Local generation of internal solitary waves in a pycnocline: numerical simulations

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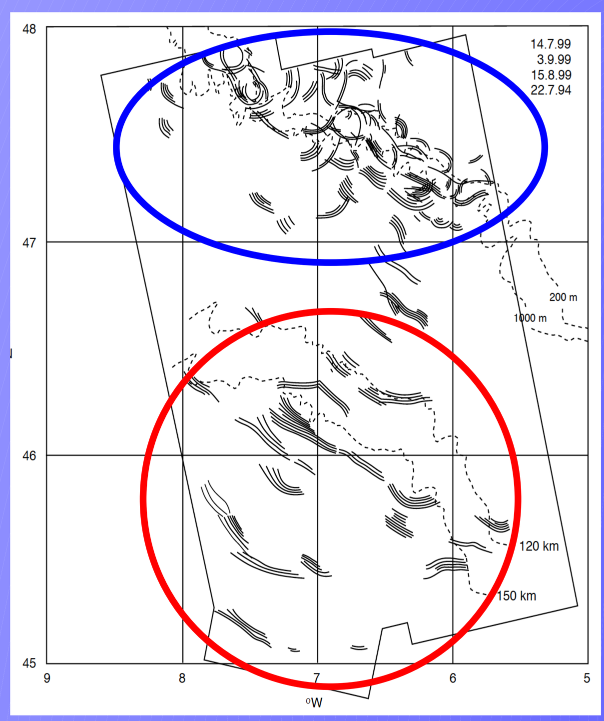
Introduction

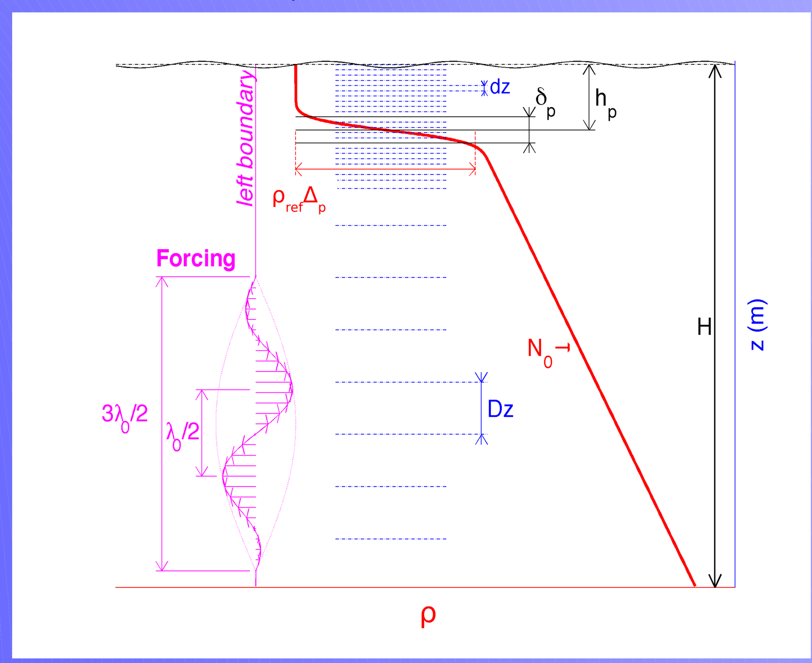
Non-Linear Internal waves (NLIW) have been detected in the Bay of Biscay far from the coast, at a distance too large to be explained by generation at the shelf break (cf. [1], [2], [3]). A mechanism for their presence far from the coast has been proposed ([2]): the **local interaction of an internal wave beam** (IWB, here the internal tide) **with the seasonal thermocline**, inducing an interfacial displacement that degenerates into a train of solitons, or NLIW. Such a local generation is rare however, implying very **restrictive conditions for it to occur**. Theoretical works ([4], [6]) have concluded that the value of the density jump across the pycnocline has to be of moderate strength so that the phase speeds of the interfacial wave and of the forcing internal wave (IW) field have the same order of magnitude. This mechanism has never been observed nor simulated numerically up to now. In this poster, we present **preliminary** results of **non-rotating, Direct Numerical Simulations which confirm this mechanism** and further analyze it. We thus show that various modes of NLIW can be generated, whereof occurrence can be controlled.

↡ NLIW packets observed in SAR images of the Bay of Biscay which are traveling approximately directly away from the shelf break. Packets close to the shelf break (blue) are to be distinguished from those emerging far from it (red). Courtesy of J. Da Silva.



