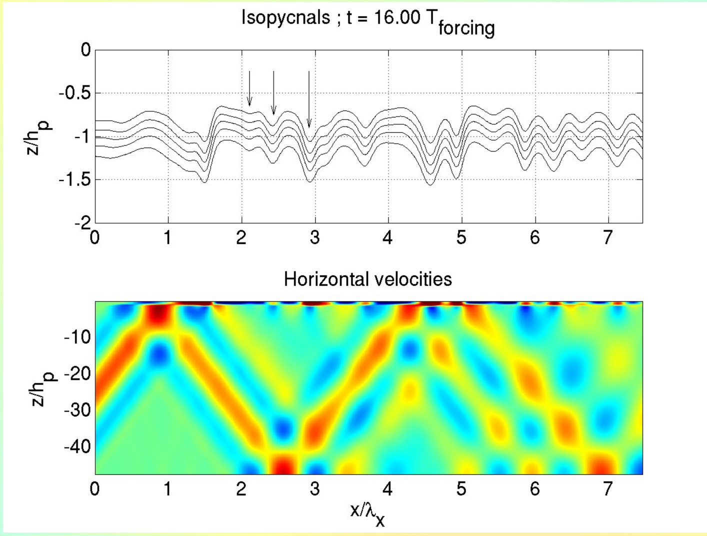
H	~ 1 m
h_p	2 cm
δ_p	1 cm
dz_m, dz_M	0.4, 4 mm
L (total length)	1.2 – 6 m
θ (IWB angle)	45°
viscosity	10^{-7} m.s^{-2}
Reynolds (IWB)	~ 10^5
Prandtl	70

↟ *Common parameters*
↞ *Vertical set-up and definitions of most of the parameters used in the text.*

References
[1] New, A. L. & Pingree, R. D. 1990. *Deep-Sea Res.*, **37**, 513–524.
[2] Pingree, R. D. & New, A. L., 1991. J. Phys. Oceanogr., **21**, 28-39
[3] New, A. L. & Pingree, R. D. 1992. *Deep-Sea Res.*, **39**, 1521-1534.
[4] New, A. L. & Da Silva, J. C. B. 2002. *Deep-Sea Res. (I)*, **49**, 915–934.
[5] Gerkema, T. 2001. *J. Mar. Res.* **59**, 227–255.
[6] Akylas, T. R., Grimshaw, R. H. J., Clarke, S. R. & Tabaei, A. 2007. *J. Fluid Mech.* **593**, 297–313.
[7] Mathur, M. and Peacock, T. 2009, *J. Fluid Mech.*, **639**, 133-152.
[8] Grisouard, N., Staquet, S. & Gerkema, T., 2010. *Submitted to J. Fluid Mech.*

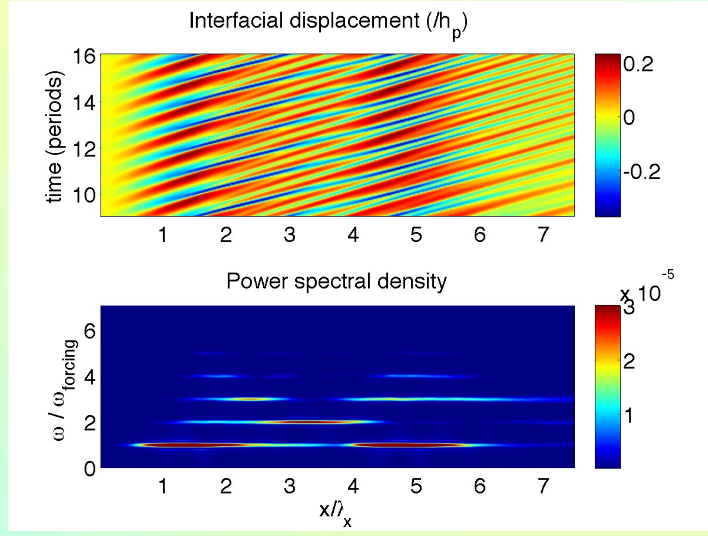
Acknowledgements
We thank the team that realized the experiments in Coriolis in 2008 for the fruitful discussions: M. Mercier, L. Gostiaux, M. Mathur, J. Magalhães, J. Da Silva & T. Dauxois. N.G. is supported by a grant from the Délégation Générale de l'Armement. This work has been supported by ANR contracts TOPOGI-3D (# 05-BLAN-0176) and PIWO (# 08-BLAN-0113). Numerical experiments were performed on the French supercomputer center IDRIS, through contracts 0705890 and 080580.

