OS31C-1009 - Damping of Balanced Motions during Critical Reflection of Inertial Waves Off the Sea Surface at Ocean Fronts

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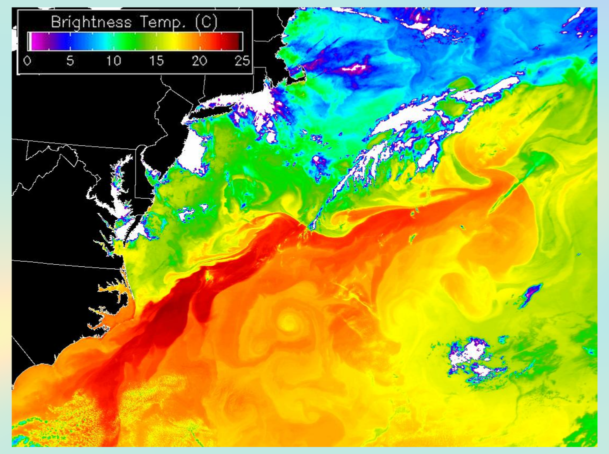
1. Introduction

- Mesoscale vortices (~100 km):
  - * contain 90% of the ocean's kinetic energy
  - * strong geostrophy, very hard to dissipate (inverse energy cascade)
- Oceanic fronts: horizontal boundaries between water masses (e.g. Gulf Stream separating sub-polar from sub-tropical waters), ~10 km wide, characterized by:
  - * strong lateral density gradient, thermal wind shear,
  - * strong ageostrophic, vertical motions, enhanced turbulence,
  - * strong internal wave activity.
- Oceanic fronts: hotspots for the dissipation of geostrophic energy?

Can internal waves extract geostrophic energy from fronts?

↓ *The Gulf Stream, a strong oceanic front (Sea surface temperature, MODIS, 25 May 2007)*

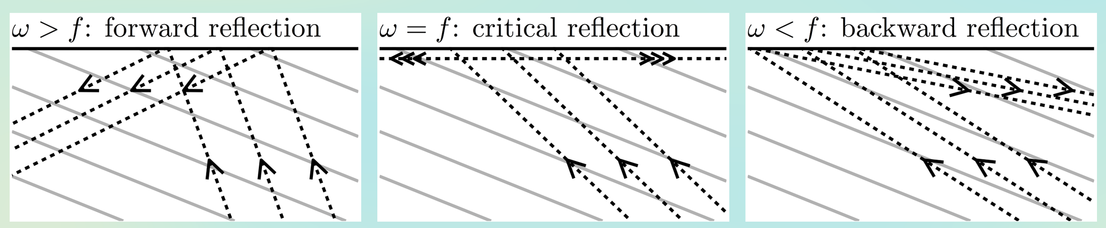


2. Critical, Forward and Backward Reflections

- Our oceanic front: strong lateral density gradient: $S^2 = -(g/\rho_0)(d\bar{\rho}/dx)$ (and a thermal wind shear with $Ri_G = f^2N^2/S^4$) and no lateral shear (≠ Dan Whitt's poster nearby)
- Unusual dispersion relationship for internal waves: $m^2\omega^2 = k^2N^2 + m^2f^2 - 2kmS^2$
- Waves can oscillate at $\omega < |f|$: $\omega_{min} = f\sqrt{1-1/Ri_G}$
- The slope of wave phase lines are symmetric around the isopycnal slope:

$$(k/m)_\pm = S^2/N^2 \pm \sqrt{S^4/N^4 + (\omega^2 - f^2)/N^2}$$

- For $\omega = f$, critical reflection against the ocean surface: *slope* = 0.
- **Similar to classical internal waves reflecting off a slope, frontal internal waves reflecting off the ocean surface can experience critical reflection for $\omega = f$.**



- However, viscosity is about to change this picture drastically...