Numerical Set-Up

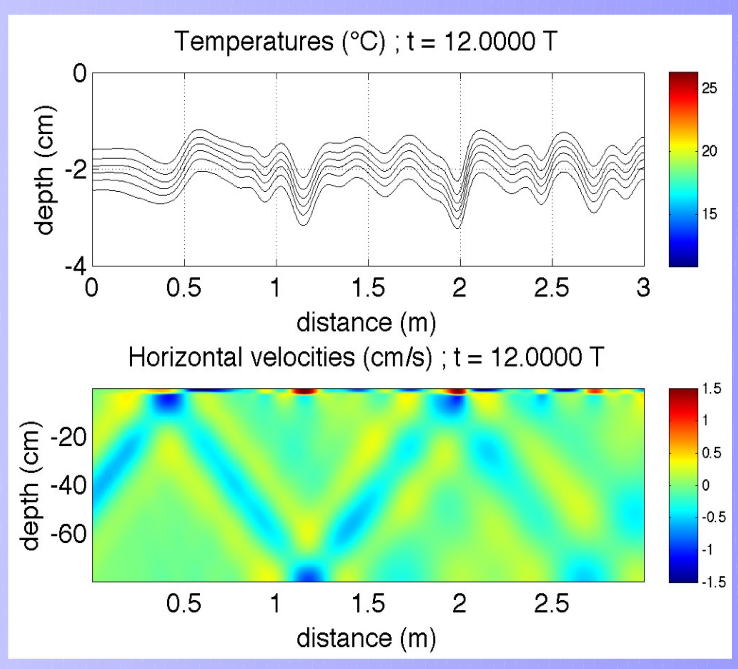
The set-up reproduces the **geometry of the Coriolis turntable** in Grenoble, where experiments on this subject have been performed in sep. 2008. The **MITgcm** code is used in a 2D configuration. The total height of the numerical domain is H=80cm with a length L of a few meters. The incompressible Boussinesq equations are solved by a finite volume method (involving a second order finite difference scheme) and are advanced in time with a 3rd order Adams-Bashforth scheme. The background stratification is an idealized continuous temperature profile representing the stratification in the Bay of Biscay in summer. Forcing occurs through a temporally oscillating velocity field, close to a Thomas & Stevenson profile, which is imposed at the left boundary of the domain (see figure below). The parameters that will be varied next will be Δ_p, L, λ_0 and the resolution.



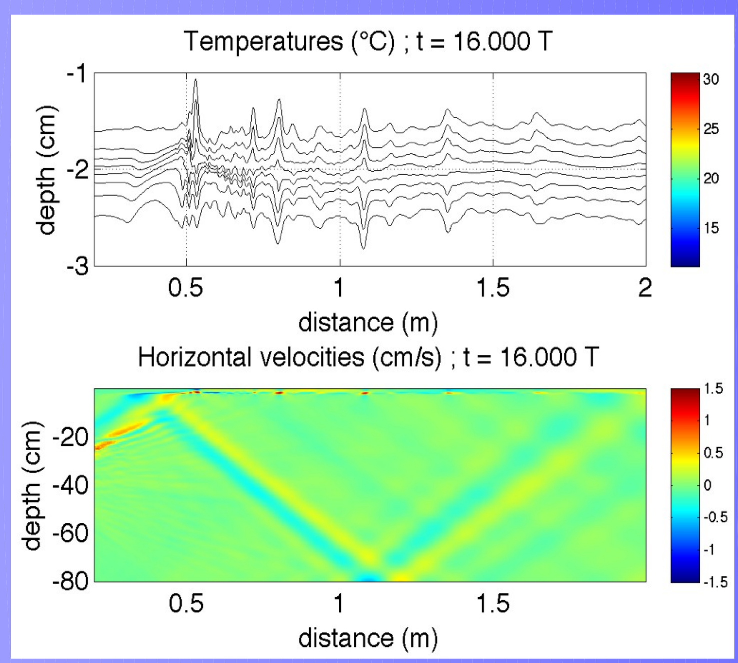
N_0	0.6 rad/s
T (forcing period)	14.81 s
ω (= $2\pi/T$)	0.424 rad/s
h_p	2 cm
δ_p	1 cm
ν (viscosity)	10^{-7} m²/s

↟ Values of various parameters
↞ Vertical set-up and definitions of most of the parameters used here.

