Local generation of internal solitary waves in an oceanic pycnocline

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Abstract

Oceanic observations have provided evidence of the generation of internal solitary waves due to an internal tidal beam impinging on a seasonal pycnocline from below, a process referred to as *local generation*. We shall present the first direct numerical simulations of such a generation process with a fully nonlinear non-hydrostatic model (MITgcm) for an idealised two-dimensional configuration. The forcing mechanism is a simple resonance so that, for a mode-n wave to be generated in the pycnocline, the horizontal phase speed of the beam has to match the phase velocity of that mode. This mode next evolves into an internal solitary wave. We shall provide examples of internal solitary waves trapped in the pycnocline as first, second and third modes. This work has been done in collaboration with Theo Gerkema (NIOZ, NL).

We shall also present two-dimensional numerical simulations of situations that are relevant for the Bay of Biscay in summer.

1 Introduction

Oceanic observations from the Bay of Biscay, Portugal seas, Mozambique Channel or Mascarene Ridge have provided evidence of the generation of internal solitary waves due to an internal tidal beam impinging on a seasonal pycnocline from below, a process referred to as *local generation* (as opposed to the more direct generation over topography). This process was proposed by New and Pingree (1990) to explain observations of internal solitary waves (ISWs) in the central Bay of Biscay. They argued that these ISWs are not directly related to topography (the continental slope), but are generated by an internal tidal beam hitting the seasonal pycnocline from below. Measurements of the beam location were consistent with ray paths computation based upon the local stratification and upon the bathymetry of the Bay of Biscay (Pingree and New, 1991). Further evidence of local generation of ISWs was presented by New and Pingree (1992). Since the beginning of the 2000s, new remote sensing techniques – namely SAR (New and da Silva, 2002; da Silva et al., 2002) and ocean colour imaging (Azevedo et al., 2006) – have confirmed the occurrence of this mechanism in spring and summer, when the seasonal pycnocline is present.

Few attempts have so far been made to explain quantitatively the underlying physical mechanism. The effect on a pycnocline of an incident plane internal gravity wave was studied experimentally and theoretically by Delisi and Orlanski (1975) for the first time. Following the work of New & Pingree in the 90's, the theoretical model of Delisi and Orlanski (1975) was considered again by Thorpe (1998). This work was further extended by Akylas et al. (2007), in the limit of long waves and when an internal wave beam (instead of a simple plane wave of infinite horizontal extent) impinges on the pycnocline. Akylas et al. (2007) addressed both the linear development of the interfacial waves and proposed a model equation for the far-field evolution involving weakly nonlinear, weakly nonhydrostatic and radiative effects which admits soliton-like solutions. Their result agreed with the finding of Gerkema (2001), namely that the initial phase is essentially linear, nonlinear (and nonhydrostatic) effects becoming important in the second phase, when the perturbation of the pycnocline propagates away, steepens and may break up into ISWs. Gerkema (2001) solved the linear problem analytically in terms of vertical modes. All parameters were representative of the oceanic configuration of the Bay of Biscay and were fixed, except for the density jump across the interface, which was varied. Gerkema (2001) concluded that the density jump across the pycnocline has to be moderate for the displacement amplitude of the interface to be maximum. If the density jump is stronger, the beam is reflected at the interface; if weaker, the beam is transmitted across the discontinuity as an evanescent wave.

All these studies are limited in that they address at best weakly nonlinear and weakly nonhydrostatic effects and treat the pycnocline as an interface, *i.e.* a layer of infinitesimal thickness (with the exception of Maugé and Gerkema (2008) who used stratification profiles obtained from measurements in the Bay of Biscay). In the present paper, we present numerical results obtained with the MIT general circulation model; we are thus able to relax all these constraints at once: we allow a finite thickness of the pycnocline as well as fully nonlinear and nonhydrostatic effects. Our aim is to investigate when ISWs can be obtained in this more general situation, what the influence is of the finite thickness of the pycnocline upon the wave structure and whether the criteria derived by Delisi and Orlanski (1975), Gerkema (2001) and Akylas et al. (2007) for maximum pycnocline displacement still hold. As a second step, we consider a configuration guided by measurements in the Bay of Biscay to investigate the occurence of ISWs in a realistic context.