3. Near-critical Linear Reflection: Theory

In the inviscid case, simple: $\left((f^2-\omega^2)\partial_z^2 - 2ikS^2\partial_z - k^2N^2\right)\hat{\varphi} = 0$

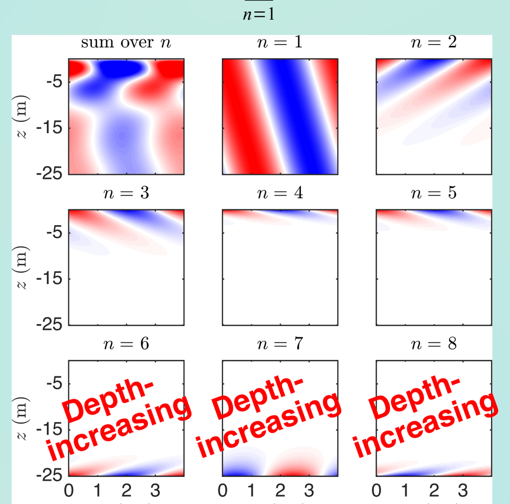
with $\varphi = \hat{\varphi}(z)\exp i(kx - \omega t)$, $\varphi = u, v, w, b, p, \psi$ or else…

But with viscosity, not so much:

$$\left(i\omega + \nu\partial_z^2\right)\left[\nu^2\partial_z^6 + 2i\nu\omega\partial_z^4 + (f^2-\omega^2)\partial_z^2 - 2ikS^2\partial_z - k^2N^2\right]\hat{\varphi} = 0,$$

with $\varphi = \hat{\varphi}(z)\exp i(kx-\omega t)$, $\varphi = v$, b or p (but not u, w or ψ).
Made possible because $\omega \equiv \omega_{incident}$ and $k \equiv k_{incident}$ for linear reflections.
$\tilde{\varphi} = e^{rz}$, $r \in \mathbb{C}$ ⇒ eigth possible r's, four of them >0 (⇔ decay with depth).

$$\varphi = \sum_{n=1}^{8}\hat{\varphi}_n(x,z,t), \qquad \hat{\varphi}_n = \tilde{\varphi}_n \exp(r_n z + ikx - i\omega t)$$



← *top left: an example of a near-critical reflection ()*
Other panels: exp(ikx + r_nz);
Four r_n (n = 1, 6, 7 and 8) increase with depth: n=1 is the incident wave, the rest is unphysical for a surface reflection.

Viscous solutions matter only for near-critical reflections.

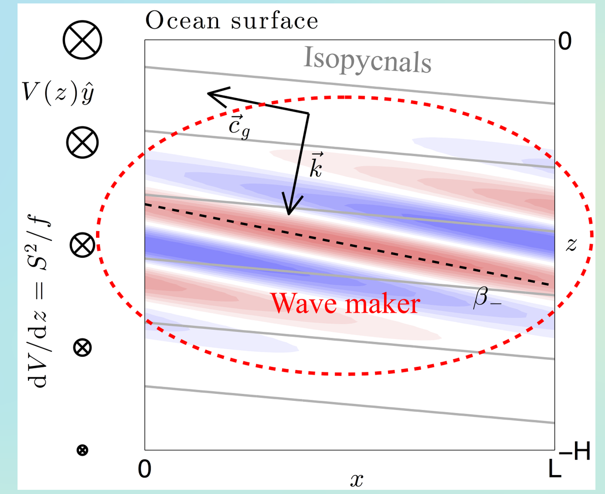
Boundary conditions:

$$\partial_z u|_{top} = \partial_z v|_{top} = w|_{top} = b|_{top} = 0 \quad \Rightarrow \quad \sum_{n=1}^{5}\tilde{u}_n r_n = \sum_{n=1}^{5}\tilde{v}_n r_n = \sum_{n=1}^{5}\tilde{w}_n = \sum_{n=1}^{5}\tilde{b}_n = 0$$

Thanks to the polarization relations, we have four equations and four unknowns (if we assume the incident wave is known)
=> **we can compute the full flow analytically**

4. Numerical Set-Up

- Two-dimensional (x, z) simulations,
- nx = 256, nz = 1024, or 2048 or 2096,
- Δx = 15.6 m, Δz = 48.8 cm,
- $\nu = 5\times10^{-5}$ m² s⁻¹
- $N^2 = 10^{-4}$ s⁻¹, $S^2 = 9.8\ 10^{-7}$ s⁻¹, $f = 10^{-4}$ s⁻¹,
- Geostrophic Richardson number: $Ri_G = f^2N^2/S^4 = 1.05$
- Background PV : $f^2N^2(1-1/Ri_G)>0$,
- Waves forced in the volume (cf. Figure), minimal generation of PV
- Forcing amplitude tuned such that the incident wave has a given Richardson number when reaching the surface