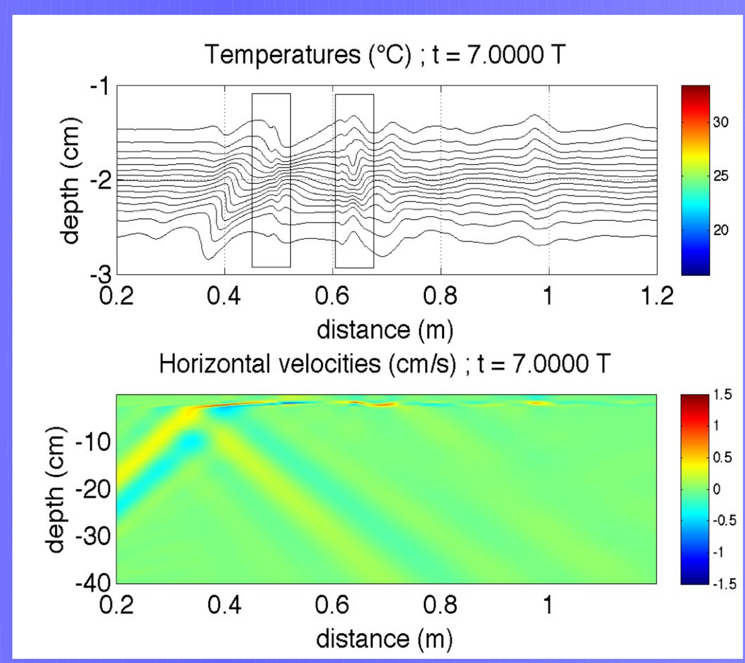
Results of the simulations: generation of mode-*n* NLIW



↟ Generation of ***mode-1*** *NLIW: the vertical displacements is in phase across the pycnocline. Δ_p=2.05%, λ_0=60cm. Resulting relevant velocities are $v_{\phi,1}$=4.05cm/s and $c_1(4\omega)$=4.03cm/s.*



↟ Generation of ***mode-2*** *NLIW: the vertical displacement has a node in the middle of the pycnocline. Δ_p=3.38%, λ_0=20cm. Resulting relevant velocities are $v_{\phi,2}$=1.33cm/s and $c_2(4\omega)$=1.37cm/s.*



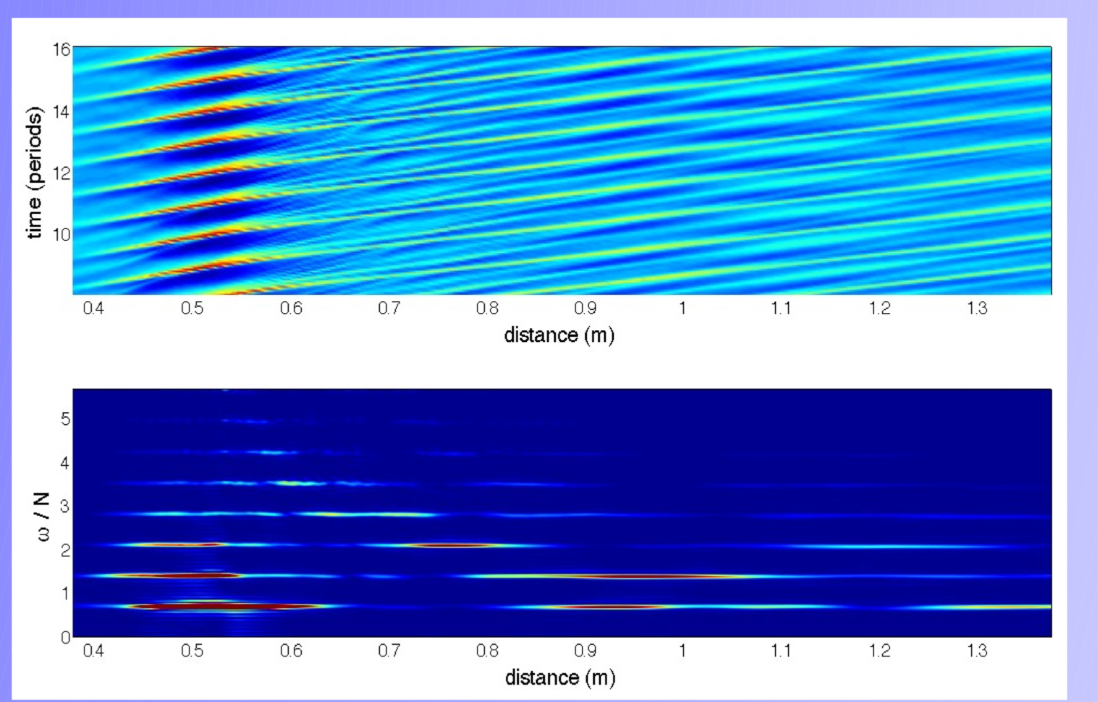
↟ Generation of ***mode-3*** *NLIW: the vertical displacement has two nodes in the pycnocline. Δ_p=4%, λ_0=14.1cm, $v_{\phi,3}$=9.52mm/s and $c_3(3\omega)$=9.52mm/s. The presence of mode-2 NLIW can be explained by the non-monochromaticity of the IWB.*