This work has been published in two recent papers, which we summarize below.

2 Generation of internal solitary waves in a pycnocline by an internal wave beam: a numerical study

In Grisouard et al. (2011), we addressed the local generation of ISWs, when an internal wave beam impinges on a pycnocline. As just mentioned, nonlinear, nonhydrostatic numerical experiments were conducted for this purpose, the vertical density profile being continuous and made up of a lower layer with constant value N_0 of the Brunt-Väisälä frequency, a pycnocline of finite thickness and a thin homogeneous upper layer.

We showed that ISWs can be generated and that the finite thickness of the pycnocline allows different vertical modes to be excited, depending on the parameters. We ran numerical experiments showing that mode-1, -2 or -3 internal solitary waves can develop (Figure 1). We next proposed two different approaches to predict the conditions of occurrence of a mode-n pycnocline wave, which depend fundamentally upon the finite thickness of the pycnocline.

In the "far-field" approach, that is when harmonics of the forcing frequency have appeared, we showed that the observed mode-n weakly nonlinear pycnocline waves have the same horizontal phase speed as the incident wave beam. We demonstrated this result heuristically by using a modal decomposition based upon the actual density profile and a frequency of value higher than N_0. For a mode-1 wave, this result extends to a pycnocline of finite thickness the linear criterion found for Akylas et al. (2007). This result allowed us to design numerical experiments so that a mode-1, -2 or -3 pycnocline wave develops, the density jump across the pycnocline increasing with the mode number (or equivalently, the horizontal wavelength of the wave beam decreasing with the mode number).

One might wonder why the phase speed of a beam should be matched with the phase speed of trapped waves, that are physically some distance away from the beam. We observed in Section 3.2 that the evolution towards ISWs actually takes one or two wavelengths only, a much shorter distance than for weakly nonlinear waves described *e.g.* by a KdV model. This is very

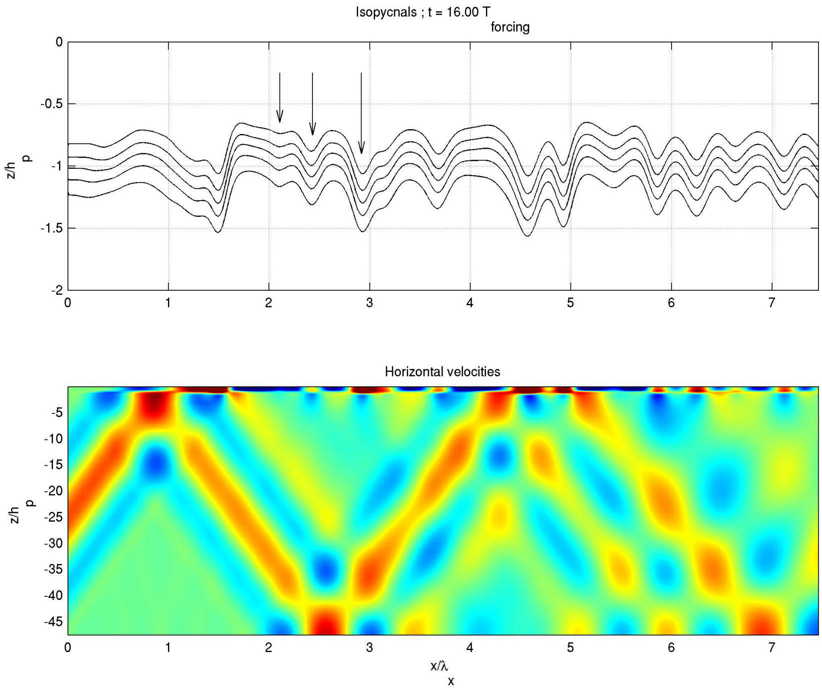

Figure 1: Generation of a mode-1 ISW by an internal wave beam. (a): Magnification of the isopycnals around the center line of the pycnocline, with a train of 3 deformations explicitly indicated with arrows (the leading deformation has almost passed by the rear deformation of the preceeding train). (b): Horizontal velocity field allowing to locate the internal wave beam; values range from -7.5 mm.s^{-1} (dark blue) to 7.5 mm.s^{-1} (red).