Mode-1 solitary waves



↟ *IW field (snapshot). Top: isopycnals around the pycnocline, a train of 3 ISWs is indicated. Bottom: horizontal velocities, whole field.*

- Exp. E1: Δ_p=2% ⇒ interfacial Φ-speed in a 2-layer fluid with same density jump: c^*=6.3 cm.s^{-1}
- $\lambda_x \approx \Lambda$=60cm, horiz. Φ-speed of the IWB: v_ϕ=4.1 cm.s^{-1}.
- In [5], $\gamma = c^*/N_0 H$ should be 0.12; here: γ=0.11.
- In [6], $\alpha = N_0 \lambda_x / \pi c^*$ should be 1; here: α=1.3.
- In one forcing period, **3 stages** can be distinguished:
  1. impact of the IWB ⇒ partial reflection, **partial transmission** in the pycnocline
  2. **nonlinear steepening** of the transmitted part ⇒ IW trapped in the upper layer,
  3. **nonhydrostatic disintegration** into 3 ISWs.
- Partial transmission back in the lower layer ⇒ **scattering of the IWB** (*cf.* [5], [7]).



↟ *Top: space-time representation of the vertical displacement of the pycnocline. Bottom: temporal Fourier analysis of top.*

Mode-*n* solitary waves: a modal resonance condition

- Taylor-Goldstein equation for frequency Ω ⇒ IW modes of frequency Ω

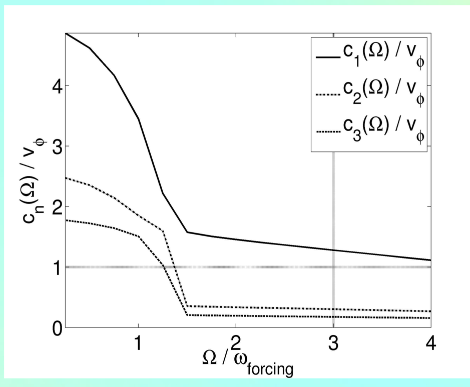
$$\frac{d^2 W_n}{dz^2} + \frac{N^2(z) - \Omega^2}{c_n^2(\Omega)} W_n = 0$$

W_n: vertical structure of n^{th} mode
$c_n(\Omega)$: Φ-speed of n^{th} mode

- When $\Omega > N_0$ *(trapped waves)*: only small variations of c_n with $\Omega \Rightarrow c_n(\Omega) \approx \hat{c}_n$ ~~(Ω)~~

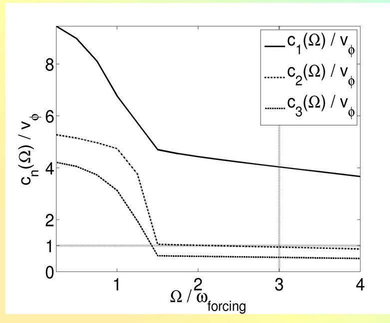
Resonance condition to generate mode-*n* ISWs: $v_\phi \sim \hat{c}_n$

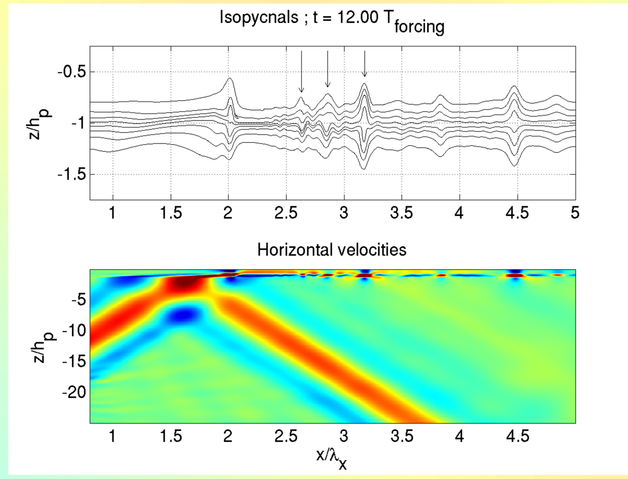
↠ *Plot of the Φ-speeds (scaled by v_ϕ) versus Ω (scaled by the forcing frequency) for the 1st three modes (experiment E1). v_ϕ is indeed close to $\hat{c}_1$.*



- Illustration of the resonance condition for **mode-2 ISWs**. (Experiment E2)
- Δ_p=3.4%, $\lambda_x \approx$26 cm, v_ϕ=1.8 cm.s^{-1}.

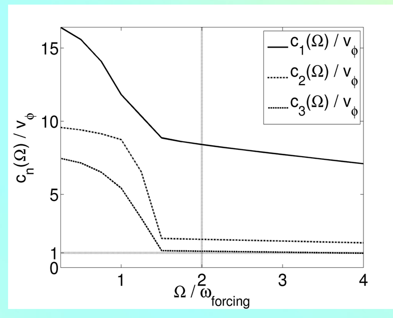
↠ *Plot of the Φ-speeds: now $v_\phi \sim \hat{c}_2$.*
↡ *Developed wave field & zoom on the pycnocline (top)*