- Free-slip, rigid lids on top & bottom, constant density on top & bottom, periodic in x
- **Equations solved by the code (Winters, MacKinnon & Mills 2004):**

$$\partial_t\vec{u} + f\hat{z}\times\vec{u} + (S^2/f)w\hat{y} - b\hat{z} + \vec{u}\cdot\vec{\nabla}\vec{u} + \vec{\nabla}p = \mathcal{D}\vec{u},$$
$$\partial_t b + S^2u + N^2w + \vec{u}\cdot\vec{\nabla}b = \mathcal{D}b,$$
$$\partial_x u + \partial_z w = 0.$$
$$\vec{u} + V(z)\hat{y} = (u,\ v + S^2z/f,\ w)$$
$$\mathcal{D} = \nu^z\partial_{zz} - \nu^h\partial_x^4$$

ν^h: keeps the code stable, no influence on the dynamics.

5. Energy Extraction

Methodology

Energy evolution equation:

$$\frac{\partial\langle\bar{E}\rangle}{\partial t} + \langle pw\rangle|_{z=z_{box}} + S^2\left\langle\frac{\overline{ub}}{N^2} + \frac{\overline{vw}}{f}\right\rangle - \nu\left\langle\overline{\vec{u}_h\cdot\frac{\partial^2\vec{u}_h}{\partial z^2} + \frac{b^2}{N^2}\frac{\partial^2 b}{\partial z^2}}\right\rangle = 0$$

Vertical flux

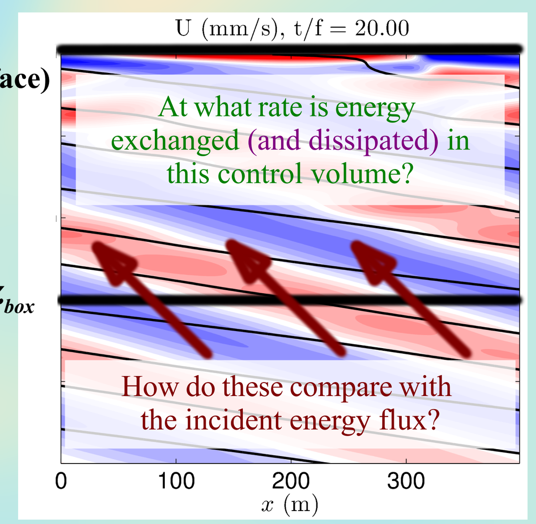
Dissipation

Exchange with front (available potential + kinetic)

$$E = \frac{1}{2}\left(|\vec{u}|^2 + b^2/N^2\right);\quad \langle\Phi\rangle = \frac{1}{L}\int_0^L \Phi\,dx;\quad \bar{\Phi}(z) = \int_0^{z_{box}}\Phi\,dz$$

$z = 0$ (surface)