likely due to the large amplitude of the forcing which allows the development of nonlinear structures in the vicinity of the beam impact, that propagate with the same speed as the forcing phase speed.

The "near-field" approach consists of deriving simple geometrical conditions ensuring that a mode-n wave develops at the beam impact (the model holds for $n \geq 2$ only). In this approach, the dynamics is linear, a simple plane wave is considered in place of a wave beam and a piecewise three-layer stratification is assumed. We showed that a mode-n wave is forced if a simple relation is satisfied which involves the thickness of the pycnocline, the angle of the refracted internal wave in the pycnocline and the horizontal wavelength of this wave. This model was originally designed as a qualitative argument but in spite of its great simplicity, it also gave a good quantitative agreement with our numerical data for $n \geq 2$. It explains why a beam with a short wavelength generates high mode waves or, equivalently, why such high mode waves are favoured by a strongly stratified pycnocline. When related to the conclusion of the far-field approach, this model provides a simple analytical expression for the phase speed of a mode-n ISW.

We discussed the more general situation in which the wave beam has an intermediate phase speed between those of modes n and $n+1$. With the help of an example, we conjectured that both modes could be selected in this case and that the lower the mode-number is, the larger its bandwidth is. The computation of the amplitude of those modes would be required to answer precisely this problem. This requires to solve the nonlinear nonhydrostatic Boussinesq equations forced by the wave beam, a rather involved task.

3 Numerical simulations of the local generation of internal solitary waves in the Bay of Biscay

In Grisouard and Staquet (2010), we presented two-dimensional numerical experiments using parameters representative of the Bay of Biscay when local generation is observed. Two first experiments addressed the problem of an internal tide beam impinging on a pycnocline initially at rest. The first experiment E1, using a mid to late summer stratification that is typical of the Bay of Biscay when local generation is observed and a realistic internal tide beam did not lead to a convincing generation of ISWs, disagreeing with the observations. The second experiment E2 used a slightly reduced density jump across the seasonal pycnocline similar to that observed in late spring. In this case mode-1 ISWs were generated. The results of E1 and E2 agreed with the predictions of the far-field condition derived in GSG10. Nonetheless, the distance between two trains of ISWs in E2 displays significant discrepancies with the observations. In the Bay of Biscay, this distance is about 50 km, which is also the dominant wavelength of the internal tide in the upper 100 m (referred to as the upper tide), regardless of the characteristics of the beam. In E2 by contrast, the ISW trains were separated by 25 to 30 km, close enough to the horizontal wavelength of the forcing beam for it to generate mode-1 ISWs.

A possible explanation for this discrepancy relies upon the interaction of the beam with the upper tide generated at the shelf break and propagating throughout the domain. A third numerical experiment E3 was thus designed, forced by the internal tide generated by a barotropic flow on the continental slope. The stratification was the same as in E1 and as in the Bay of Biscay when local generation occurs. This experiment showed that mode-1 ISWs are generated, each train being now separated by about 55 km, in agreement with observations. To investigate the role of the upper tide in the generation process of these ISWs, a fourth and last experiment E4 was designed, which forcing included the upper tide but not the beam. No ISWs appeared but the interfacial displacement could be superposed on the carrier wave of the ISWs observed in the third experiment.