Mode-*n* NLIW are generated by matching the horizontal phase speed $v_{\phi,n}$ of the IWB with the phase speed $c_n(\Omega)$ of the n-th mode of the internal wave field of frequency Ω (>ω). $c_n(\Omega)$ is deduced from the classical equation of the normal modes:

$$\frac{d^2\Phi_n}{dz^2}+\frac{N^2(z)-\Omega^2}{c_n^2(\Omega)}\Phi_n=0$$

Ω is not well defined and is chosen as $m\omega$, m being the number of emerging solitons of the train. This can be confirmed by the pair of figures shown below. This way of processing, mixing linear and non-linear arguments, is of course an approximation.

↠ Top: distance vs. time diagram. The generation of solitons in the trains of NLIW can be seen. Bottom: distance vs. temporal spectrum. The growth of harmonics can be seen in comparison with the top figure, as the distances match. These two diagrams are related to mode-2 NLIW.



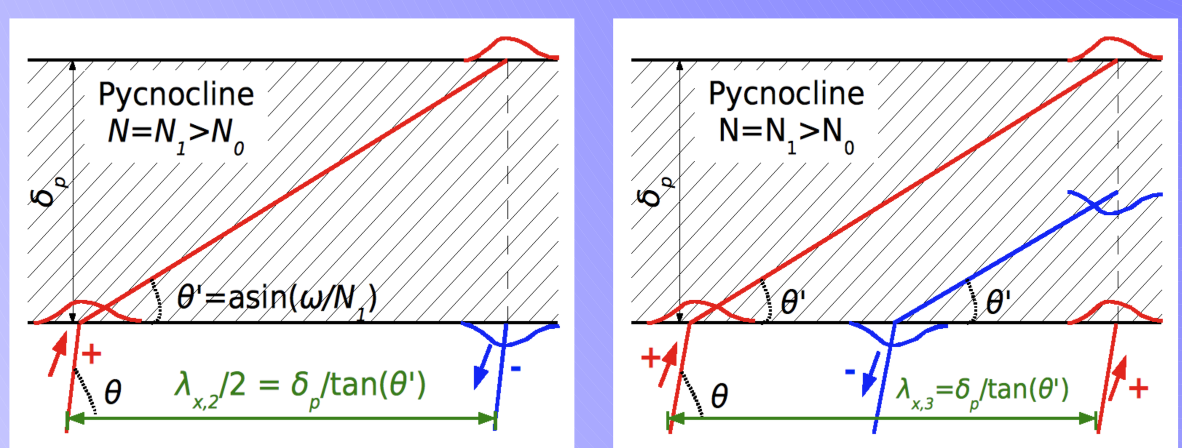
How is a specific mode forced?

Simplifying hypotheses:
- **Linearized problem (valid in the early stage of the generation)**
- Forcing wave is plane (instead of beam-like), horizontal wavelength is λ_x
- **3-layer system**: 1/ Mixed layer with N=0; 2/ Pycnocline with $N=N_1$; 3/ Deep-water layer with $N=N_0$. N_1 is determined by conserving $\int N^2(z)dz = g.\ln(\rho_{bot}/\rho_{surf})$.

A **simple condition** can be found to generate a mode-*n* IW in the pycnocline:

$$v_{\phi,n}=\frac{\delta_p\sqrt{N_1^2-\omega^2}}{\pi(n-1)}$$

Here, this gives $v_{\phi,2}$=1.78cm/s and $v_{\phi,3}$=8.9mm/s (*cf.* prev. section).



↟ Condition on λ_x to get a mode-2 (left) or a mode-3 (right) IW. The phase difference between red & blue lines is 180° and their associated displacements are indicated.

First conclusions and perspectives

Summary:
- **successful simulation** of the interaction between an IWB and pycnocline waves and successful reproduction of the generation of NLIW,
- **method to get NLIW of mode-*n*** based on the matching of the phase speeds of the IWB and the phase speed of the mode-*n* internal gravity wave that has the frequency of the NLIW,
- simple argument to **understand what vertical structure is selected**.

Perspectives:
- better understanding of the generation: what model would best describe the NLIW (KdV, eKdV, BO...?) + influence of the amplitude => description of the **transition to non-linear dynamics** and determination of the number of emerging solitons (inverse scattering method),
- application to the **oceanic case**: ~400x4 km² with rotation,
- quantification of the **energetic transfers**.

References
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