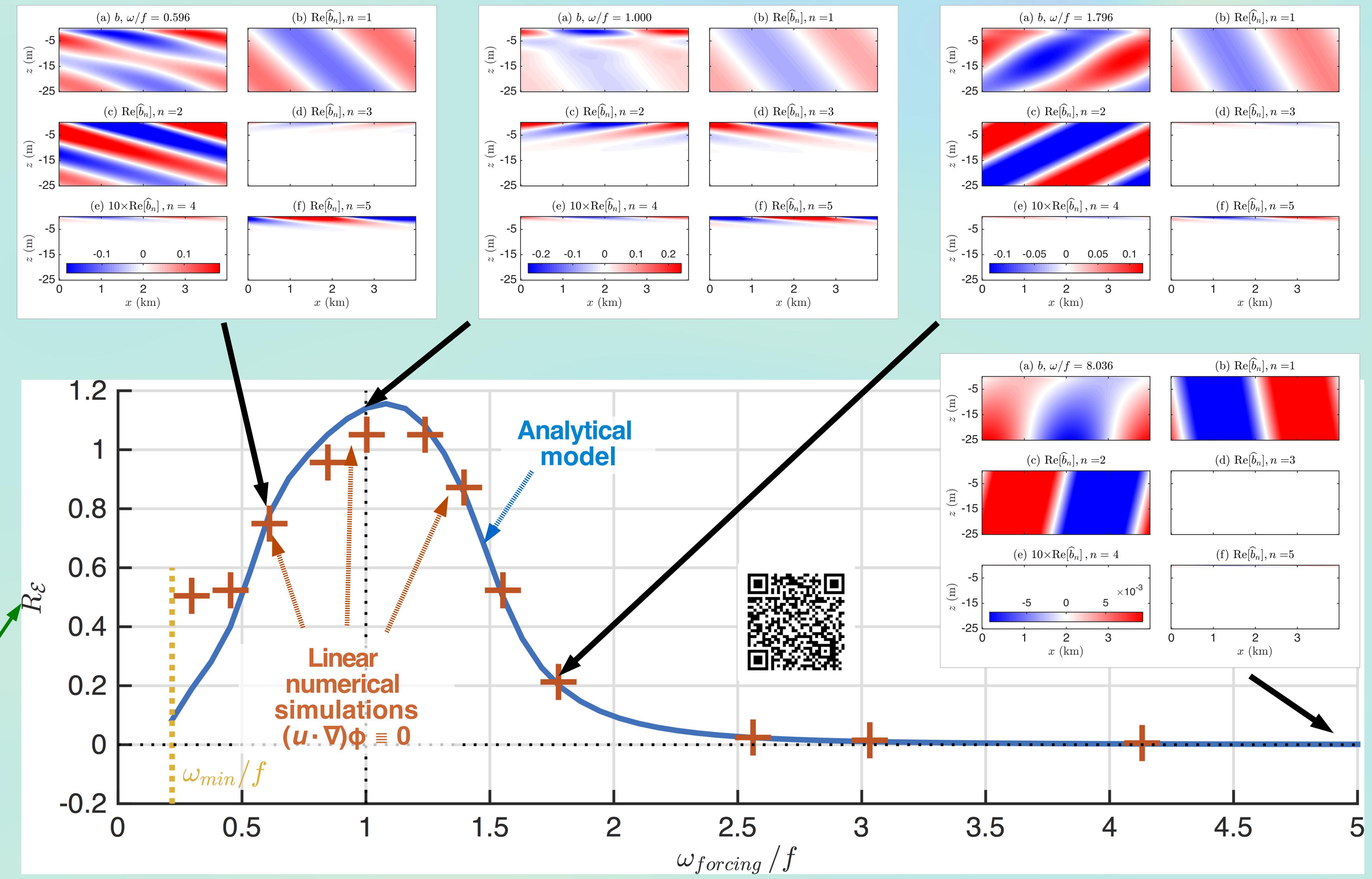
$z = z_{box}$



We compute the following two ratios:

Exchange ratio: $R_\mathcal{E} = \dfrac{S^2\langle\overline{ub}/N^2 + \overline{vw}/f\rangle}{\langle pw\rangle|_{z=z_{box}}}$

Dissipation ratio: $R_\mathcal{D} = \dfrac{\nu\langle\overline{\vec{u}\cdot\partial_z^2\vec{u} + b\partial_z^2 b/N^2}\rangle}{\langle pw\rangle|_{z=z_{box}}}$