To sum up, E1 and E2 showed that an internal tide beam impinging on a seasonal pycnocline can generate ISWs but that the mechanism is very sensitive to the amplitude of the density jump across the seasonal pycnocline, in agreement with GSG10 predictions, as well as the predictions of previous works (Gerkema, 2001; Akylas et al., 2007). E2 also showed that mode-1 ISWs are generated but with quantitatively different features than those of the ISWs observed in the Bay of Biscay. On the other hand, the upper tide alone does not evolve into trains of ISWs. Only when an internal tide beam interacts with the upper tide is the formation of ISWs observed, with physical characteristics close to the observations. The propagation speeds and distance between trains of ISWs are now controlled by the upper tide but the existence of these ISWs is most likely due to an energy input from the inernal tide beam in the pycnocline.

Conjectures on the mechanism of how energy is transferred to the pycnocline in the presence of the upper tide can be proposed. For instance, the upper tide can be modelled as an oscillating horizontal shear flow. As this oscillating motion is synchronous with those induced in the pycnocline by the beam, the phase shift between the beam and the upper tide may be such that the phase speed of mode-1 ISW would be reduced by Doppler effect at the location of the impact of the beam and hence brought closer to the beam phase speed. If it is brought close enough, the selection condition discussed in Grisouard and Staquet (2010) would predict that mode-1 ISWs are generated. Further analytical work is required to address the validity of this conjecture.

We are quite aware that several effects have been neglected. First, our numerical experiments are two-dimensional. Also, Gerkema et al. (2008) showed that the internal tide beam

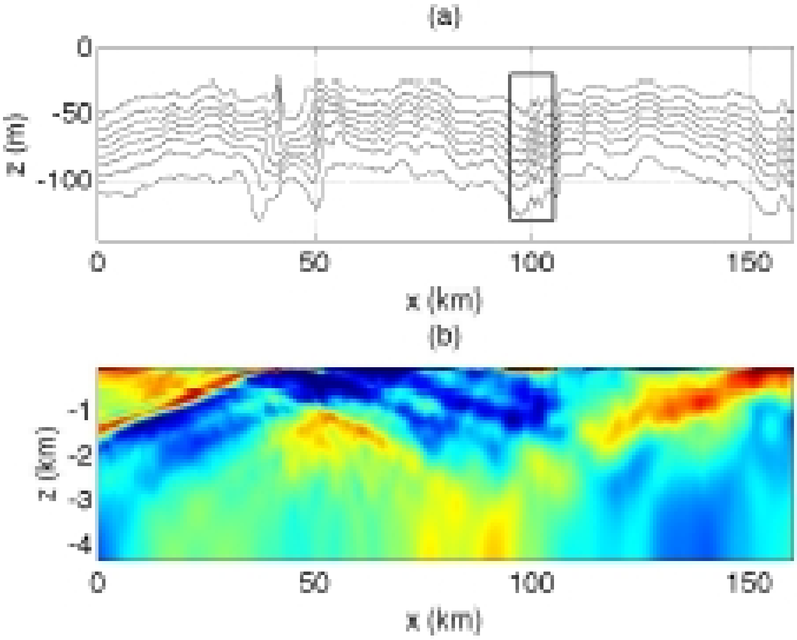

Figure 2: Snapshot at 16 periods of the barotropic tide after the start of the simulation. (a): Contours of a few isopycnals around the pycnocline. A mode-2 ISW is framed. (b): Horizontal velocity field in the domain, the colour scale ranging from -12 (blue) to 12 cm/s (red).

in the Bay of Biscay is very sensitive to non-traditional effects. Including these effects would modify the initialisation of experiments E3 and E4. Also, the relative amplitudes of the beam and of the upper tide might be different than those set in E3. Performing three-dimensional realistic non-hydrostatic (and nonlinear) numerical experiments of the Bay of Biscay, still a computational challenge nowadays, appears to be necessary to assess the respective roles of the internal tide beam and the upper tide in the generation of ISWs and in the control of their spatial structure.

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