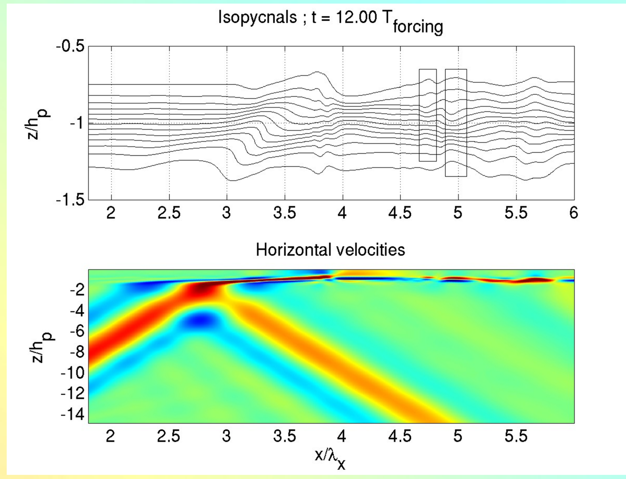




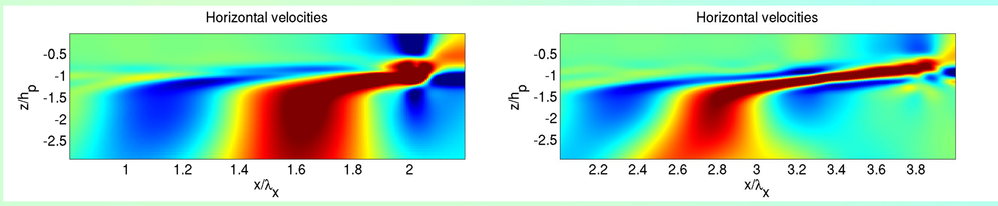
- Illustration of the resonance condition for **mode-3 ISWs**. (Experiment E3)
- Δ_p=4%, $\lambda_x \approx$15 cm, v_ϕ=1 cm.s^{-1}.

↠ *Plot of the Φ-speeds: now $v_\phi \sim \hat{c}_3$.*
↡ *Developed wave field & zoom on the pycnocline (top)*



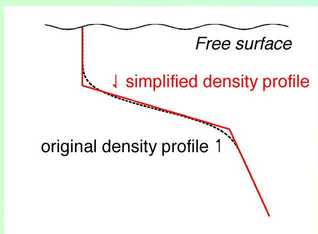


Mode-n ($n \geq 2$) solitary waves: a Bragg-like resonance condition



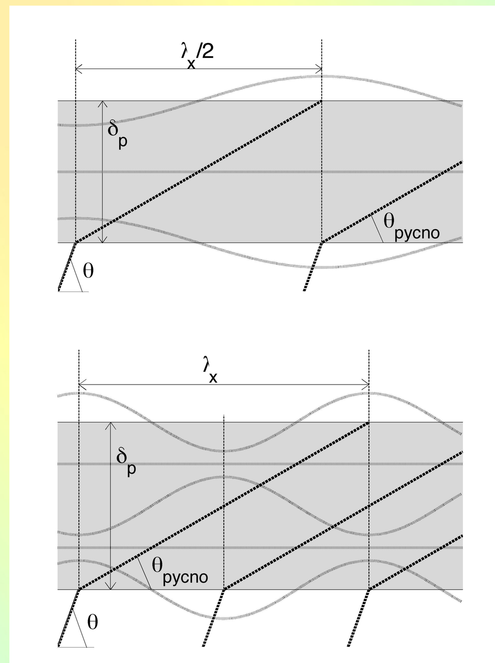
↟ *Magnification of the velocity fields at the beam impact in the pycnocline for E2 (left) and E3 (right)*

- Careful observation of the IWB transmission in the pycnocline reveals **refraction** and **modification of its vertical structure.**
- Simple model, approximations:
  1. Stratification ⇝ 3 layers (see ↠)
  2. IWB ⇝ plane wave
  3. WKB-like Ansatz (rays)



- If $\lambda_x \tan(\theta_{pycno}) = 2\delta_p$: IW field in the pycnocline ≈ mode-2 trapped IW.
- If $\lambda_x \tan(\theta_{pycno}) = \delta_p$: same for a mode-3 trapped IW.

↠ *Illustration for n=2 (top) and n=3 (bottom). The grey area is the pycnocline.*



Resonance condition to generate mode-*n* ISWs ($n \geq 2$):

$$\mu_n = \frac{n-1}{2} \frac{\lambda_x \tan(\theta_{pycno})}{\delta_p} = 1$$

- For E2: μ_2 = 0.88; for E3: μ_3 = 0.95 ⇒ **excellent agreement of this model with the numerics**

Conclusions and perspectives

Summary

- **First numerical evidence of the generation of internal solitary waves by an internal wave beam** .
- Possibility of generating ISWs of any mode.
- **Two different resonance conditions** to select the mode, both show that **the slower the phase speed of the beam, the higher the ISW mode**.

Perspectives

- **Validation against experiments** (ongoing).
- **Application to realistic simulations** (ongoing).
- Integration of realistic effects (shear flow, background IW field...).
- Validation against *in situ* measurements.