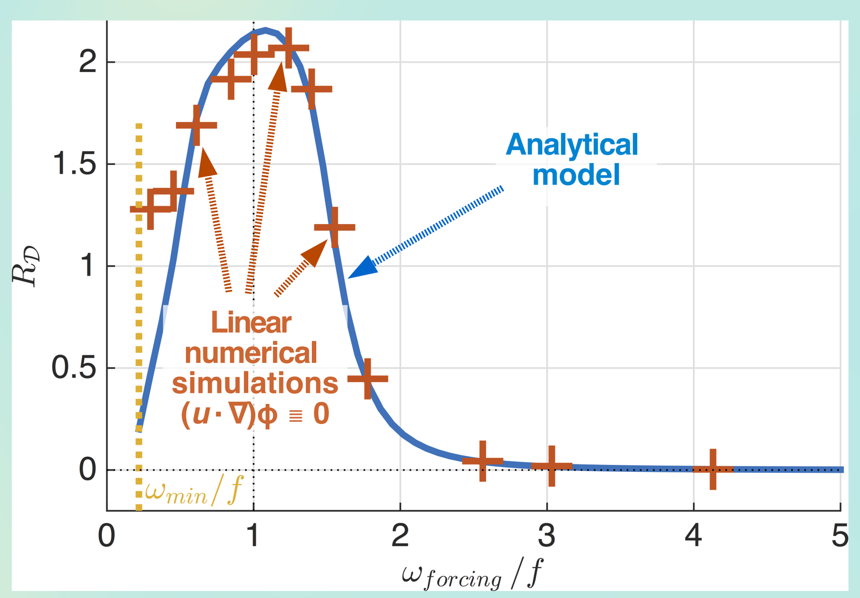


Near-inertial waves reflecting off the surface in strong fronts ($Ri_G = O(1)$) extract and dissipate a significant amount* of geostrophic energy.

* at a rate of the same order of magnitude as the wave's energy flux

Dissipation ratio

See below: the dissipation ratio is higher than one!
Here, for near-inertial waves:
(Energy dissipation) ≈ 2× (energy brought by the waves)
⇒ **Geostrophic energy reservoir is tapped, and dissipated**



If $\chi_{nq}(z) = S^2\left(\dfrac{\tilde{u}_n\tilde{b}_q^*}{N^2} + \dfrac{\tilde{v}_n\tilde{w}_q^*}{f}\right)\exp(r_n + r_q^*)z$,

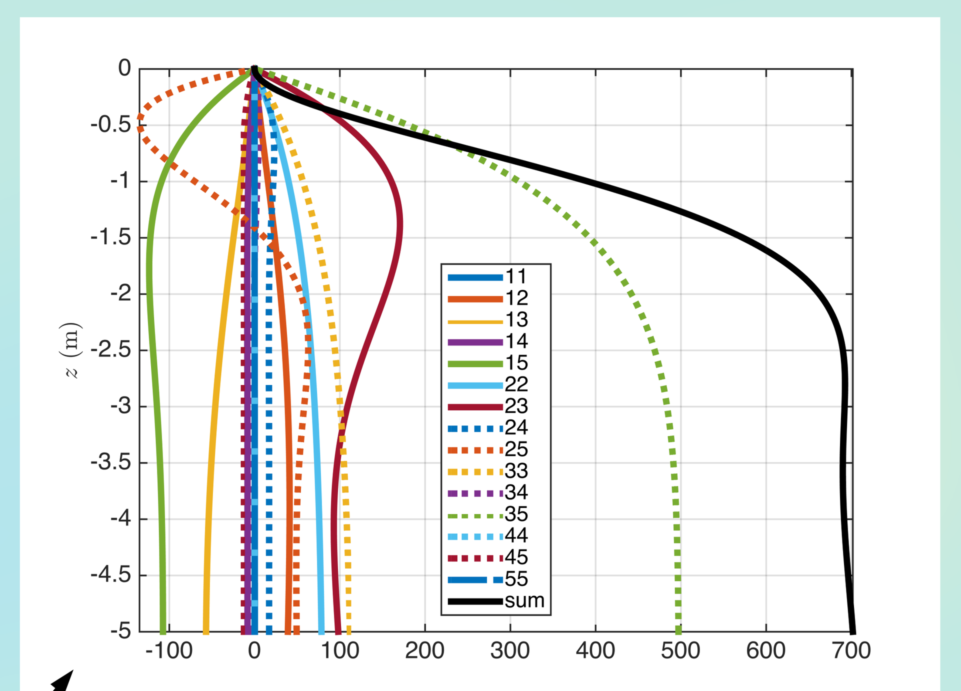
then $S^2\left\langle\dfrac{\overline{ub}}{N^2} + \dfrac{\overline{vw}}{f}\right\rangle = \dfrac{1}{2}\sum_{n=1}^{5}\sum_{q=1}^{5}\mathrm{Real}\left[\dfrac{\chi_{nq}(z) - \chi_{nq}(0)}{r_n + r_q^*}\right]$

The energy exchanges are the result of several contributions.
We break down these exchanges solution-by-solution:

$$EX_{nq}(z) = \frac{1}{2}\left[\chi_{nq}(z) + (1-\delta_{qn})\chi_{qn}(z)\right]$$

Avoids double-counts

No single contribution dominates the interactions.

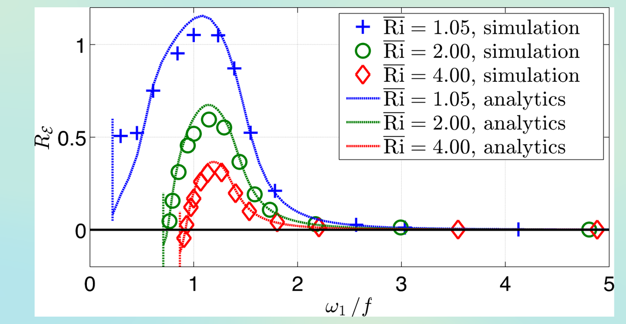


↑ *Exchange terms due to the interactions of the n^{th} and q^{th} solutions (n and q are given in the legend)*

6. How robust is this process?

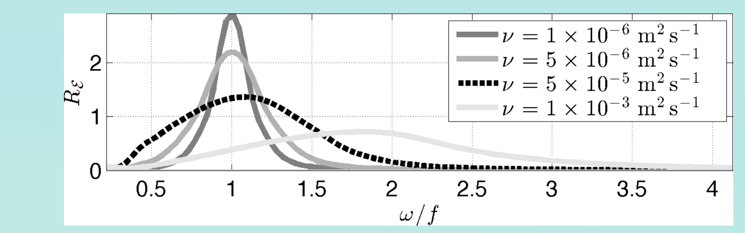
6.1 Sensitivity to Ri_G (analytical & numerical study)

Critical reflections are exacerbated for low Ri_G (strong fronts)
⇒ **There is relatively less energy extraction in weak fronts** (high Ri_G)



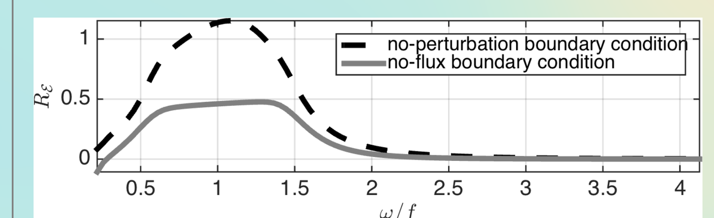
Computation of $R_\mathcal{E}$ in linear numerical simulations and analytical model for various Ri_G ↑

6.2 Sensitivity to the viscosity (analytical study)



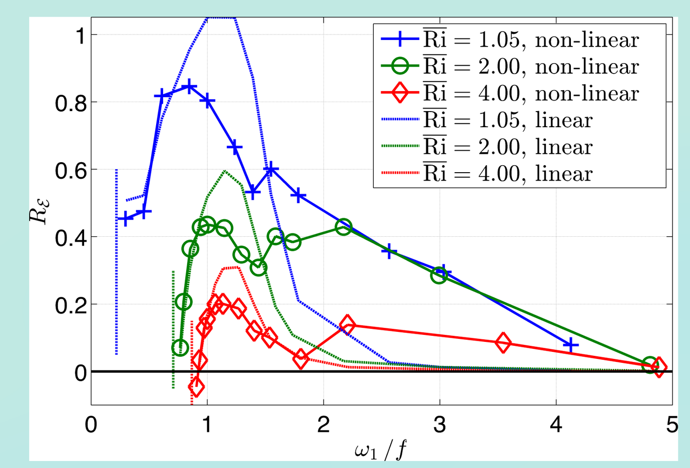
- Lower viscosity: narrower peak (viscosity is active over a narrower region), more extraction around the critical frequency.
- Higher viscosity: broader and smaller peak, extraction shifted toward higher frequencies.
- **Across a wide range of viscosities, only moderate changes in magnitude and frequency dependence of $R_\mathcal{E}$**

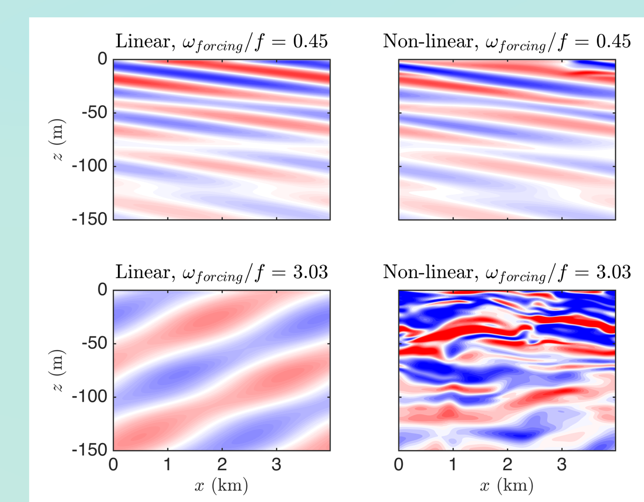
6.3 Sensitivity to the boundary condition for b (analytical study)



- In our numerical code: $b|_{surface} = 0$; somewhat unphysical (isopycnals "pinned" to the surface, meaning large buoyancy fluxes to compensate)
- **Using a no-flux B.C. ($\partial_z b|_{surface} = 0$) does not qualitatively change the picture.**

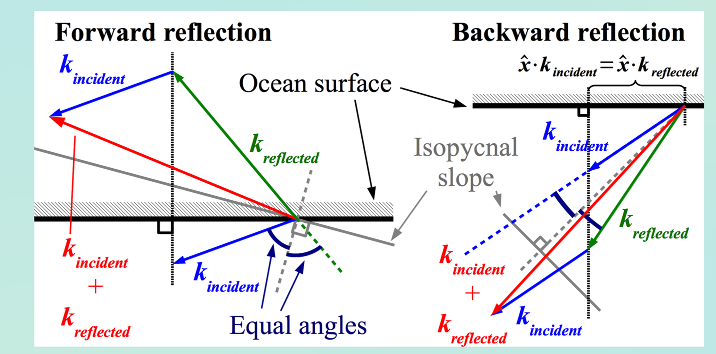
6.4 Sensitivity to non-linear effects (numerical study)





↑ *Vorticity u_z-w_x in four numerical simulations*

- Linear and non-linear backward reflections are similar.
- Linear and non-linear forward reflections are very different.
- Backward reflections do not favor triadic resonances (cf. explanation below), while forward reflections do.
- Triadic resonances trigger weak, then full, turbulent cascades.
- Higher frequencies propagate faster vertically: reduce $R_\mathcal{E}$
- Smaller scales propagate slower, pin down energy under the surface, and dissipate: increase $R_\mathcal{E}$
- Net effect: increase of $R_\mathcal{E}$ for forward reflections.



Non-linearly interacting incident and reflected waves create:
- frequencies $\omega = 2\omega_{forcing}$
- forward reflections: shallow $\vec{k}$, steep $\vec{c}_g$, resonances favored
- backward reflections: steep $\vec{k}$, shallow $\vec{c}_g$, resonances unlikely

7. Conclusions

- In fronts, inertial waves ($\omega = f$) experience critical reflections against the ocean surface.
- Linear, viscous theory predicts that eight solutions are allowed instead of two in the inviscid theory.
- In a linear reflection, the full flow can be analytically predicted thanks to the boundary conditions.
- For a strong front ($Ri_G = O(1)$), the energy extracted is of the same order as the incident wave energy flux.
- Viscous effects and the interaction of wave modes are ultimately responsible for the irreversible energy exchange.
- These statements are validated by linear numerical simulations.
- The process is weaker for higher Richardson numbers, and is only moderately sensitive to boundary conditions for the buoyancy anomaly and to viscosity.
- Non-linear effects do (not) induce qualitative changes for forward (backward) reflections.

References:
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